CHIJ ST. THERESA'S CONVENT
PRELIMINARY EXAMINATION 2024
SECONDARY 4 EXPRESS / 5 NORMAL (ACADEMIC)

CANDIDATE NAME			
CLASS		INDEX NUMBER	

ADDITIONAL MATHEMATICS **4049/2**

Paper 2

23 Aug 2024
2 hours 15 minutes

Candidates answer on the Question Paper as well as on the graph paper provided.

READ THESE INSTRUCTIONS FIRST

Write your name, class and index number in the spaces at the top of this page.
Write in dark blue or black pen.
You may use a pencil for any diagrams or graphs.
Do not use staples, paper clips, glue or correction fluid.

Answer **all** the questions.
Give non-exact numerical answers correct to 3 significant figures, or 1 decimal place in the case of angles in degrees, unless a different level of accuracy is specified in the question.
The use of an approved scientific calculator is expected, where appropriate.

The number of marks is given in brackets [ ] at the end of each question or part question.

The total number of marks for this paper is 90.

Mathematical Formulae

1. ALGEBRA

Quadratic Equation

For the equation $ax^2 + bx + c = 0$,

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

Binomial expansion

$$(a+b)^n = a^n + \binom{n}{1}a^{n-1}b + \binom{n}{2}a^{n-2}b^2 + \ldots + \binom{n}{r}a^{n-r}b^r + \ldots + b^n,$$

where n is a positive integer and $\binom{n}{r} = \frac{n!}{(n-r)!r!} = \frac{n(n-1)...(n-r+1)}{r!}$.

2. TRIGONOMETRY

Identities

$$\sin^2 A + \cos^2 A = 1$$

$$\sec^2 A = 1 + \tan^2 A$$

$$\operatorname{cosec}^2 A = 1 + \cot^2 A$$

$$\sin(A \pm B) = \sin A\cos B \pm \cos A\sin B$$

$$\cos(A \pm B) = \cos A\cos B \mp \sin A\sin B$$

$$\tan(A \pm B) = \frac{\tan A \pm \tan B}{1 \mp \tan A\tan B}$$

$$\sin 2A = 2\sin A\cos A$$

$$\cos 2A = \cos^2 A - \sin^2 A = 2\cos^2 A - 1 = 1 - 2\sin^2 A$$

$$\tan 2A = \frac{2\tan A}{1 - \tan^2 A}$$

Formulae for ΔABC

$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$

$$a^2 = b^2 + c^2 - 2bc\cos A$$

$$\Delta = \frac{1}{2}ab\sin C$$

1 The equation of a curve is $y = 5\sin^2\left(x - \frac{\pi}{6}\right)$, where $0 \le x \le \frac{\pi}{2}$.

(a) Given that y is decreasing at a rate of 0.3 units per second, find the rate of change of x at $x = \frac{5\pi}{12}$. [3]

(b) The normal to the curve at $x = \frac{5\pi}{12}$ intersects the vertical axis at $(0, k)$.
Find the exact value of k. [3]

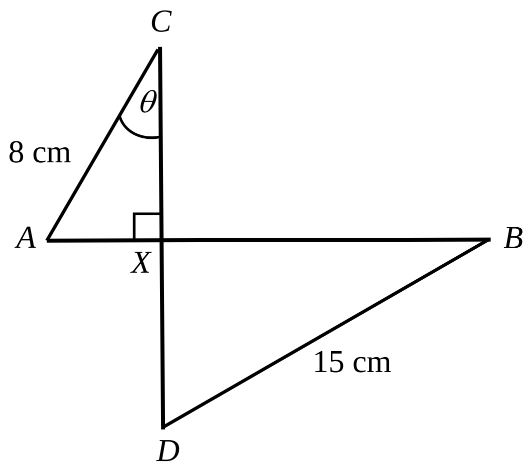
2 **(a)** Find the values of x and y which satisfy the equations

$$8^x - 2^{-y} = 0,$$
$$\left(\sqrt{125^x}\right)^y = \frac{1}{\sqrt{5}}.$$

[3]

(b) Show that the equation $3\left(2^{x+2}\right)-1=35\left(2^{-x}\right)$ has only one solution and find its value correct to 2 significant figures. [5]

3



The diagram shows two perpendicular lines *AB* and *CD* which intersect at *X*. The points *A*, *B*, *C* and *D* lie on the circumference of a circle. *AC* = 8 cm, *BD* = 15 cm, and angle *ACD* equals to $\theta°$.

(a) Show that the length of *AB* is $8\sin\theta + 15\cos\theta$. [2]

(b) Express *AB* in the form $R\sin(\theta + \alpha)$, where *R* > 0 and $0° \le \alpha \le 90°$. [4]

(c) Find the value(s) of θ if AB = 16 cm. [3]

4 A calculator must not be used in this question.

It is given that $\dfrac{\cos(A+B)}{\cos(A-B)} = \dfrac{1}{3}$.

(a) Show that $\tan A \tan B = \dfrac{1}{2}$. [3]

(b) If $\tan A = 2+\sqrt{3}$, find an expression for $\tan B$, in the form $a+b\sqrt{3}$, where a and b are constants. [3]

(c) Hence, express $\sec^2 B$ in the form $c + d\sqrt{3}$, where c and d are constants. [3]

5 The equation of a circle is $x^2 + y^2 - 6x + 16y + 48 = 0$.

(a) Find the radius and coordinates of the centre of the circle. [4]

(b) The point $A(0, -4)$ lies on the circle.
Given that *AB* is a diameter of the circle, find the coordinates of *B*. [2]

(c) A line with equation $y = mx$, where $m > 0$, does not intersect the circle. A is the point on the circle closest to the line.

(i) Find the value of m. [2]

(ii) Hence, find the coordinates of the point on the line that is closest to the circle. [2]

6 It is given that $f(x) = 4x^p + qx^2 - 3x + 1$, where p and q are constants, has a factor of $x - 1$ and leaves a remainder of -33 when divided by $x + 2$.

(a) Find the values of p and q. [3]

(b) Using the values of p and q found in part **(a)**, solve the equation $f(x) = 0$ completely, leaving non-integer roots in their simplest surd form. [4]

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7 **(a)** Prove the identity $\frac{1-2\cos^2\theta}{\sin\theta\cos\theta}=-2\cot 2\theta$. [3]

(b) Hence, solve the equation $\frac{1-2\cos^2\theta}{\sin\theta\cos\theta}+\tan 2\theta+1=0$, for $0°\le\theta\le 90°$. [5]

8 The blood alcohol concentration, C mg/L, in a person t minutes after he consumes a bottle of wine can be modelled by the formula

$$C = 1250\left(e^{kt} - e^{-0.1t} \right).$$

(a) In Singapore, a driver can be charged with drink driving if he drives when his blood alcohol concentration exceeds 800mg/L. When Jonathan consumes a bottle of wine, his blood alcohol concentration will only fall to 800mg/L after three hours.

Show that $k = -0.0025$ when corrected to 2 significant figures, and find his blood alcohol concentration after 1 hour. [4]

Using $k = -0.0025$,

(b) Find the rate of change of Jonathan's blood alcohol concentration after 1 hour. [2]

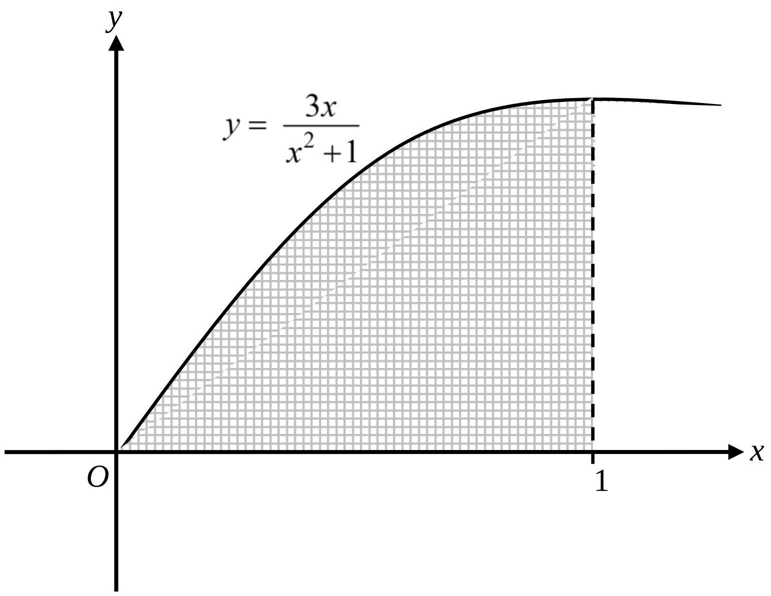
(c) After consumption of alcohol, the blood alcohol concentration will rise to a peak before decreasing slowly over time. Explain why the blood alcohol concentration found in part **(a)** is not the peak level, and find the peak blood alcohol concentration level. [5]

9 **(a)** The equation of a curve is $y = \ln\sqrt{\dfrac{x^2+1}{2x+1}}$.

Show that $\dfrac{\mathrm{d}y}{\mathrm{d}x} = \dfrac{x}{x^2+1} - \dfrac{1}{2x+1}$. [4]

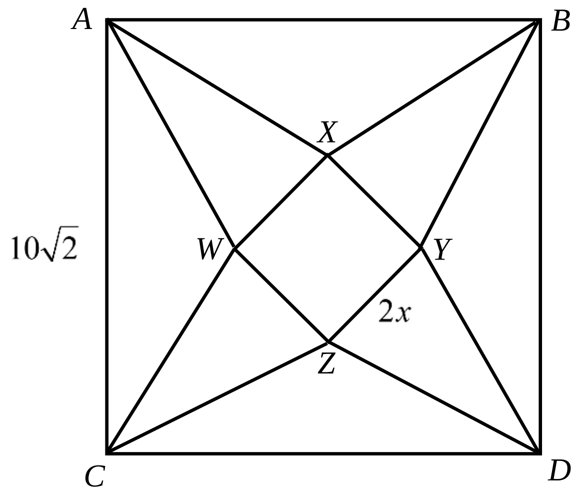
(b) The diagram below shows part of the graph of $y = \dfrac{3x}{x^2+1}$.

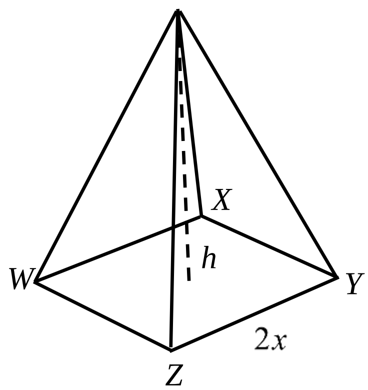
Using the result from part **(a)**, find the area of the shaded region bounded by the curve, the x- axis and the line $x = 1$. Express your answer in the form $a\ln b$, where a and b are constants. [6]



Continuation of working space for Question 9.

10 In the diagram below, $ABCD$ is a square paper with side $10\sqrt{2}$ cm. The net of a regular pyramid with square base $WXYZ$ was cut from the paper. AB is parallel to WY and the base of the pyramid has sides $2x$ cm.





(a) By expressing the perpendicular height, h cm, of the pyramid in terms of x, show that $V = \frac{8}{3}x^2\sqrt{25-5x}$. [4]

(b) Given that x can vary, find the value of x for which the volume of the pyramid is stationary. [6]

(c) Determine whether this value of x gives a maximum or minimum value for the volume of the pyramid. [2]

~ ~ ~ **End of Paper** ~ ~ ~