GCE ‘N’ AND ‘O’ LEVEL ELEMENTARY MATHEMATICS NOTES

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Notes to All Students

This book is written according to the syllabus of GCE O Level Elementary Mathematics (Subject code: 4016 and 4048) for Year 2016 taken from SEAB website. The purpose in writing this book is to help students break down Mathematics into simpler parts and to achieve a better understanding of the individual concepts. By studying this book solely, you are not guaranteed to get A1 in the O levels. Intensive practise is advised.

Notes written in red emphasises the common mistakes committed by students in the exams. Do make sure that you understand everything that is written. Students are advised to write their units clearly in all workings and final answer. Failure to do so can lead to a loss of marks. Students are also advised to write their workings and statements clearly. Writing the statements alone does not earn you marks but by showing clear statements and workings, it reflects your understanding towards the question. In the event that you did not get the final answer correct, it will be easier for the examiner to allocate marks through the workings.

Even though some of the examples used are adapted from 'N' level ten year series, students taking 'O' levels are still advised to read through them thoroughly and make sure that every concept is understood clearly as similar questions may appear in the O levels paper. Being complacent will lead to nothing but failure. If possible, cover the examples with a paper and attempt it first before looking at the answers.

Students are recommended to contact me at 9232 2940 as soon as possible should they spot any errors during the process of revisions.

Good luck to all of you taking the O and N levels ☺

Lai Zhi Yin

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N1. Numbers and Four Operations

N1.1 Basic Definitions of Mathematical Terms

- ☺ Prime number: Number that can only be divided by itself and 1. One is not a prime number. Two is the only even prime number. Example: 2, 3, 5, 7, 11 …
- ☺ Composite Number: Number that have more than one factor (opposite of prime number). Example: 6, 8, 9, 10, 12, 14…
- ☺ Integer: Number that is a whole number. An integer can be either positive or negative. Example: -2, -1, 1, 2, 3…
- ☺ Rational number: Number that can be expressed as a fraction with the denominator not equal to zero. Recurring decimals are rational numbers. Example: $\sqrt{4}$, $\frac{1}{2}$, $3.\dot{5}6$
- ☺ Irrational number: Number that cannot be expressed as a fraction. Example: π, $\sqrt{3}$
- ☺ Real number: Includes all rational number, irrational number, whole number and fraction
- ☺ Natural Number: Positive integer not including 0. Example: 1, 2, 3, 4, 5…
- ☺ Perfect square: A number that can be square rooted to give a whole number. Example: 1, 4, 9, 16, 25 …
- ☺ Perfect cube: A number that can be cube rooted to give a whole number. Example: 1, 8, 27, 64, 125…
- ☺ Consecutive number: Numbers that follow each other in order. Example: 3, 4, 5… Therefore, if the smallest consecutive number is x, the next integer will be $x + 1$
- ☺ Consecutive odd number: Odd numbers that follow each other in order Example: 1, 3, 5… Therefore, if the smallest consecutive odd number is x, the next integer will be $x + 2$, $x + 4$…
- ☺ Consecutive even numbers: Even numbers that follow each other in order Example: 2, 4, 6… Therefore, if the smallest consecutive odd number is x, the next integer will be $x + 2$, $x + 4$…
- ☺ Negative number: Real number that is less than 0. The bigger the negative number, the smaller the value i.e. -3 is smaller than -2. Example: -1, -2, -3, -4 …

Note: Students usually refer π to $\frac{22}{7}$, thereby inferring that π is a rational number. However, $\frac{22}{7}$ is just a estimate of π used in primary schools. It is important to note that π is an irrational number. The accurate form of π is found in the calculator.

N1.2 Number Line

Number becomes bigger when moving from left to right of the number line. Number becomes smaller when moving from right to left of the number line.

Example: N Level 2013 Paper 1 Question 5

The values of p, q, r and s are $\frac{8}{5}$, 0.6, $\frac{5}{8}$, $\frac{\pi}{2}$. Find p, q, r and s.

$\frac{8}{5} = 1.6$
$\frac{5}{8} = 0.625$
$\frac{\pi}{2} = 1.571$ (4 s.f.)

Note: Always convert fractions to decimals first and answer in its original form

Therefore, p = 0.6, q = $\frac{5}{8}$, r = $\frac{\pi}{2}$ and s = $\frac{8}{5}$

N1.3 Basic Mathematical Symbols

$<$: Less than; does not include the number stated
Example: $x < 6$ means 5, 4, 3, 2, 1 …

$>$: More than; does not include the number stated
Example: $x > 6$ means 7, 8, 9, 10, 11 …

$\leq$: Less than or equal to; include the number stated
Example: $x \leq 6$ means 6, 5, 4, 3, 2, 1 …

$\geq$: More than or equal to; include the number stated
Example: $x \geq 6$ means 6, 7, 8, 9, 10 …

Example: N Level Paper 2 Question 6 (a)

List all the solutions to $-4 \leq 2x < 8$

$-4 \leq 2x < 8$
$-2 \leq x < 4$

Therefore, the solutions are -2, -1, 0, 1, 2 and 3.

N1.4 Expressing Recurring Decimals as Fraction

Recurring decimals are rational numbers where it can be converted into fractions.

Example: Express $3.\dot{4}\dot{2}$ as fraction.

$3.\dot{4}\dot{2} = 3.424242424$
$x = 3.424242424$
$100x = 342.42424$
$99x = 342.42424 - 3.42424$
$99x = 339$
$x = \frac{339}{99}$

N1.5 Basic Calculator Skills (CASIO fx-96SG PLUS)

Solving Quadratic Equation:

Example: Solve $2x^2 + 5x - 3$
Step 1: Press: Mode > 3 > 3
Step 2: Press: a = 2, b = 5, c = -3
Step 3: Calculator will show $x_1 = \frac{1}{2}$, $x_2 = -3$
Step 4: Answer for the above example will be $x = \frac{1}{2}$ or $x = -3$
Note: You have to show your working i.e. completing the square, factorisation or general formula if the question is worth more than 2 marks.

Factorising Quadratic Equation:

Example: Factorise $3x^2 + 10x - 8$
Step 1: Press: Mode > 3 > 3
Step 2: Press: a = 3, b = 10, c = -8
Step 3: Calculator will show $x_1 = \frac{2}{3}$, $x_2 = -4$
Step 4: Answer for the above example will be $(3x - 2)(x + 4)$

Solving Simultaneous Equations Involving 2 Variables:

Example: $3x + 5y = 165$
$2x - 5y = 10$

Step 1: Press: Mode > 3 > 1
Step 2: Key in the following order:

	a	b	c
1	3	5	165
2	2	-5	10

Step 3: Calculator will show $x = 35$, $y = 12$
Step 4: Answer for the example will be $x = 35$, $y = 12$
Note: When using this method, always make sure that x and y are aligned in the same column.

Solving Time Related Problems:

Example: The time is 08 45. Find the time after 4 hours and 36 minutes.

Step 1: Press: 8 $\boxed{\circ\,'\,''}$ 45 $\boxed{\circ\,'\,''}$ + 4 $\boxed{\circ\,'\,''}$ 36 $\boxed{\circ\,'\,''}$
Step 2: Calculator will show: 13 $\boxed{\circ\,'\,''}$ 21 $\boxed{\circ\,'\,''}$

Answer for the above question is 13 21 (24 hours format) or 1.21 pm (12 hours format)

Solving Simultaneous Equation Involving 3 Variables:

Example: a + b + c = 6
4a + 2b + c = 15
9a + 3b + c = 28

Step 1: Press: Mode > 3 > 2
Step 2: Key in the following order:

	a	b	c	d
1	1	1	1	6
2	4	2	1	15
3	9	3	1	28

Step 3: Calculator will show x = 2, y = 3, z = 1
Step 4: Answer for the example will be a = 2, b = 3, c = 1

Calculating Mean and Standard Deviation

Example: O Level 2014 Paper 2 Question 10 (a)
Calculate the estimated mean and standard deviation of the times taken by 100 males to complete a 10 km race.

Time (t min)	$30 \le t < 40$	$40 \le t < 50$	$50 \le t < 60$	$60 \le t < 70$	$70 \le t < 80$
Frequency	15	32	30	16	7

To calculate mean, the estimated mean has to be derived first before the data can be input into the calculator.

Estimated Time (t min)	35	45	55	65	75
Frequency	15	32	30	16	7

Step 1: Before continuing, ensure that your calculator allows you to key in frequency by following the subsequent steps. DO NOT CONTINUE if this step is not done: Shift > Mode > Arrow Down > 4 (STAT) > 1 (ON)
Step 2: Press Mode > 2 (STAT) > 1 (1-VAR)
Step 3: Key in the following order:

x	Y
35	15
45	32
55	30
65	16
75	7

Step 4: Press (ON)
Step 5: To calculate mean, press Shift > 1 (STAT)> 4 (VAR)> 2 ($\bar{x}$) > =
Step 6: Calculator will show 51.8, which is the answer for mean.
Step 7: For standard deviation, press Shift > 1 (STAT) > 4 (VAR) > 3 (σx) > =
Step 8: Calculator will show 11.21427662 which is the answer for standard deviation. Round off the answer to 3 s.f.

N1.6 Approximation and Estimation

When rounding off, do note that the zeros in front is not significant. However, the zeros in the middle or at the end is significant.

Example: Round off 0.0948039 to:

- ☺ 1 significant figure: 0.09 (1 s.f.)
- ☺ 1 decimal place: 0.1 (1 d.p.)
- ☺ 2 significant figures: 0.095 (2 s.f.)
- ☺ 2 decimal places: 0.09 (2 d.p.)
- ☺ 3 significant figures: 0.0948 (3 s.f.)
- ☺ 3 decimal places: 0.095 (3 d.p.)
- ☺ 4 significant figures: 0.09480 (4 s.f.) Note: there must be a 0 at the end
- ☺ 4 decimal places: 0.0948 (4 d.p.)

Format required in the exams:

- ☺ Workings: At least 4 significant figures. Preferably 5 significant figures and above.
- ☺ Final Answer (except angles): 3 significant figures (3 s.f.)
- ☺ Angles/ Bearings: 1 decimal place (1 d.p.)
- ☺ Money: round off to the nearest cents (2 d.p.)
  Note: Many students tend to round off to the nearest 10 cents (1 decimal place) instead of nearest cents (2 decimal places) when question ask to round off answer to the nearest cents.
- ☺ Other format: Nearest kg, nearest dollar, nearest cubic centimetres etc. means correct to the nearest whole number

Example: N Level 2014 Paper 1 Question 3

By writing each number correct to 1 significant figure, estimate the value of $\frac{28.19 \times 623.1}{0.3285}$. You must show your working.

$$\frac{28.19 \times 623.1}{0.3285} \approx \frac{30 \times 600}{0.3}$$
$$= 60\,000 \text{ (1 s.f)}$$

N1.7 Finding Highest Common Factor (HCF) and Lowest Common Factor (LCM)

Example: Determine the HCF and LCM of 150 and 126

Step 1: Do prime factorisation of 150 and 126 and express them in the product of prime factors

2	150
3	75
5	25
5	5
	1

$150 = 2 \times 3 \times 5^2$

2	126
3	63
3	21
7	7
	1

$126 = 2 \times 3^2 \times 7$

Step 2: For HCF, take lower power of common factors; do not include uncommon factors

$150 = 2 \times 3 \times 5^2$
$126 = 2 \times 3^2 \times 7$
$HCF = 2 \times 3$
$= 6$

Note: If question did not state leave in index notation or product of prime factors, evaluate your answer!

For LCM, take higher power of common factors; include uncommon factors

$150 = 2 \times 3 \times 5^2$
$126 = 2 \times 3^2 \times 7$
$LCM = 2 \times 3^2 \times 5^2 \times 7$
$= 3150$

Finding smallest integer, k, that makes 150k the square of a number

$150 = 2 \times 3 \times 5^2$
$k = 2 \times 3$

Note: Ensure that powers of the factors are to the multiples of 2

Finding smallest integer, m, that makes 126 the cube of a number

$126 = 2 \times 3^2 \times 7$
$m = 2^2 \times 3 \times 7^2$

Note: Ensure that powers of the factors are to the multiples of 3

N1.8 Determining Square Root of Number by Prime Factorisation

Example: Determine $\sqrt{324}$ by prime factorisation

Step 1: Do prime factorisation and leave the answer in index notation.

2	324
2	162
3	81
3	27
3	9
3	3
	1

$324 = 2^2 \times 3^4$

Step 2: Divide powers of the factors by 2.

$\sqrt{324} = 2 \times 3^2$

$= 18$

N1.9 Determining Cube Root of Number by Prime Factorisation

Example: Determine $\sqrt[3]{216}$ by prime factorisation

Step 1: Do prime factorisation and leave the answer in index notation.

2	216
2	108
2	54
3	27
3	9
3	3
	1

$216 = 2^3 \times 3^3$

Step 2: Divide powers of the factors by 3.

$\sqrt[3]{216} = 2 \times 3$

$= 6$

N1.10 Word Problems Involving HCF and LCM

If the word 'greatest number' appear in the question, it is most likely to be a HCF question. If the word 'together' or 'least number' appear in the question, it is most likely to be a LCM question.

Example: O level 2008 Paper 1 Question 17 (b)

The lights of three lightships flash at regular intervals. The first light flashes every 12 seconds, the second every 27 seconds and the third every 90 seconds. The three lights flash together at 0900. At what time do they next flash together?

Step 1: Express all numbers in index notation.

$12 = 2^2 \times 3$
$27 = 3^3$
$90 = 2 \times 3^2 \times 5$

$\text{LCM} = 2^2 \times 3^3 \times 5$
$= 540$

Step 2: Since question ask for the time that the lights will flash together, LCM is determined.

Time taken = 540 seconds
= 9 min

Time taken for the three lights to flash together = 0900 + 9 min
= 09 09

Example: There are 300 blue beads, 156 yellow beads and 126 red beads. Given that the beads are distributed equally among the girls with no left over. What is the greatest possible number of girls in the group?

Step 1: Express all numbers in index notation.

$300 = 2^2 \times 3 \times 5^2$
$156 = 2^2 \times 3 \times 13$
$126 = 2 \times 3^2 \times 7$

Step 2: Since question ask for the greatest possible number of girls, HCF is determined.

$\text{HCF} = 2 \times 3$
$= 6$

N1.11 Standard Form

To express a number in standard form, always ensure that the number is more than 1 and less than 10 i.e. $1 < x < 10$. When changing numbers to standard form, if the number becomes bigger, the power will become smaller. When the number becomes smaller, the power will become bigger.

Prefix for Standard Form

Factor	Prefix	Symbol
10^{-1}	deci	d
10^{-2}	centi	c
10^{-3}	milli	m
10^{-6}	micro	µ
10^{-9}	nano	n
10^{-12}	pico	p
10^{3}	kilo	k
10^{6}	mega	M
10^{9}	giga	G
10^{12}	tetra	T
10^{6}	million	
10^{9}	billion	
10^{12}	trillion	

Example: N Level 2012 Paper 2 Question 1

The time-keeper in a race sees a starting gun fire 334 nanoseconds after it has been fired.

(a) Write 334 nanoseconds in seconds in standard form.

334 nanoseconds = 334×10^{-9} s

$= 3.34 \times 10^{-7}$ s

Note: -7 is bigger than -9

(b) The time-keeper hears the gun fire 0.294 seconds after it has been fired. Given that 0.294 seconds = k x 334 nanoseconds, find the value of k. Give your answer in standard form.

$0.294 = k \times 3.34 \times 10^{-7}$

$k = \frac{0.294}{3.34 \times 10^{-7}}$

$= 8.80 \times 10^{5}$ (3 s.f.)

Example: N level 2014 Paper 1 Question 11

The table shows the population of some countries in 2011.

Country	Population
China	1.34 billion = 1.34×10^9
USA	3.14×10^8
Philippines	1.04×10^8
Germany	8.13×10^7
Ghana	2.52×10^7

(a) The population of China in 2011 was 1.34 billion. Complete the table by writing the population of China in standard form. Shown in purple.

(b) How many more people lived in Germany than in Ghana? Give your answer in millions.

Difference in the number of people = $(8.13 \times 10^7) - (2.52 \times 10^7)$
$= 5.61 \times 10^7$
$= 56.1 \times 10^6$
$= 56.1$ million

(c) The population of Finland was approximately one twentieth of the population of the Philippines. Estimate the population of Finland, in millions, correct to one significant figure.

Population of Finland $= \frac{1}{20} \times 1.04 \times 10^8$
$= 0.052 \times 10^8$
$= 5.2 \times 10^6$
$= 5.2$ million

N1.12 Law of Indices

☺ $a^m \times a^n = a^{m+n}$ — Note: Add powers when base number is the same

Example: $a^5 \times a^2 = a^{5+2}$

$= a^7$

☺ $a^m \div a^n = a^{m-n}$ — Note: Minus powers when base number is the same

Example: $a^5 \div a^2 = a^{5-2}$

$= a^3$

☺ $(a^m)^n = a^{mn}$ — Note: Multiply powers

Example: $(a^5)^3 = a^{5 \times 3}$

$= a^{15}$

☺ $a^m \times b^m = (a \times b)^m$ — Note: Multiply bases when power is the same

Example: $5^2 \times 3^2 = (5 \times 3)^2$

☺ $a^m \div b^m = (a \div b)^m$ — Note: Divide bases when power is the same

Example: $6^2 \div 3^2 = (6 \div 3)^2$

☺ $a^0 = 1$ — Note: Any number to the power of 0 is 1

Example: $5^0 = 1$

☺ $a^{-n} = \frac{1}{a^n}$ — Note: Take reciprocal of fraction to remove negative power

Example: $7^{-2} = \frac{1}{7^2}$

☺ $(\frac{a}{b})^{-n} = (\frac{b}{a})^n$ — Note: Take reciprocal of fraction to remove negative power

Example: $(\frac{3}{2})^{-5} = (\frac{2}{3})^5$

☺ $a^{\frac{m}{n}} = \sqrt[n]{a^m}$ — Note: Numerator of fraction at the top position of roots, denominator of fractions at bottom position

Example: $5^{\frac{3}{2}} = \sqrt[2]{5^3}$

Example: N Level 2013 Paper 1 Question 15

(a) Evaluate 3^{-2}.

$3^{-2} = \frac{1}{3^2}$

$= \frac{1}{9}$

(b) Given that $16^{\frac{3}{2}} = 2^x$, find x.

$16^{\frac{3}{2}} = 2^x$

$(2^4)^{\frac{3}{2}} = 2^x$

$2^x = 2^6$

$x = 6$

Example: O Level 2013 Paper 1 Question 5

(a) Given that $8 \times 16^{\frac{1}{4}} = 2^n$, find n.

$2^3 \times (2^4)^{\frac{1}{4}} = 2^n$

$2^3 \times 2^1 = 2^n$

$2^{3+1} = 2^n$

n = 4

(b) Given that $\frac{1}{9} = 3^k$, find k.

$9^{-1} = 3^k$

$3^{-2} = 3^k$

k = -2

Example: O Level 2014 Paper 1 Question 9

Some bacteria were introduced into a culture. The number of bacteria, B, t hours after being introduced is given by $B = 1000 \times 3^t$

(a) How many bacteria were introduced into the culture?

When t = 0,

Number of bacteria = 1000×3^0

= 1000

Note: when bacteria is introduced initially, t = 0.

(b) Find the percentage increase in the number of bacteria at the end of the first hour.

When t = 1,

Number of bacteria = 1000×3^1

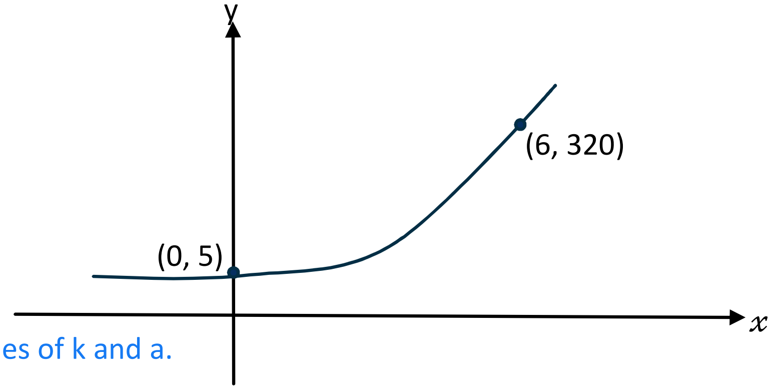
= 3000

% increase = $\frac{3000-1000}{1000} \times 100\%$

= 200%

Example: O level 2012 Paper 1 Question 5

The sketch shows the graph of $y = ka^x$. The points (0, 5) and (6, 320) lie on the graph.



Find the values of k and a.

$y = ka^x$

When $x = 0$, $y = 5$,

$5 = ka^0$
$k = 5$

When $x = 6$, $y = 320$, $k = 5$

$320 = 5a^6$
$64 = a^6$
$2^6 = a^6$
$a = 2$

Therefore, $a = 2$ and $k = 5$.

N2. Ratio and Proportion

N2.1 Word Problems Involving Ratio and Proportion

Example: N Level 2014 Paper 1 Question 1 (b)

Ruth mixes 250 ml of mango juice with 2 litres of apple juice. Write the ratio of mango juice: apple juice in its simplest form.

2 litres = 2000 ml   Note: Ensure that the units are the same before comparing

Mango juice: Apple juice

= 250: 2000
= 1 : 8

Example: N Level 2012 Paper 2 Question 5

(a) Which of these ratios are equivalent to the ratio a : b?

$a^2 : b^2$ 3a : 3b $\frac{1}{b} : \frac{1}{a}$ a + 1: b + 1

3a : 3b
= 3 x a : 3 x b
= a : b   Note: Divide both sides by 3

$\frac{1}{b} : \frac{1}{a}$
$= \frac{a}{b} : 1$ Note: Multiply by a
= a : b   Note: Multiply by b

Therefore, 3a : 3b and $\frac{1}{b} : \frac{1}{a}$ are equivalent to the ratio a : b.

(b) Ahmed and Bijan share the costs of running their car in the ratio 5 : 4 respectively. How much should they each pay when the total costs are $3690?

Amount that Ahmed paid = $\frac{5}{9}$ x 3690
= $2050

Amount that Bijan paid = $\frac{4}{9}$ x 3690
= $1640

N2.2 Basic Conversion between Length, Area and Volume

Conversion Between cm^2 and m^2

1 m = 100 cm
$(1\text{ m})^2 = (100\text{ cm})^2$
$1\text{ m}^2 = 10\,000\text{ cm}^2$

$10\,000\text{ cm}^2 = 1\text{ m}^2$
$1\text{ cm}^2 = \frac{1}{10\,000}\text{ m}^2$

Note: Common mistake student makes is by assuming $1\text{ m}^2 = 100\text{ cm}^2$. Please note the above proper method for conversion.

Conversion Between cm^3 and m^3

1 m = 100 cm
$(1\text{ m})^3 = (100\text{ cm})^3$
$1\text{ m}^3 = 1\,0000\,000\text{ cm}^3$

$1\,000\,000\text{ cm}^3 = 1\text{ m}^3$
$1\text{ cm}^3 = \frac{1}{1\,000\,000}\text{ m}^3$

Note: Common mistake student makes is by assuming $1\text{ m}^3 = 1000\text{ cm}^3$. Please note the above proper method for conversion.

N2.3 Maps and Scales

The representative fraction (R.F.) expresses the linear scale of a map 1: n in the form $\frac{1}{n}$, noting that the numerator is always 1.

Example: O level 2011 Paper 1 Question 9

A map is drawn to a scale of 1: 25 000

(a) This scale can be expressed as 1 cm represents n km. Find n.

1: 25 000
1 cm: 25 000 cm
1 cm: 250 m
1 cm: 0.25 km

Therefore, n = 0.25

(b) The distance between two towns on the map is 30 cm. Find the actual distance, in kilometres, between the two towns.

1 cm: 0.25 km
30 cm: (0.25 x 30) km
30 cm: 7.5 km

Therefore, the distance between the two towns is 7.5 km.

(c) A lake has an actual area of 2.5 km^2. Find the area, in square centimetres, of the lake on the map.

1 cm : 0.25 km
$(1\ cm)^2 : (0.25\ km)^2$
$1\ cm^2 : 0.0625\ km^2$

$0.0625\ km^2 : 1\ cm^2$
$1\ km^2 : \frac{1}{0.0625}\ cm^2$
$2.5\ km^2 : (\frac{1}{0.0625} \times 2.5)\ cm^2$
$2.5\ km^2 : 40\ cm^2$

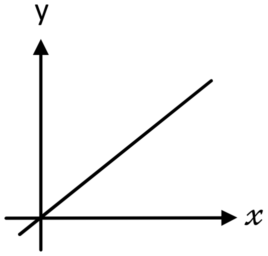
Therefore, the area of the lake on the map is 40 cm^2.

Note: When doing area, always make sure the actual area is changed to whatever unit that is required before squaring both sides for easier conversion.

N2.4 Direct Proportion

When y is directly proportional to x, when x increases, y increases i.e. y = kx

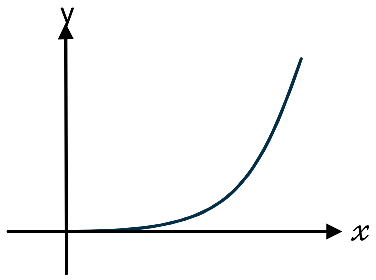
If x and y are in direct proportion, the graph is a straight line which passes through the origin.



Example: O level 2014 Paper 1 Question 19

An object starts from rest and travels in a straight line. The distance travelled, s metres, is directly proportional to the square of the travelling time, t seconds.

(a) Sketch a distance-time graph for the object



Note: Since it is the square of the travelling time, it will be a curve instead of a straight line

In the first 4 seconds the object travels 36 m.

(b) Find the equation for s in terms of t

S α t^2
s = kt^2
when t = 4 and s = 36,
36 = $k(4)^2$
k = 2.25

Therefore, s = $2.25t^2$

Note: Always find constant, k first

(c) Calculate the time taken to travel 20 m.

When s = 20,
20 = $2.25t^2$
$t^2 = \frac{80}{9}$
$t = \sqrt{\frac{80}{9}}$ or $t = -\sqrt{\frac{80}{9}}$
= 2.98s (3 s.f.) (Rejected since t > 0)

Therefore, time taken to travel 20 m is 2.98 seconds.

Example: O level 2008 Paper 1 Question 13

The braking distance of a car is directly proportional to the square of its speed. When the speed is p metres per second, the braking distance is 6 m. When the speed is increased by 300%, find

(a) An expression for the speed of the car

Speed of the car = p + 3p

= 4p m/s

Note: The new speed of the car is not 3p as the speed is **INCREASED** by 300%; the speed is not 300%.

(b) The braking distance

Let the speed of the car be s; the braking distance of the car be d.

Note: Make constant the subject of the formula if the actual constant cannot be determined.

$d \propto s^2$

$d = ks^2$

When s = p and d = 6,

$6 = kp^2$

$k = \frac{6}{p^2}$

When s = 4p,

$d = ks^2$

$d = \frac{6}{p^2} \times (4p)^2$

$d = \frac{6}{\cancel{p^2}} \times 16\cancel{p^2}$

$d = 6 \times 16$

$= 96$ m

Therefore, the braking distance is 96 m.

(c) The percentage increase in the braking distance

$$\% \text{ increase} = \frac{\text{distance increased}}{\text{original distance}} \times 100\%$$

$$= \frac{96 - 6}{6} \times 100\%$$

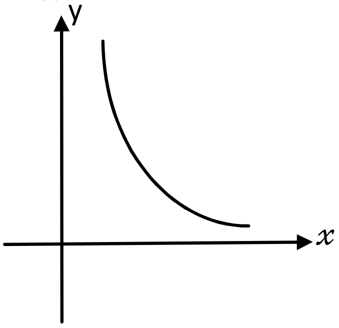
$$= 1500\%$$

N2.5 Inverse Proportion

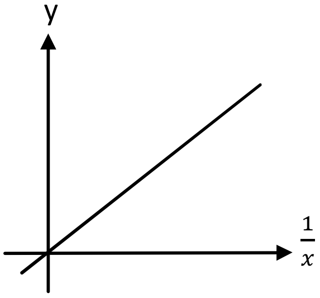
When y is indirectly proportional to x, when x increase, y decreases.

$y = \frac{k}{x}$

If x and y are in inverse proportion, the graph against x is part of a curve which is also called the hyperbola.



If x and y are in inverse proportion, the graph against $\frac{1}{x}$ is a straight line passing through the origin.



Example: N Level 2008 Paper 1 Question 18 (a)

The time taken to fill a fish tank is inversely proportional to the rate at which the water is flowing through the tap. It takes 4 minutes to fill the fish tank when the water is flowing at a certain rate. How long will it take if the rate is halved?

Let the time taken for to fill the fish tank be t and the rate of the water be r.

Note: Make constant the subject of the formula if the actual constant cannot be determined.

$t \propto \frac{1}{r}$

$t = \frac{k}{r}$

when t = 4,

$4 = \frac{k}{r}$

k = 4r

When $r = \frac{1}{2}r$,

$t = \frac{4r}{\frac{1}{2}r}$

t = 8 minutes

N3. Percentage

N3.1 Conversion between Percentage, Decimal and Fraction

To convert percentage into fraction or decimal, divide the value by 100.

Example: Express 0.88% as a fraction

$$0.88\% = \frac{0.88}{100}$$
$$= \frac{11}{1250}$$

Example: Express 1.64% in decimal

$$1.64\% = \frac{1.64}{100}$$
$$= 0.0164$$

To convert fraction or decimal in percentage, multiply the value by 100.

Example: Express $\frac{35}{100}$ in percentage

$$\frac{35}{100} \text{ x } 100\% = 35\%$$

Example: Express 0.76 in percentage

$$0.76 \text{ x } 100\% = 76\%$$

N3.2 Expressing One Quantity as a Percentage of Another

Example: O Level 2014 Paper 1 Question 8

Two bottles are geometrically similar. The smaller bottle has a capacity of 1 litre and the larger bottle has a capacity of 2 litres. Calculate the height of the smaller bottle as a percentage of the height of the larger bottle.

Let the height of the smaller bottle be h_1 and the larger bottle be h_2.

Ratio of the height of the 2 bottles = $\frac{h_1}{h_2}$

Since $(\frac{h_1}{h_2})^3 = \frac{1000}{2000}$

$\frac{h_1}{h_2} = \sqrt[3]{\frac{1000}{2000}}$

Therefore, percentage of $\frac{h_1}{h_2} = \sqrt[3]{\frac{1000}{2000}}$ x 100%

= 79.37

= 79.4% (3 s.f.)

N3.3 Comparing Two Quantities by Percentage

Example: Jug A is sold for \$8 at Supermarket A while Jug B is sold for \$6.50. During an economic recession, the prices of both jugs were marked up by \$0.50 and \$0.80 respectively. Determine which jug has a higher percentage increase in price.

Percentage increase in price of Jug A = $\frac{0.50}{8}$ x 100%

= 6.25%

Percentage increase in price of Jug B = $\frac{0.80}{6.50}$ x 100%

= 12.3% (3 s.f.)

Therefore, the percentage increase in price of Jug B is higher.

N3.4 Calculating % Profit or Loss

Basic Formula:

% Profit or Loss = $\frac{New\ Price - Original\ Price}{Original\ Price}$ x 100%

Example: N Level 2014 Paper 1 Question 16

Daniel owns an electrical shop

(a) The cost of a washing machine was $420. Daniel made a profit of 30% on the cost price when he sold it. Find the selling price

Selling price = $\frac{100 + 30}{100}$ x 420

= $546

ALTERNATIVE WORKING:

100% → $420

1% → $\frac{420}{100}$

130% → $\frac{420}{100}$ x 130

= $546

(b) Daniel paid $180 for a vacuum cleaner. He sold it in a sale for $153. Find the loss as a percentage of the cost price.

% Loss = $\frac{180 - 153}{180}$ x 100%

= 15%

N3.5 Increasing or Decreasing a Quantity by a Given Percentage

Example: N Level 2011 Paper 2 Question 7

In July 2007, Goods and Services Tax (GST) was raised from 5% to 7%.

(a) Before the increase, the price of the television without GST was $850. Calculate the increase in the price of the television due to the increase in GST.

To calculate the price of the television with 5% GST:

100% → 850

1% → $\frac{850}{100}$

105% → $\frac{850}{100}$ x 105

= $892.50

To calculate the price of the television with 7% GST:

107% → $\frac{850}{100}$ x 107

= $909.50

Increase in price = $909.50 – $892.50

= $17

(b) Before the increase, the price of a washing machine including GST was $630. What was the price of the washing machine after the increase to 7% GST?

To calculate selling price excluding 5% GST:

105% → 630

1% → $\frac{630}{105}$

100% → $\frac{630}{105}$ x 100

= $600

To calculate price of the washing machine after the increase to 7% GST

100% → 600

1% → $\frac{600}{100}$

107% → $\frac{600}{100}$ x 107

= $642

Therefore, the price of the washing machine was $642.

N4. Rate and Speed

Average speed = $\frac{Total\ Distance}{Total\ Time}$

Note: When calculating average speed, total time includes time stopped at a certain location

N4.1 Conversion of Units

Example: Convert 30 km/h into m/s

$\frac{30\text{ km}}{\text{h}} = \frac{30\,000\text{m}}{3600\text{s}}$

$= 8\frac{1}{3}$ m/s

Example: Convert 7 m/s into km/h

$\frac{7\text{ m}}{\text{s}} = \frac{0.007\text{ km}}{\frac{1}{3600}\text{h}}$

= 25.2 km/h

N4.2 Word Problems Involving Speed

Example: N Level 2011 Paper 1 Question 24

A car is travelling at a constant speed of 72 km/h.

(a) Find the distance the car takes to travel in 40 minutes

1h = 60 min

40 minutes = $\frac{40}{60}$ h

Distance = 72 x $\frac{40}{60}$

= 48 km

(b) Find how many minutes the car takes to travel 126 km.

60 min → 72 km

1 min → $\frac{72}{60}$ km

Time taken to travel 126 km = 126 ÷ $\frac{72}{60}$

= 105 min

(c) Another car travels 196 km in 2 hours 27 min.

2 hours 27 min = $2\frac{27}{60}$ h

Average speed = $\frac{Total\ Distance}{Total\ Time}$

$= \frac{196\ km}{2\frac{27}{60}\ h}$

= 80 km/h

N4.3 Word Problems Involving Rate

Example: If it takes 3 days for 10 workers to finish building one house, how many days will it take 15 workers to finish building 4 houses?

10 workers → 3 days → 1 house
1 worker → 30 days → 1 house
15 workers → 2 days → 1 house

Note: 1 worker will take more days to complete the job as compared to 10 workers.

1 house → 2 days
4 houses → 8 days

Example: Tap A can fill up a tank in 5h. Tap B can fill up a tank in 4h. If both the taps are turned on simultaneously, how long would it take to fill the tank?

Tap A

5h → 1 tank
1h → $\frac{1}{5}$ tank

Tap B

4h → 1 tank
1h → $\frac{1}{4}$ tank

Tap A + B

1h → $\frac{1}{5}+\frac{1}{4}$ tank
$= \frac{9}{20}$ tank

$\frac{9}{20}$ tank → 1 h
$\frac{1}{20}$ tank → $\frac{1}{9}$ h
$\frac{20}{20}$ tank → $\frac{20}{9}$ h
$= 2\frac{2}{9}$ h

N5. Algebraic Representation and Formulae

N5.1 Evaluation of Algebraic Expressions and Formulae

Basic algebraic expression

- ☺ ab = a x b
- ☺ $\frac{a}{b}$ = a ÷ b
- ☺ a^2 = a x a
- ☺ $\frac{1}{5}x = \frac{x}{5}$
- ☺ $-\frac{1}{3}x = \frac{-x}{3} = \frac{x}{-3}$
- ☺ ab^2 = a x b x b
- ☺ $(ab)^2 = a^2b^2$ = a x a x b x b

Note: Negative sign can be on the numerator or denominator, but not both.

To solve linear equations, there are a few rules to take note of:

1. Always open the brackets first in the order of (, [and {. When expanding brackets, do take note of the possible existence of the negative sign. The number in front of the bracket will only affect the numbers inside the bracket.
2. Do multiplication and division next, depending on the order that comes first. Always move from left to right of the equation
3. Lastly, do addition and subtraction. Always move from left to right of the equation

Example: Simplify -2 {a + 2b x 4 – [3(5a – b) – 5(b – 3) + 8] – 5}:

-2 {a + 2b x 4 – [3(5a – b) – 5(b – 3) + 8] – 5}
= -2 {a + 2b x 4 – [15a – 3b – 5(b – 3) + 8] – 5} Note: Change in sign when bracket is opened
= -2 {a + 2b x 4 – [15a – 3b – 5b + 15 + 8] – 5} Note: Simplify expression in bracket first
= -2 [a + 2b x 4 – (15a – 8b + 23) – 5] Note: Order of the brackets can be changed
= -2 [a + 2b x 4 – 15a + 8b – 23 – 5]
= -2 [a + 8b – 15a + 8b – 23 – 5]
= -2 [– 14a + 16b – 28]
= 28a – 32b + 56

N5.2 Changing the Subject of the Formula

Put whatever you need on the left hand side, the rest on the right hand side. Cross multiply to obtain a linear equation whereas required. When a square root is involved do no forget to insert your $\pm$ sign. Failing to do so will cause you to lose a mark from the question.

Example: O level 2014 Paper 2 Question 1 (bii)

It is given that W = $\frac{1}{2}$ m (v^2 - u^2). Express u in terms of W, m and v.

$W = \frac{1}{2} m (v^2 - u^2)$

$2W = m (v^2 - u^2)$

$\frac{2W}{m} = v^2 - u^2$

$u^2 = v^2 - \frac{2W}{m}$

$u = \pm\sqrt{v^2 - \frac{2W}{m}}$

N5.3 Addition and Subtraction of Algebraic Expressions

Ensure that the denominators are the same before you combine the fractions. When combining the fractions, do take note of the negative sign before you open the brackets. Note: Denominator need not be expanded when expressing algebraic expressions as single fraction.

Example: Modified from O Level 2012 Paper 1 Question 11 (b)

Write as a single fraction in its simplest form $\frac{5}{(x-2)^2} + \frac{1}{2-x}$.

$\frac{5}{(x-2)^2} + \frac{1}{2-x}$

$= \frac{5}{(x-2)^2} - \frac{1}{x-2}$ Note: How the signs are changed

$= \frac{5}{(x-2)^2} - \frac{1\,(x-2)}{(x-2\,)(x-2)}$ Note: Negative sign when combining fractions

$= \frac{5-(x-2)}{(x-2)^2}$

$= \frac{5-x+2}{(x-2)^2}$

$= \frac{7-x}{(x-2)^2}$

Example: O Level 2013 Paper 2 Question 1 (c)

Express as a single fraction in its simplest form $\frac{6}{3-2x} - \frac{4}{2-x}$

$\frac{6}{3-2x} - \frac{4}{2-x}$

$= \frac{6(2-x)}{(3-2x)(2-x)} - \frac{4(3-2x)}{(3-2x)(2-x)}$

$= \frac{6(2-x)-4(3-2x)}{(3-2x)(2-x)}$

$= \frac{12-6x-12+8x}{(3-2x)(2-x)}$

$= \frac{2x}{(3-2x)(2-x)}$

N5.4 Multiplication and Division of Algebraic Expressions

Always change the division sign to a multiplication sign before proceeding on. Upon changing the division sign to a multiplication sign, take the reciprocal of the fraction (flip the fraction over)

Example: O Level 2012 Paper 2 Question 1 (c)

Simplify $\frac{x}{2y} \div \frac{3x^2y}{4}$

$\frac{x}{2y} \div \frac{3x^2y}{4}$

$= \frac{\cancel{x}}{2y} \times \frac{4}{3x^{\cancel{2}}y}$

$= \frac{2}{3xy^2}$

Example: O Level 2011 Paper 1 Question 8 (a)

Simplify $\frac{5c}{2} \div \frac{20c^2}{d}$

$\frac{5c}{2} \div \frac{20c^2}{d}$

$= \frac{\cancel{5c}}{2} \times \frac{d}{_4\cancel{20c^2}}$

$= \frac{d}{8c}$

Example: Simplify $(\frac{3a}{4ab})^2 \div \frac{27a}{2b}$

$(\frac{3a}{4ab})^2 \div \frac{27a}{2b}$

$= \frac{\cancel{9a^2}}{_8\cancel{16a^2b^2}} \times \frac{\cancel{2b}}{_3\cancel{27a}}$

$= \frac{1}{24ab}$

N5.5 Word Problems involving Algebraic Expressions

Example: N level 2011 Paper 1 Question 23 (b)

A 40 cm length of ribbon costs x dollars. Find an expression for the cost, in dollars, of y metres of ribbon. Give your answer in its simplest form.

40 cm = 0.4 m
0.4 m → $\$x$
1 m → $\$\frac{x}{0.40}$
y m → $\frac{x}{0.40} \cdot y$
= $\$\frac{xy}{0.40}$

Note: There are 4 different types of units used in this question

N5.6 Basic Quadratic Formulas and Methods to Solve Quadratic Equations

<u>Basic Formulas:</u>

☺ $(a + b)^2 = a^2 + 2ab + b^2$
☺ $(a - b)^2 = a^2 - 2ab + b^2$
☺ $a^2 - b^2 = (a + b)(a - b)$

<u>Methods to Factorise Quadratic Equations</u>

☺ Cross Factorisation (Refer to page 33)
☺ Completing the Square (Refer to page 34)

N5.7 Expansion of the Product of Quadratic Expressions

To expand quadratic equations, use ‘rainbow’ method.

Example: Expand $3(2x - 1)(x + 5)$

Expand the numbers within the brackets by rainbow method first. DO NOT multiply in the number outside the bracket, in this case, 3.

$3(2x - 1)(x + 5)$

$= 3(2x^2 + 10x - x - 5)$
$= 3(2x^2 + 9x - 5)$
$= 6x^2 + 27x - 15$

N5.8 Completing the Square not involving an Equation

Example: $2x^2 - 3x - 5$

$2x^2 - 3x - 5$

$= 2\ (x^2 - \frac{3}{2}x - \frac{5}{2})$

$= 2\ [x^2 - \frac{3}{2}x + (-\frac{3}{2} \div 2)^2 - \frac{5}{2} - (-\frac{3}{2} \div 2)^2]$

$= 2\ [(x - \frac{3}{4})^2 - \frac{49}{16}]$

$= 2(x - \frac{3}{4})^2 - \frac{49}{8}$

Note: Do not put = 0 if the question does that state so!

1. Ensure that x^2 is by itself.
2. Divide the consonant in front of x by 2 and square it. Shown in green.
3. Since it is on the same side of the equation, one of them must be a plus, the other a minus.
4. Multiply the factor in accordingly.

N5.9 Factorisation of Linear Algebraic and Quadratic Expression

Example: O Level 2012 Paper 1 Question 18 (b)

Factorise fully $6x^2 - 15x - 9$

Step 1: Factorise out common factors. If the string of numbers is keyed into the calculator, it will not give an equivalent answer as calculator can only give answers in the simplest form.

$6x^2 - 15x - 9$
$= 3(2x^2 - 5x - 3)$

Step 2: Either key the numbers into the calculator (Refer to Page 7) or draw up format for cross factorisation. For cross factorisation, fill up the boxes according to the format

$2x^2$	-3	$-5x$

Step 3: Trial and error is required. Fill up the boxes with possible numbers that may fit the numbers. Note: DO NOT INCLUDE any NEGATIVE SIGNS at this stage.

$2x$	1	x
x	3	$6x$
$2x^2$	-3	$-5x$

Step 3: Determine how the number on the right can be obtained. In this case, $6x$ will have to be negative in order to obtain $-5x$ i.e. $x - 6x = -5x$. The negative number will be carried to the number directly on its left.

$2x$	1	x
x	-3	$-6x$
$2x^2$	-3	$-5x$

Step 4: Write the answer in its factorised form.

$6x^2 - 15x - 9$
$= 3(2x^2 - 5x - 3)$
$= 3(2x + 1)(x - 3)$

Example: O Level 2013 Paper 1 Question 18 (a)

Factorise completely $3x\text{y} - 6\text{ay} - 4x + 8\text{a}$

$3x\text{y} - 6\text{ay} - 4x + 8\text{a}$
$= 3\text{y}(x - 2\text{a}) - 4(x - 2\text{a})$ Note: Sign change in red!
$= (x - 2\text{a})(3\text{y} - 4)$

Other possible workings:

$3x\text{y} - 6\text{ay} - 4x + 8\text{a}$
$= 3x\text{y} - 4x + 8\text{a} - 6\text{ay}$
$= x(3\text{y} - 4) + 2\text{a}(4 - 3\text{y})$ Note: How (3y – 4) is changed into (4 – 3y)
$= x(3\text{y} - 4) - 2\text{a}(3\text{y} - 4)$
$= (x - 2\text{a})(3\text{y} - 4)$

Example: O Level 2013 Paper 2 Question 1 (b)

a) Factorise completely $18\text{p}^2 - 8$

$18\text{p}^2 - 8$
$= 2(9\text{p}^2 - 4)$ Note: Check for common factors first
$= 2(3\text{p} - 2)(3\text{p} + 2)$

b) Simplify $\frac{18p^2 - 8}{6p^2 - 14p -}$

$\frac{18p^2 - 8}{6p^2 - 14p - 12}$

$= \frac{2(3p - 2)(3p + 2)}{2(3p^2 - 7p - 6)}$

$= \frac{2\cancel{(3p - 2)}(3p + 2)}{2(p + 3)\cancel{(3p - 2)}}$

$= \frac{(3p + 2)}{(p + 3)}$

Note: Do not key the denominator into the calculator if the common factor has not been factorised

3p	2	2p
p	-3	-9p
3p^2	-6	-7p

N5.10 Number Patterns

Linear Number Pattern

The difference between the terms will always be constant. Assuming that term 0 exists, determine term 0.

Example 1: -2, 5, 12, 19 …

Since the pattern keeps increasing by 7 therefore +7n will be part of the pattern.
Term 0 = -2 – 7
= -9

Therefore, n^{th} term = -9 + 7n

Example 2: 21, 18, 15, 12 …

Note that the pattern keeps decreasing by 3, therefore -3n will be part of the pattern.
Term 0 = 21 + 3
= 24

Therefore, n^{th} term = 24 – 3n

Special Number Patterns

Example 1: 1, 4, 9, 16 …
n^{th} term: n^2

Reason: 1 x 1, 2 x 2, 3 x 3, 4 x 4…

Example 2: 1, 8, 27, 64 …
n^{th} term = n^3

Reason: 1 x 1 x 1, 2 x 2 x 2, 3 x 3 x 3, 4 x 4 x 4 …

Number Pattern Involving x^2

Example: 6, 15, 28, 45 …

Note that the difference between each terms is not the same. In this case, quadratic formula $an^2 + bn + c$ has to be used. The first number that appears in the number pattern is known as term 1.

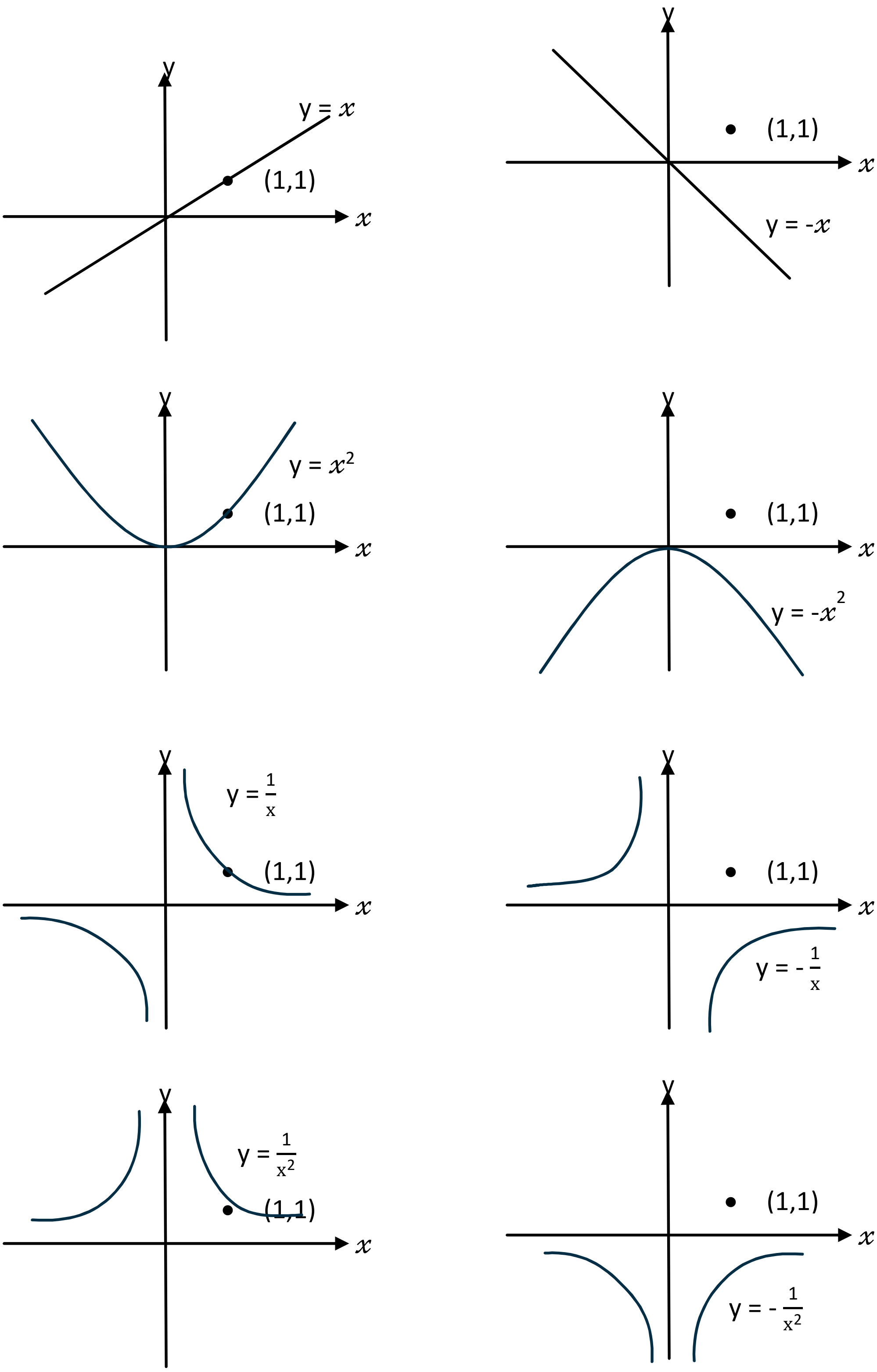
When n=1, $a + b + c = 6$
when n= 2, $4a + 2b + c = 15$
when n=3, $9a + 3b + c = 28$

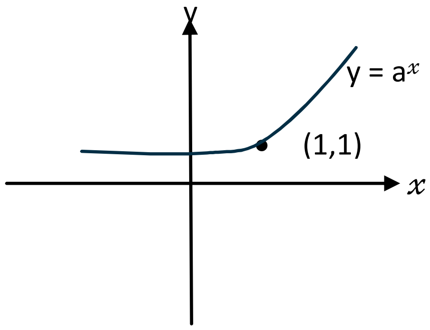
Solve the above equations using calculator, calculator will show answer as $x = 2$, y = 3, z = 1. Refer to page 8 for elaborate steps on calculator skills. Note: The numbers in red will remain the same if the first 3 terms of the number pattern is used. The last column varies according to the numbers given in the question.

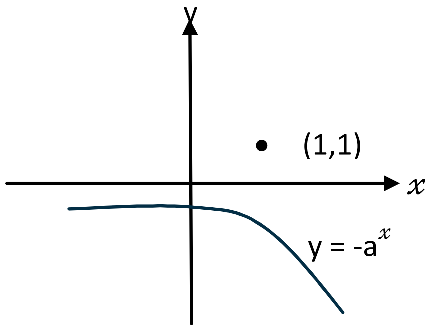
Therefore, the n^{th} term is $2n^2 + 3n + 1$

N6. Functions and Graphs

N6.1 Graphs of Common Equations







N6.2 Sketching of Curve (Quadratic Equation – Factorisation)

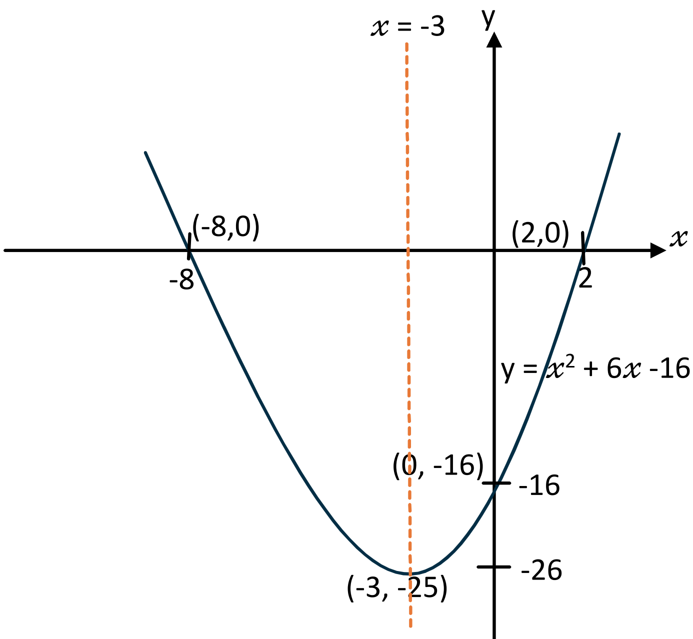
Example: Sketch the graph y = $x^2 + 6x - 16$ and show the intercepts and turning point clearly.

1. Find x intercept i.e. when y = 0
   When y = 0,
   $x^2 + 6x - 16 = 0$
   $(x + 8)(x - 2) = 0$
   Either $x + 8 = 0$ or $x - 2 = 0$
   $x = -8$ $x = 2$

2. Find y intercept i.e. when $x = 0$
   When $x = 0$,
   $y = 0^2 + 6(0) - 16$
   $= -16$

3. Find line of symmetry
   $x = \frac{(-8)+2}{2}$
   $= -3$

4. Find turning point
   When $x = -3$,
   $y = (-3)^2 + 6(-3) - 16$
   $= -25$



Note: To label all co-ordinates of the x-intercepts, y-intercept, turning point, line of symmetry and equation of the curve for the factorisation method clearly.

N6.3 Sketching of Curve (Completing the Square Method)

Example: Sketch the graph y = $x^2 + 6x - 16$ and show the intercepts and turning point clearly.

Step 1: Complete the square (Refer to page 34 for detailed workings)

$$y = x^2 + 6x - 16$$
$$y = x^2 + 6x + \left(\frac{6}{2}\right)^2 - 16 - \left(\frac{6}{2}\right)^2$$
$$y = (x + 3)^2 - 25$$

If this is positive, turning point of x – co-ordinate will be negative (Opposite power). If it is negative, turning point of the x – co-ordinate will be positive

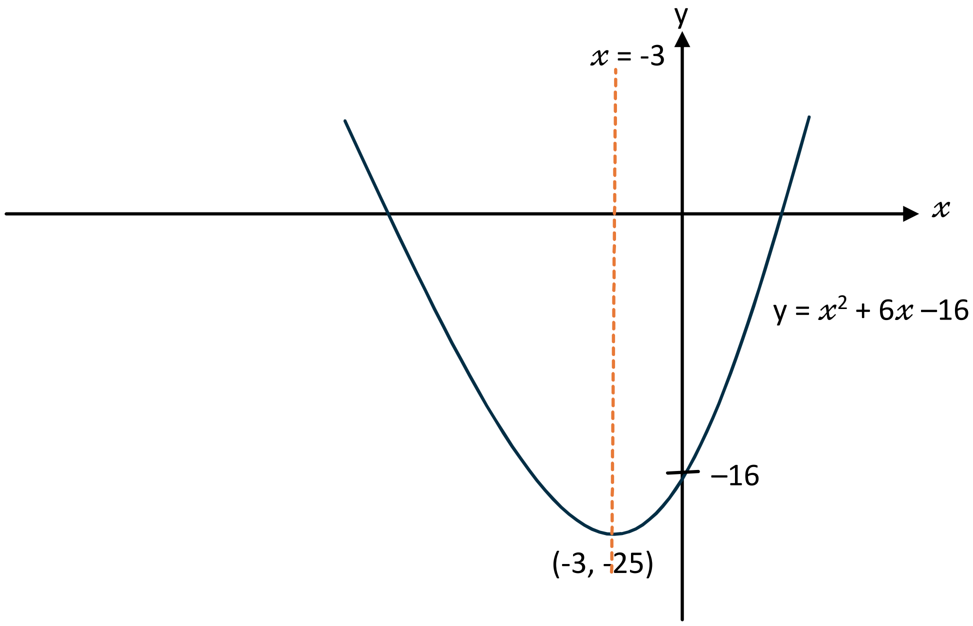
If this is negative, turning point of y-coordinate will be negative (Same power). If it is positive, turning of the y – co-ordinate will be negative.

Therefore, the turning point of curve is at (-3, 25)

Step 2: Find y – intercept i.e. when $x = 0$

When $x = 0$,
$y = (0 + 3)^2 - 25$
$= -16$

Step 3: Determine the shape of the curve.
x^2 –> U shaped. This question is a U shaped curve.
$-x^2$ –> ∩ shaped



Note: To label all co-ordinates of the y-intercept, turning point, line of symmetry and equation of the curve for completing the square method clearly. If you cannot visualise the shape of the curve, determine the x – intercepts.

N6.4 Plotting of Curve on Graph Paper

Possible Questions:

1. Determine the value of x when provided with y
2. Determine the value of y when provided with x
3. Determine the coordinates of the point where the line intersects the curve
4. The maximum point of the curve (y-axis)
5. When given the equation of a similar graph, a linear version of the graph needs to be drawn to determine the solution.
6. Draw a tangent to the graph and determine the gradient of the tangent
7. Plotting of linear graph and determining the intersection points.
8. Simultaneous equation: by combining the equations of the curve and the linear graph into one equation. Simplify the equation (usually in fractions) to a linear equation

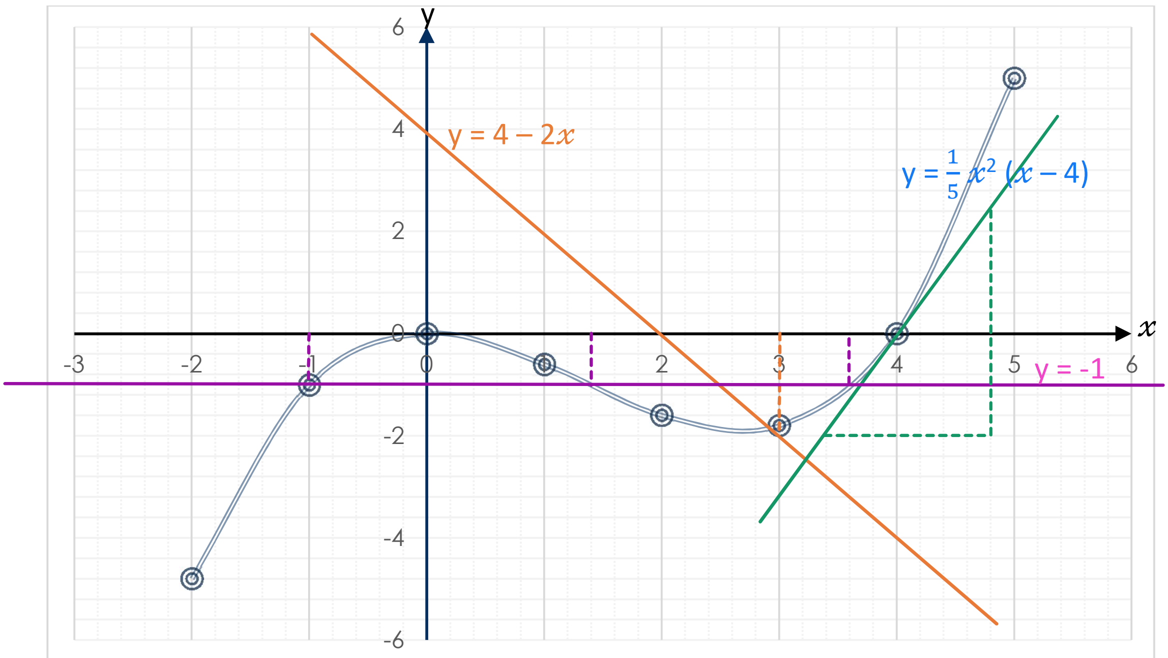
Example: O Level 2011 Paper 2 Question 9

The variables x and y are connected by the equation $y = \frac{1}{5}x^2(x - 4)$. Some corresponding values of x and y are given in the following table.

x	-2	-1	0	1	2	3	4	5
y	p	-1	0	-0.6	-1.6	-1.8	0	5

(a) Find the value of p.
   p = -4.8

(b) Using a scale of 2 cm to 1 unit, draw a horizontal x-axis for $-2 \le x \le 5$. Using a scale of 1 cm to 1 unit, draw a vertical y – axis for $-6 \le x \le 6$. On your axes, plot the points in the table and join them with a smooth curve.



(c) Use your graph to find three solutions of $\frac{1}{5}x^2(x-4) = -1$. Shown in purple.

Since $y = \frac{1}{5}x^2(x-4)$, plot equation y = -1

x = -1, 1.4 or 3.6

(d) By drawing a tangent, find the gradient of the cure at the point (4, 0). Shown in green.

Gradient = $\frac{2.4-(-2)}{4.8-3.4}$

= 3.1 (1 d.p.)

(e) On the same axes, draw the line $y = 4 - 2x$ for $0 \leq x \leq 5$. Shown in orange.

$y = 4 - 2x$

x	0	1	2
y	4	2	0

(f) Write down the x-coordinate of the point where this line intersects the curve.

x = 2.9

(g) This value of x is a solution of the equation $x^3 - 4x^2 + Ax + B = 0$

$y = \frac{1}{5}x^2(x-4)$ ---------------------(1)

$y = 4 - 2x$ ---------------------(2)

Substitute (1) into (2)

$\frac{1}{5}x^2(x-4) = 4 - 2x$

$x^2(x-4) = 20 - 10x$

$x^3 - 4x^2 + 10x - 20 = 0$

By comparing coefficients,

A = 10, B = -20

N7. Solutions of Equations and Inequalities

N7.1 Solving Linear Equations in One Unknown

Example: O Level Paper 1 Question 8

Solve the equation $\frac{2x-3}{6}+\frac{x+2}{3}=\frac{5}{2}$

$\frac{2x-3}{6}+\frac{x+2}{3}=\frac{5}{2}$

$\frac{2x-3}{6}+\frac{2(x+2)}{3\ x\ 2}=\frac{5\ x\ 3}{2\ x\ 3}$

Note: Denominator has to be the same

$\frac{2x-3}{6}+\frac{2x+4}{6}=\frac{15}{6}$

$2x - 3 + 2x + 4 = 15$

$4x = 14$

$x = 3.5$

Example: O Level 2012 Paper 2 Question 1 (e)

Solve the equation $\frac{2x-7}{3x+2} = 4$

$\frac{2x-7}{3x+2}=\frac{4}{1}$

Note: Cross multiply to obtain linear equation. Always put any whole number over one if you cannot visualise.

$1(2x - 7) = 4(3x + 2)$

$2x - 7 = 12x + 8$

$2x - 12x = 8 + 7$

$-10x = 15$

$x = -1.5$

N7.2 Simultaneous Equation

Simultaneous equation can be solved by either elimination, substitution or graphical method. Elimination method aims to remove one of the variables, substitution methods works by making one of the variables the subject of the formula while the solutions can be determined by graphical methods by the intersections of the coordinates on the graph.

Example: O Level 2007 Paper 2 Question 2 (b)
$2x = y + 6$ -----------------(1)
$6x - 2y = 13$ -----------------(2)

By Elimination Method,
(1) x 3

$6x - 3y = 18$ -----------------(3)
$6x - 2y = 13$ -----------------(2)

(3) – (2),
$(6x - 3y) - (6x - 2y) = 18 - 13$
$6x - 3y - 6x + 2y = 5$
$-y = 5$
$y = -5$

Sub y = –5 into (1)

$2x = -5 + 6$
$2x = 1$
$x = 0.5$

Therefore, $x = 0.5$, $y = -5$

By Substitution Method,
$x = \frac{y+6}{2}$ -----------------(3)

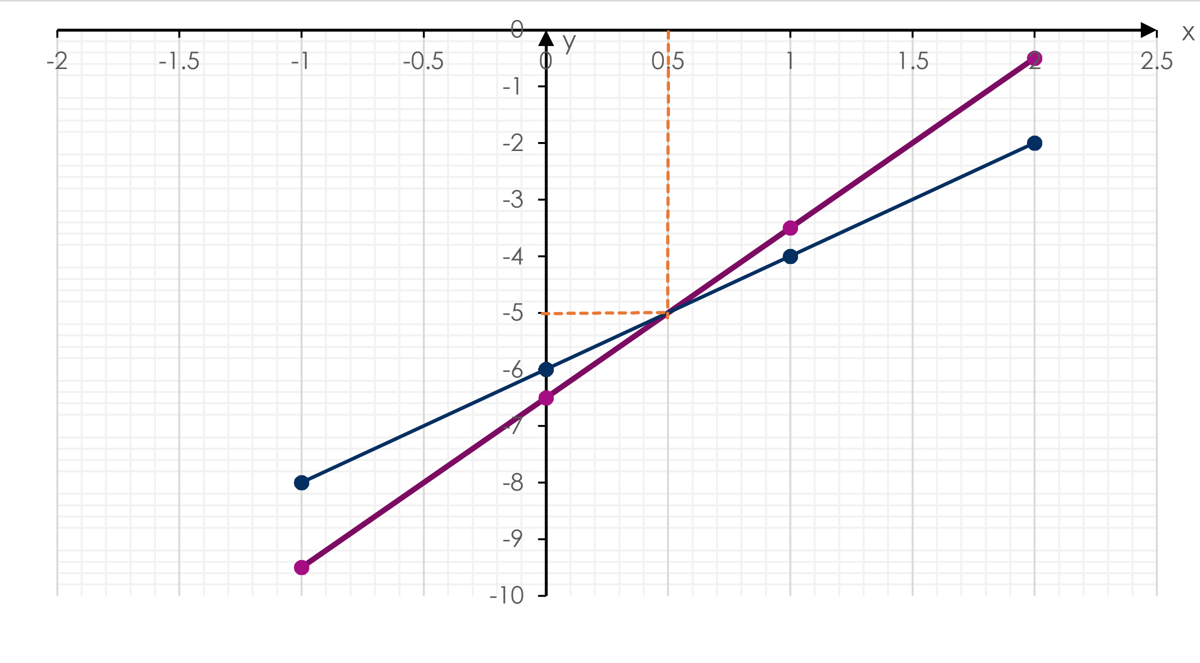
Sub (3) into (2),
$6\,(\frac{y+6}{2}) - 2y = 13$
$3y + 18 - 2y = 13$
$y = -5$

Sub y = -5 into (3)
$x = \frac{-5+6}{2}$
$x = -0.5$

Therefore, $x = 0.5$, $y = -5$

<u>By Graphical Method</u>

Plot the equations $2x = y + 6$ and $6x - 2y = 13$ on a piece of graph paper. The solutions will be at the point of intersection.



Therefore, $x = 0.5$, $y = -5$

N7.3 Solving Quadratic Equation

The three ways to solve quadratic equations not by graphical methods are factorisation, general formula and completing the square.

<u>Cross Factorisation</u>

When using this method, note that the right hand side must always be zero. If not, bring all variables to the left hand side before proceeding.

Example: N Level 2014 Paper 1 Question 14 (bii)

Solve $x^2 - 2x - 24 = 0$

$x^2 - 2x - 24 = 0$
$(x - 6)(x + 4) = 0$
$(x - 6) = 0$ or $(x + 4) = 0$
$x = 6$ or $x = -4$

x	-6	$-6x$
x	4	$4x$
x^2	-24	$-2x$

<u>General Formula</u>

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

Note: Consonant in front of x^2 is <u>a</u>, consonant in front of x is <u>b</u> and the constant term alone is <u>c</u>. Do include negative signs whereas applicable. The right hand side must always be equals to 0.

Example: $2x^2 - 3x - 5 = 0$

a = 2, b = -3, c = -5

$$x = \frac{-(-3) \pm \sqrt{(-3)^2 - 4(2)(-5)}}{2(2)}$$

$$x = \frac{3 \pm \sqrt{9 + 40}}{4}$$

$$x = \frac{3 \pm \sqrt{49}}{4}$$

$x = \frac{3 + \sqrt{49}}{4}$ or $x = \frac{3 - \sqrt{49}}{4}$

$x = 2.5$ or $x = -1$

Completing the Square

When doing completing the square in an equation, bring the constant term to the right hand side of the equation. Ensure that x^2 is doesn't have any constant term.

$2x^2 - 3x - 5 = 0$

$2x^2 - 3x = 5$

$x^2 - \mathbf{1.5}x = 2.5$

$x^2 - 1.5x + \left(\frac{-1.5}{2}\right)^2 = 2.5 + \left(\frac{-1.5}{2}\right)^2$

$(x - 0.75)^2 = \frac{49}{16}$

$x - 0.75 = \pm\sqrt{\frac{49}{16}}$

$x - 0.75 = \frac{7}{4}$ or $x - 0.75 = \frac{7}{4}$

$x = 2.5$ or $x = -1$

1. Bring the constant term over to the right hand side.
2. Ensure that x^2 is by itself.
3. Divide the consonant in front of x by 2 and square it. Shown in green.
4. Since it is on the different side of the equation, both of them will be a plus
5. Square root the equation on both side. If $\pm$ of the number square rooted is not determined, marks will be lost.
6. Solve for x.

N7.4 Solving Fractional Equations that can be reduced to Quadratic Equation

Example: O Level 2014 Paper 2 Question 1 (d)

Solve the equation $\frac{4}{x+3} + \frac{3}{x-2} = 1$

$\frac{4}{x+3} + \frac{3}{x-2} = 1$

$\frac{4(x-2)}{(x+3)(x-2)} + \frac{3\,(x+3)}{(x-2)(x+3)} = 1$

$\frac{4(x-2)+3(x+3)}{(x+3)(x-2)} = \frac{1}{1}$

$4(x - 2) + 3(x + 3) = (x + 3)(x - 2)$

$4x - 8 + 3x + 9 = x^2 - 2x + 3x - 6$

$7x + 1 = x^2 + x - 6$

$x^2 - 6x - 7 = 0$

$(x - 7)(x + 1) = 0$

$(x - 7) = 0$ or $(x + 1) = 0$

$x = 7$ or $x = -1$

N7.5 Solving Quadratic Equations Related Problems

Example: O level 2010 Paper 2 Question 6

John and Peter took part in a marathon race. They each ran 42 km.

(a) John ran at a constant speed of x kilometres per hour. Write down an expression in terms of x, for the number of hours he took.

Number of hours John took = $\frac{42}{x}$ hours

(b) Peter ran a constant speed which was $\frac{1}{2}$ km/h less than John's speed. Write down an expression, in terms of x, for the number of hours he took.

Speed of Peter = $x - \frac{1}{2}$

= $\frac{2x-1}{2}$ km/h

Note: Units used in this question

Number of hours Peter took = $42 \div \frac{2x-1}{2}$

= 42 x $\frac{2}{2x-1}$

= $\frac{84}{2x-1}$ hours

(c) The difference between their times was 10 minutes. Write down an expression in x to represent this information, and show that it reduces to $2x^2 - x - 252 = 0$

$$\frac{84}{2x-1} - \frac{42}{x} = \frac{10}{60}$$

$$\frac{84(x) - 42(2x-1)}{(2x-1)(x)} = \frac{10}{60}$$

$$\frac{84x - 84x + 42}{2x^2 - x} = \frac{10}{60}$$

$$\frac{42}{2x^2 - x} = \frac{10}{60}$$

$(42)(60) = 10(2x^2 - x)$

$2520 = 20x^2 - 10x$

$20x^2 - 10x - 2520 = 0$

$2x^2 - x - 252 = 0$ (Shown)

Note: When doing speed related quadratic equations, determine who is faster and who is slower. The object that is faster will take a longer time. Thus, to find the difference between the times taken, always take the object take is slower to minus the object that is faster.

Take note of your units. The question can mix dollars and cents together; same goes for hours and minutes.

(d) Solve the equation $2x^2 - x - 252 = 0$, giving the answers correct to three decimal places.

a = 2, b = 1, c = –252

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$x = \frac{-(-1) \pm \sqrt{(-1)^2 - 4(2)(-252)}}{2(2)}$$

$$= \frac{1 \pm \sqrt{2017}}{4}$$

$x = 11.478$ or $x = -10.978$ (3 d.p)

(e) Calculate the time that John took to complete the race, giving your answer in hours, minutes and seconds.

Since speed cannot be negative, x = 11.478

Time taken for John to complete the race

$= \frac{42}{x}$

$= \frac{42}{11.47776}$

= 3.65925 h

= 3 h 39 min 33s

Example: O Level 2014 Paper 2 Question 8

Daniel went on a journey of 90 km.

(a) Daniel took x minutes to drive the first 40 km at a constant speed. Write down an expression, in terms of x, for his speed in km/h for the first 40 km.

Speed $= \frac{Distance}{Time}$

$= \frac{40}{\frac{x}{60}}$

$= 40 \div \frac{x}{60}$

$= 40 \text{ x } \frac{60}{x}$

$= \frac{2400}{x}$ km/h

(b) It took Daniel (x + 15) minutes to drive the rest of the distance at a different constant speed. Write down an expression, in terms of x, for his speed in km/h for this part of the journey.

Remaining distance = 90 – 40

= 50 km

Speed $= \frac{Distance}{Time}$

$= \frac{50}{\frac{(x+15)}{60}}$

$= 50 \div \frac{x+15}{60}$

$= 50 \text{ x } \frac{60}{x+15}$

$= \frac{3000}{x+15}$ km/h

(c) Daniel's speed for the first part of the journey was 9 km/h faster than the second part. Write down an equation in x to represent this information and show that it reduces to $3x^2 + 245x - 12000 = 0$.

$\frac{2400}{x} - \frac{3000}{x+15} = 9$

$\frac{2400(x+15) - 3000x}{x(x+15)} = 9$

$2400x + 36000 - 3000x = 9(x^2 + 15x)$

$-600x + 36000 = 9x^2 + 135x$

$9x^2 + 735x - 36000 = 0$

$3x^2 + 245x - 12000 = 0$ (shown)

(d) Solve the equation $3x^2 + 245x - 12000 = 0$, giving your solutions correct to 2 decimal places.

a = 3, b = 245, c = -12000

$x = \frac{-245 \pm \sqrt{245^2 - 4(3)(-1200\ \ )}}{2(3)}$

$x = \frac{-24\ \ \pm\sqrt{204025}}{2(3)}$

$x = 34.449$ or $x = -116.115$

$= 34.45$ (2 d.p) or $x = -116.12$ (2 d.p)

(e) Calculate Daniel's average speed, in km/h, for the whole journey.

Since $x > 0$, $x = 34.449$ min,

Average speed for the whole journey

$= \frac{Total\ distance}{Total\ time}$

$= \frac{90}{\frac{x}{60} + \frac{x+15}{60}}$

$= 90 \div \frac{2x+15}{60}$

$= 90 \times \frac{60}{2x+15}$

$= \frac{5400}{2x+15}$

When $x = 34.449$,

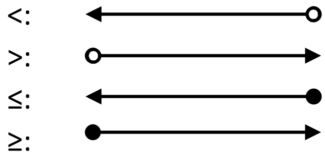
Average speed $= \frac{5400}{2(34.449) + 15}$

$= 64.3647$ km/h

$= 64.4$ km/h (3 s.f.)

N7.6 Solving Linear Inequalities

When drawing number lines for inequalities,



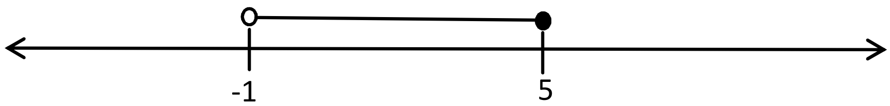
Note: The inequality sign should be flipped when x is on the right hand side instead of the left or when x is negative

Example: Modified from O Level 2013 Paper 1 Question 9 (b)

Solve the inequalities $-5 < 2x - 3 \leq 7$. Represent the solutions on a number line.

$-5 < 2x - 3$	or	$2x - 3 \leq 7$
$-5 + 3 < 2x$	or	$2x \leq 7 + 3$
$-2 < 2x$	or	$2x \leq 10$
$x > -1$	or	$x \leq 5$

Therefore, $-1 < x \leq 5$.



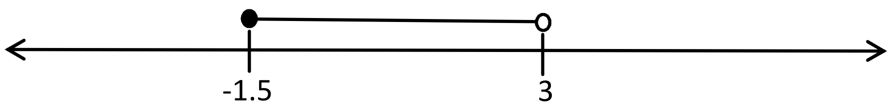
Note: The format in O Level E Maths is 99%: ___< x ≤ ____ of the time. If your inequality signs do not match, please check your answer again. The number on the left MUST BE smaller than the number on the right while fulfilling the inequality signs.

Example: Modified from O Level 2012 Paper 1 Question 13 (b)

Solve the inequalities $-9 \leq 4x - 3 < 9$. Represent the solutions on a number line.

$-9 \leq 4x - 3$	or	$4x - 3 < 9$
$-9 + 3 \leq 4x$	or	$4x < 9 + 3$
$-6 \leq 4x$	or	$4x < 12$
$x \geq -1.5$	or	$x < 3$

Therefore, $-1.5 \leq x < 3$.



N8. Set Language and Notation (Not Tested in N Levels)

N8.1 Definitions of Set Language and Notation

$\mathcal{E}$	Universal set
$\in$	is an element of
$\notin$	Is not an element of
A’	Complement of set A
n(A)	Number of elements in set A
$\emptyset$ or { }	Empty or null set
$\subseteq$	Is a subset of
$\nsubseteq$	Is not a subset of
$\subset$	Is a proper subset of
$\not\subset$	Is not a proper subset of
$\cup$	Union of A and B
$\cap$	Intersection of A and B

A **set** is a collection of objects. A set can be set to defined specific items required. Example: A = {x: x is a prime number from $1 < x < 20$}

The **universal set**, denoted by $\mathcal{E}$ is the set that contains all elements being considered in a given discussion

$\mathcal{E}$ = {x: x is a prime number from $1 < x < 20$}
$\mathcal{E}$ = {2, 3, 5, 7, 11, 13, 17, 19}

An **element** of a set denoted by the symbol $\in$ means that it is an object that belongs to the set. For the example above, 2 is an element of A or $2 \in A$. If the object does not belong to the set, we say that it is **not the element** of the set, which is denoted by the symbol $\notin$. In the above example, 6 is not an element of A or $6 \notin A$.

A’ is the **complement** of the A. It is the numbers in the universal set that is not in A. In the above example, A’ = {4, 6, 8, 9, 10, 12, 14, 15, 16, 18}

An **empty set** or a null set is a set containing no elements. It is denoted by $\emptyset$ or { }. Example A = {x: x is an even prime number from $5 < x < 20$}. Since none of the numbers are able to fulfil the requirements, A = $\emptyset$. Note: It is WRONG to write {$\emptyset$}.

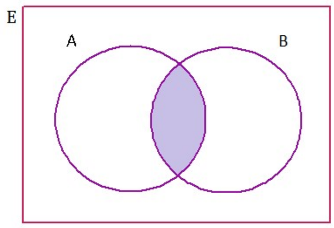
If every element of a set A is also an element of a set B, then A is the **subset** of B, which is denoted by A ⊆ B. In short, it is A has some or all of the elements that can be found in B. Example: If we have the set B = {2, 3, 5, 7, 11, 13, 17, 19}. Possible subsets (A) will be {2, 3, 5, 7, 11, 13, 17, 19} (equal set), {2, 3, 5}, {13, 17}, {11}, ∅ etc. An equal set can also be a subset of B. If A is **not the subset of** B denoted by A ⊈ B, it contains elements that cannot be found in B. Examples of A ⊈ B is A = {1, 2, 3}. Since 1 is not part of B, A ⊈ B.

Set A is a **proper subset** of B if every element of set A is also an element of set B and set B has more elements than set A (they are not equal). In short, A has only some of the elements that can be found in B. We write A ⊂ B to denote A is a proper subset of B. Example: A = {1, 2, 3} and B = {1, 2, 3, 4, 5, 6}. Since all the elements of A are present in B, it is said that A ⊂ B.

N8.2 Venn Diagram

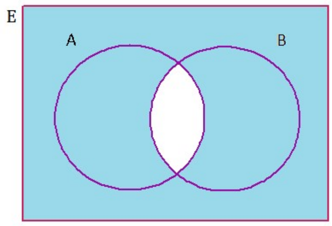
Example: Shade (A∩B)’

Step 1: Draw A∩B



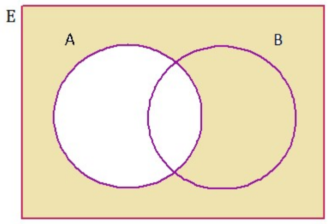
Note: Workings need not be shown if you can visualise

Step 2: Draw (A∩B)’

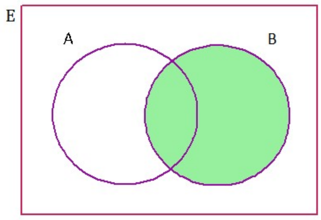


Example: Shade A' ∩ B

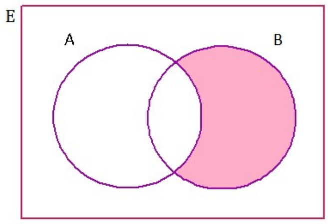
Step 1: Draw A'



Step 2: Draw B



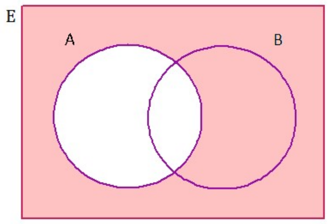
Step 3: Draw A' ∩ B



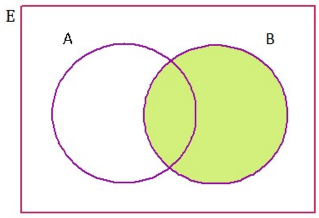
Note: When doing Venn diagram on exam scripts, lines are used to depict the shaded parts instead. Draw number 1 with a colour pencil using diagonal lines. Draw number 2 with another colour pencil using diagonal lines not the same as number 1. The answer will be the region with both colours.

Example: Shade A’ ∪ B

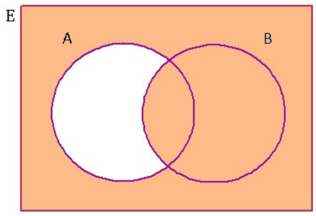
Step 1: Shade A’



Step 2: Shade B



Step 3: Shade A’ ∪ B



Note: When doing Venn diagram on exam scripts, lines are used to depict the shaded parts instead. Draw number 1 with a colour pencil using diagonal lines. Draw number 2 with another colour pencil using diagonal lines not the same as number 1. As long as any region is coloured, that will be the answer.

N8.3 Word Problems Involving Set Language and Notation

Example: CASCO O Level Mathematics Companion Book 2 Page 185 Question 24

$\mathcal{E}$ = {x: x is an integer and $3 \leq x \leq 14$}
A = {x: x is a multiple of 3}
B = {x: x is a factor of 12}
C = {x: x is a prime number}

(a) List the elements of the set

$\mathcal{E}$ = {3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14}
A = {3, 6, 9, 12}
B = {3, 4, 6, 12}
C = {3, 5, 7, 11, 13}

Note: Some numbers are excluded as you are required to follow the range of the universal set.

(i) B∩C
B∩C = {3}

(ii) A'∩B
A' = {4, 5, 7, 8, 10, 11, 13, 14}
A'∩B= {4}

(iii) (A∪C)'
A∪C = {3, 5, 6, 7, 9, 11, 12, 13}
(A∪C)' = {4, 8, 10, 14}

(b) Find the value of

(i) n(A∪B∪C)
A∪B∪C = {3, 4, 5, 6, 7, 9, 11, 12, 13}
n(A∪B∪C) = 9

(ii) n(A∩B∩C)
A∩B∩C = {3}
n(A∩B∩C) = 1

(iii) n(A∩B)'
A∩B = {3, 6, 12}
(A∩B)' = {4, 5, 7, 8, 9, 10, 11, 13, 14}
n(A∩B)' = 9

(iv) n(B'∩C)
B' = {5, 7, 8, 9, 10, 11, 13, 14}
B'∩C = {5, 7, 11, 13}
n(B'∩C) = 4

N9. Matrices (Not Tested in N Level)

N9.1 Order of Matrices

The order of the Matrices is written as a x b, where a denotes the number of rows while b denotes the number of columns.

$$\begin{bmatrix} a & b \\ c & d \\ e & f \end{bmatrix} \rightarrow \text{Row}$$

$\downarrow$ Column

In this example, the order is 3 x 2

N9.2 Addition and Subtraction of Matrices

$$\begin{bmatrix} a & b \\ c & d \end{bmatrix} + \begin{bmatrix} e & f \\ g & h \end{bmatrix} = \begin{bmatrix} a+e & b+f \\ c+g & d+h \end{bmatrix}$$

$$\begin{bmatrix} a & b \\ c & d \end{bmatrix} - \begin{bmatrix} e & f \\ g & h \end{bmatrix} = \begin{bmatrix} a-e & b-f \\ c-g & d-h \end{bmatrix}$$

N9.3 Multiplication of Matrices

$$a\begin{bmatrix} b \\ c \\ d \end{bmatrix} = \begin{bmatrix} ab \\ ac \\ ad \end{bmatrix}$$

When multiplying matrices together, the order of the matrices plays an important role. For example, when multiplying two matrices of order 2 x 3 and 3 x 1 together,

2 x 3 and 3 x 1

The numbers in purple must be the same before can multiply the matrices together. The numbers in blue will determine the order of the final matrix. In this case the order of the final matrix is 2 x 1. The multiplying of the matrices always goes from left to right →, then up to down ↓.

Example: $\begin{bmatrix} a & b \\ c & d \end{bmatrix} \begin{bmatrix} e & f \\ g & h \end{bmatrix} = \begin{bmatrix} ae+bg & af+bh \\ ce+dg & cf+dh \end{bmatrix}$

(2 x 2) (2 x 2) (2 x 2)

Example: $\begin{bmatrix} a & c & e \\ b & d & f \end{bmatrix} \begin{bmatrix} g & h \\ i & j \\ k & l \end{bmatrix} = \begin{bmatrix} ag+ci+ek & ah+cj+el \\ bg+di+fk & bh+dj+fl \end{bmatrix}$

(2 x 3) (3 x 2) (2 x 2)

Example: $\begin{bmatrix} a & b \\ c & d \\ e & f \end{bmatrix} \begin{bmatrix} g & h & i \\ j & k & l \end{bmatrix} = \begin{bmatrix} ag+bj & ah+bk & ai+bl \\ cg+dj & ch+dk & ci+dl \\ eg+fj & eh+fk & ei+fl \end{bmatrix}$

(3 x 2) (2 x 3) (3 x 3)

Example: $\begin{bmatrix} d & e & f \end{bmatrix} \begin{bmatrix} a \\ b \\ c \end{bmatrix} = \begin{bmatrix} da+eb+fc \end{bmatrix}$

(1 x 3) (3 x 1) (1 x 1)

N9.4 Problems Involving Matrices

Example: O Level 2014 Paper 1 Question 21

Lin and Hadi save 10 cent, 20 cent and $1 coins. The number of coins they saved last month is given by the matrix **C**.

$$C = \begin{array}{c} \\ \\ \end{array}\begin{array}{ccc} 10c & 20c & \$1 \end{array}$$
$$C = \begin{pmatrix} 29 & 10 & 5 \\ 30 & 6 & 8 \end{pmatrix} \begin{array}{l} \text{Lin} \\ \text{Hadi} \end{array}$$

(a) A 10c coin has a mass of 2.6 grams
A 20c coin has a mass of 4.5 grams
A $1 coin has a mass of 6.3 grams
Represent these masses in a 3 x 1 column matrix D.

$$D = \begin{bmatrix} 2.6 \\ 4.5 \\ 6.3 \end{bmatrix}$$

(b) Evaluate the matrix E = CD

E = CD

$$= \begin{bmatrix} 29 & 10 & 5 \\ 30 & 6 & 8 \end{bmatrix} \begin{bmatrix} 2.6 \\ 4.5 \\ 6.3 \end{bmatrix}$$
$$= \begin{bmatrix} (29\ x\ 2.6) + (10\ x\ 4.5) + (5\ x\ 6.3) \\ (30\ x\ 2.6) + (6\ x\ 4.5) + (8\ x\ 6.3) \end{bmatrix}$$
$$= \begin{bmatrix} 151.9 \\ 155.4 \end{bmatrix}$$

(c) State what the elements of E represent.

The elements of E represents the total mass, in grams, of the coins saved last month by Lin and Hadi respectively.

(d) The elements of the matrix V, where V = CF, represents the values, in cents of the coins saved last month by Lin and Hadi. Write down the matrix F.

$$F = \begin{bmatrix} 10 \\ 20 \\ 100 \end{bmatrix}$$

(e) At the next month, the girls saved more coins. The total mass of the coins increased by 20%. Calculate the new mass of the coins saved by the two girls respectively [not in O level TYS].

$$E = \begin{bmatrix} 1.2 & 0 \\ 0 & 1.2 \end{bmatrix} \begin{bmatrix} 151.9 \\ 155.4 \end{bmatrix}$$
$$= \begin{bmatrix} (1.2\ x\ 151.9) + (0\ x\ 155.4) \\ (0\ x\ 151.9) + (1.2\ x\ 155.4) \end{bmatrix}$$
$$= \begin{bmatrix} 182.28 \\ 186.47 \end{bmatrix}$$

N10. Problems in Real World Context

N10.1 Simple Interest

$I = \frac{PRT}{100}$

Where P = principal ($)
R = rate of interest (%)
T = time (years)

Example: Ahmad invested $8000 in a savings plan that promises a return of 7% simple interest per annum. Given that Ahmad invested his money in this saving plans for 9 years 9 months, what is the total amount he can get from the saving plans?

$$\text{Interest} = \frac{PRT}{100}$$
$$= \frac{8000 \times 7 \times 9\frac{9}{12}}{100}$$
$$= \$5460$$

Total amount = 8000 + 5460
= $13460

Note: Question involving simple interest usually requires you to calculate total amount, so please read the question properly. If years and months are involved in a question, you are required to convert it to years only.

N10.2 Compound Interest

$\text{Amount} = P\left(1 + \frac{R}{100}\right)^n$

Where P = principal ($)
R = rate of interest (%)
T = time (years, unless otherwise stated)

Example: Mary invested her savings of $5000 in Money to Riches Bank which provides an interest that is compounded quarterly at a rate of 1% per annum. She invested her savings for 3 years. What is the total amount of interest Mary can obtain?

$$\text{Total amount} = P\left(1 + \frac{R}{100}\right)^n$$
$$= 5000\left(1 + \frac{1 \div 4}{100}\right)^{3 \times 4}$$
= $5152.08 (corrected to the nearest cents)

Interest obtained = $5152.08 - $5000
= $152.08

Note: When question state that interest is 1% per annum, compounded quarterly. Interest rate must divide by 4 as it is compounded quarterly. Time (period) will have to multiply by 4 since there are 4 accounting cycles in a year given that it is compounded quarterly. Question involving compound interest usually requires you to calculate interest so please read the question carefully.

N10.3 Money Exchange

Example: O level 2014 Paper 1 Question 2

Hasim is travelling from Singapore to Switzerland.
In Singapore, the exchange rate is 1 Singapore dollar = 0.765 Swiss Franc.
In Switzerland, the exchange rate is 1 Swiss Franc = 1.294 Singapore Dollars

Hasim wants to change 500 Singapore Dollars into Swiss Francs. How many more Swiss Francs will he get by changing his money in Switzerland?

SGD \$1 = 0.765 Swiss Francs
SGD \$500 = 382.50 Swiss Francs

1 Swiss Franc = \$1.294 SGD
\$1.294 SGD = 1 Swiss Franc
1 SGD = $\frac{1}{1.294}$ Swiss Franc
\$500 SGD = $\frac{1}{1.294}$ x 500 Swiss Franc
= 386.40 Swiss Franc (2 d.p.)

Amount of money he will get more = 386.40 – 382.50
= 3.90 Swiss Francs

Example: O Level 2012 Paper 2 Question 4 (a)

One day the exchange rate between pounds (£) and Singapore dollars (\$) was £1 = \$2.10. On the same day, the exchange rate between euros (€) and pounds was €1 = £0.93.

a) Ann changed £200 into Singapore dollars. Calculate how many dollars she received.

£1 = \$2.10
£200 = \$420

b) Bill converted \$500 into euros. Calculate how many euros he received, correct to the nearest euro.
£1 = \$2.10
\$2.10 = £1
\$1 = £$\frac{1}{2.10}$
\$500 = £$\frac{1}{2.10}$ x 500
= £238.095 (5 s.f.)
€1 = £0.93
£0.93 = €1
£1 = €$\frac{1}{0.93}$
£238.095 = €$\frac{1}{0.93}$ x 238.095
= €256 (nearest euros)

N10.4 Hire Purchase

Example: O level 2014 Paper 1 Question 11

The cash price of a washing machine is \$980. The hire purchase price of the washing machine is \$1089. The hire purchase price is a deposit of 15% of the cash price plus 12 equal monthly payments.

Calculate one monthly payment.

Deposit = $\frac{15}{100}$ x \$980
= \$147

Remaining amount = \$1089 – \$147
= \$942

Monthly payment = $\frac{942}{12}$
= \$78.50

Example: Xiao Ming borrowed a sum of money from the bank. He is due to repay the bank with interest of an amount totalling to \$15079.23 within a year. Given that his **LAST** instalment is \$1255.75, how much was his monthly instalments?

Monthly instalment = $\frac{15079.23 - 1 \quad .75}{12 - 1}$
= \$1256.68

Note: When dealing with money, it can only involves 2 decimal places. That is why for this question, it cannot be 15079.23 ÷ 12 as it will give an answer of 4 decimal places.

N10.5 Taxation

When doing questions on tax, always determine the chargeable income first.

Chargeable income = Total income – Total relief

Example: CASCO 3B Revised Edition (2010) Page 259 Question 32

Mr Ng earns $3250 a month. He has two children and a wife to support. He donates 2% of his monthly salary to charitable organisations. He contributes $7800 towards CPF, pay $600 personal life insurance. Calculate the amount of tax Mr Ng has to pay if he is entitled to the following reliefs: Personal $3000; wife $2000; children, $2000 each

Chargeable Income ($)	Tax Rate (%)	Tax Payable ($)
On the first 20 000 On the next 10 000	0 3.50	0 350
On the first 30 000 On the next 10 000	 5.50	350 550

Annual income = 3250 x 12
= $39 000

Total reliefs = ($\frac{2}{100}$ x 39000) + 7800 + 600 + 3000 + 2(2000) + 2000
= $18 180

Annual chargeable income = 39000 – 18180
= $20 820

Since first $20000 is tax free,

Tax = $\frac{3.5}{100}$ x (20820 – 20000)
= $28.70

N10.6 Utilities Bills

Calculating Water and Electricity Consumption

Example: O level 2014 Paper 2 Question 2

The table shows the electricity consumption, in gigawatts hours (GWh), in Singapore in 2010.

Domestic	Manufacturing	Other industries	Total
7304.5	16693.0	17202.3	41199.8

(a) Convert the total amount of electricity consumed into kilowatts hours (kWh). Give your answer in standard form correct to 3 significant figures.

41 199.8 GWh = 41199.8 x 10^9 Wh
= 4.11998 x 10^{13} Wh
= 4.11998 x 10^{10} x 10^3 Wh
= 4.12 x 10^{10} KWh (3 s.f.)

(b) Calculate the percentage of the total that was consumed by manufacturing.

Percentage that was consumed by manufacturing = $\frac{16693\ GWh}{411998\ GWh}$ x 100%
= 40.5% (3 s.f.)

(c) The population of Singapore in 2010 was 5.077 million. Calculate the average amount of domestic electricity consumed per person. Give your answer to the nearest kWh.

Average amount of domestic electricity consumed per person

= $\frac{7304.5\ GWh}{5.077\ x\ 10^6}$

= $\frac{7304.5\ x\ 10^9\ Wh}{5.077\ x\ 10^6}$

= $\frac{7304.5\ x\ 10^6\ kWh}{5.077\ x\ 10^6}$

= 1438.74 kWh

= 1439 kWh

(d) The amount of electricity consumed by other industries in 2009 was 13628.0 GWh. Calculate the percentage increase in electricity consumption by other industries from 2008 to 2010.

% increase = $\frac{17202.3-13628.0}{13628.0}$ x 100%
= 26.2% (3 s.f.)

(e) From 2000 to 2010, the amount of domestic electricity consumption increased by 27.6%. Calculate the domestic electricity consumption in 2000. Give your answer to the nearest GWh.

Let the amount of domestic electricity consumption in 2000 be x GWh.

$\frac{7304.5 - x}{x}$ x 100% = 27.6%

$\frac{7304.5 - x}{x}$ = 0.276x

7304.5 – x = 0.276x

1.276x = 7304.5

x = 5725 GWh (nearest GWh)

ALTERNATIVE WORKINGS

127.6% → 7304.5 GWh

1% → $\frac{7304.5}{127.6}$ GWh

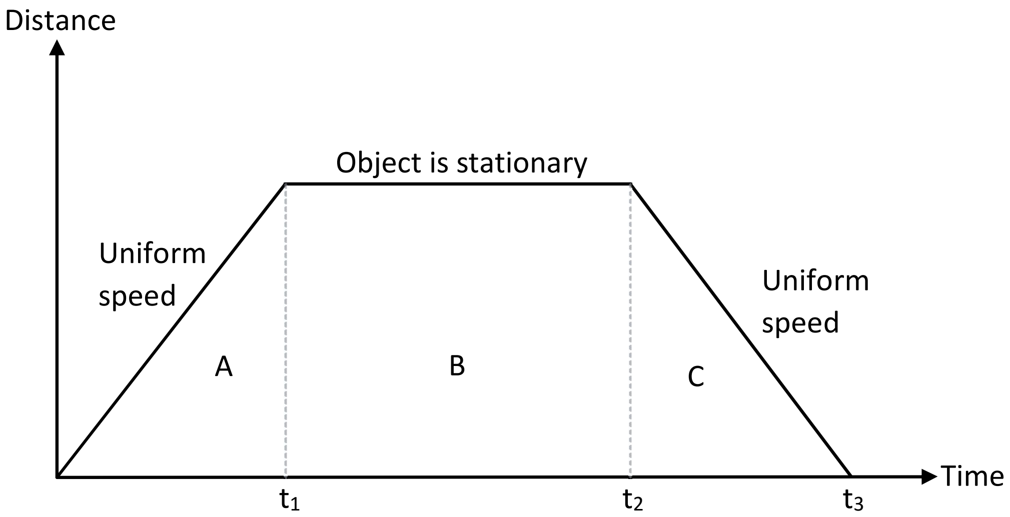
100% → $\frac{7304.5}{127.6}$ GWh x 100%

= 5725 GWh (nearest GWh)

N10.7 Kinematics (Not Tested in N Levels)

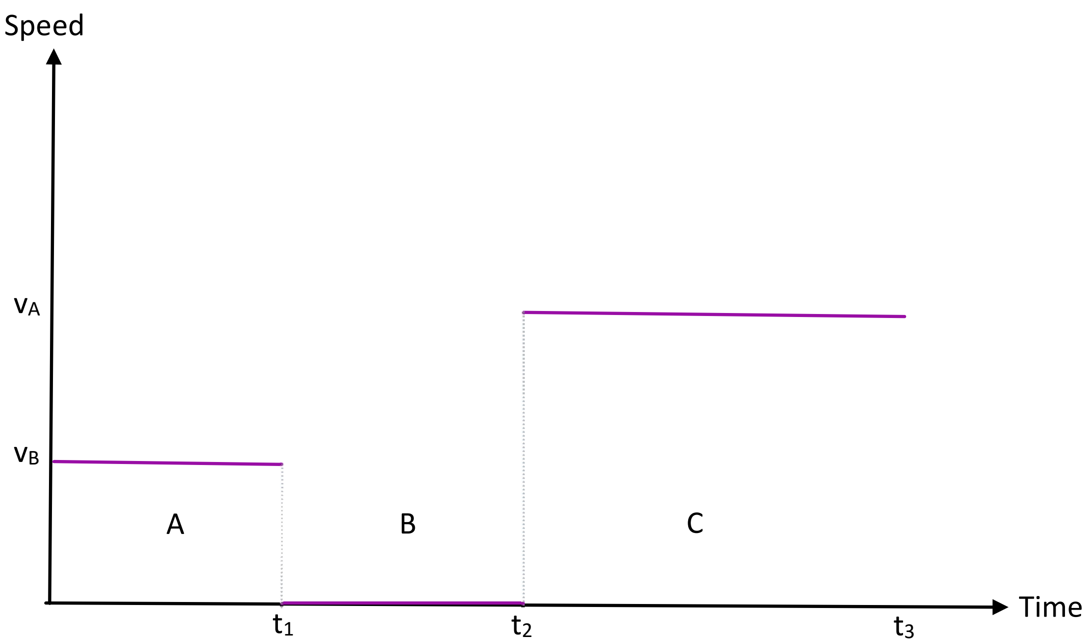
Distance-Time Graph Involving Constant Speed

In a distance-time graph, the speed of the graph can be determined by finding the gradient of the straight line (constant speed).



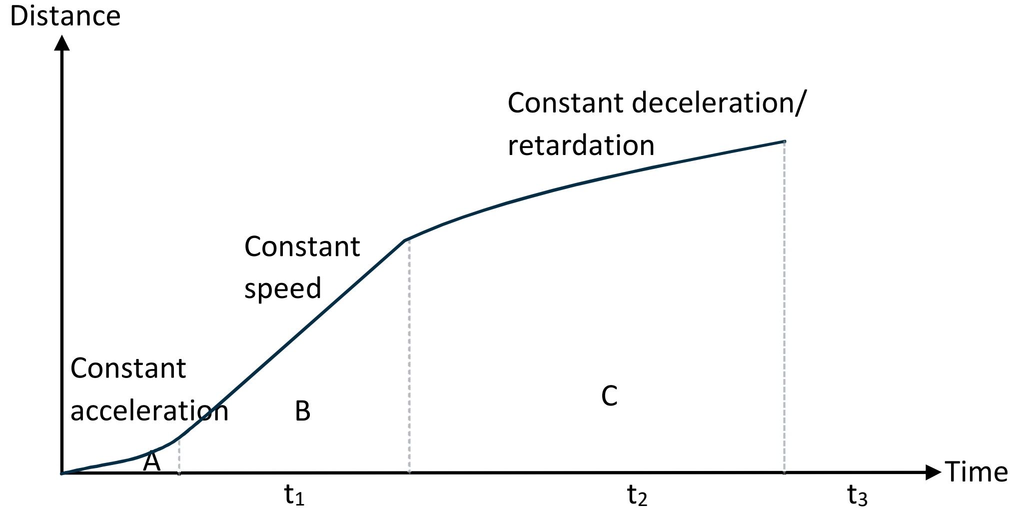
Conversion of Distance-Time Graph to Speed-Time Graph

For part A and C of the graph, since the object is travelling at a uniform speed, it will appear as a straight line. For part B of the graph, since the object is stationary, no speed is involved.



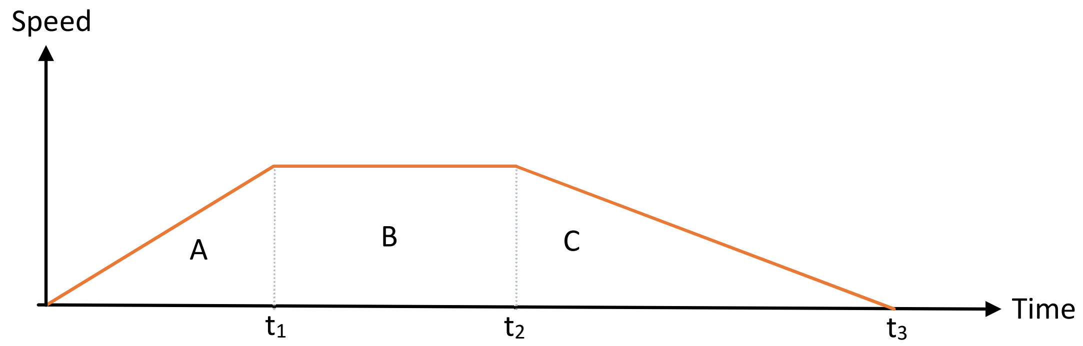
<u>Distance-Time Graph Involving Constant Acceleration</u>

In a distance-time graph, the speed of the graph can be determined by drawing a tangent line since it involves a curve (constant acceleration).



<u>Conversion of Distance-Time Graph to Speed-Time Graph</u>

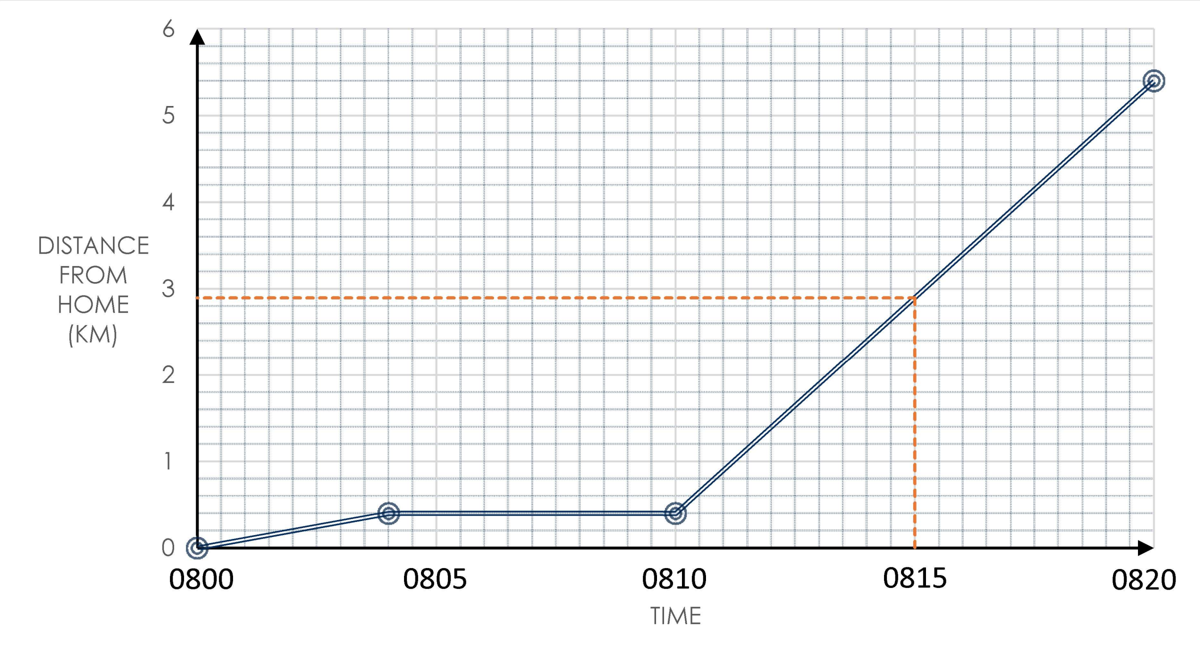
For part A of the graph, since the object is travelling at a constant acceleration, the straight line will have a positive gradient. For part B of the graph, since the object is travelling at constant speed, it will be a horizontal line. For part C of the graph, since the object is retarding constantly, a negative gradient is formed.



Word Problems Involving Distance-Time Graphs

Example: O Level 2010 Paper 1 Question 17

The graph shows Alison's journey from home to school. She left home at 0800, walked to the bus stop and travelled the rest of the way on the bus.



(a) How long did she wait at the bus stop?

Time waited at the bus stop = 08 10 – 0804
= 6 min

(b) How far was she from school at 08 15? Shown in orange.
Distance from school = 5.4 – 2.9
= 2.5 km

(c) Find the speed of the bus.

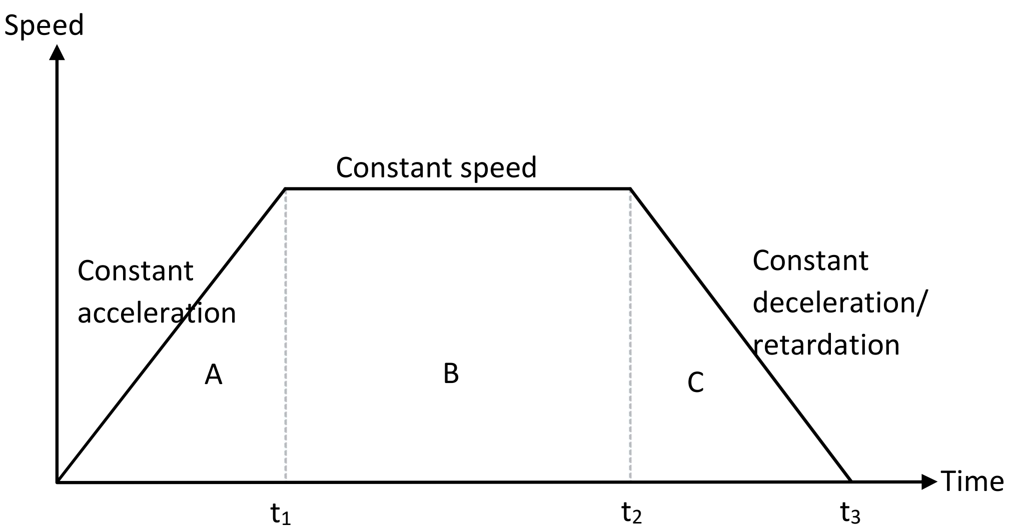
Speed of the bus = $\frac{(5.4-0.4)km}{\frac{10}{60}h}$
= 30 km/h

(d) Alison's brother, Sam, also left home at 0800. He cycled to school at a constant speed of 20 km/h. Show his journey on the above graph.

Time taken = $\frac{5.4\ km}{20\ km/h}$
= 0.27 h
= 16.2 min

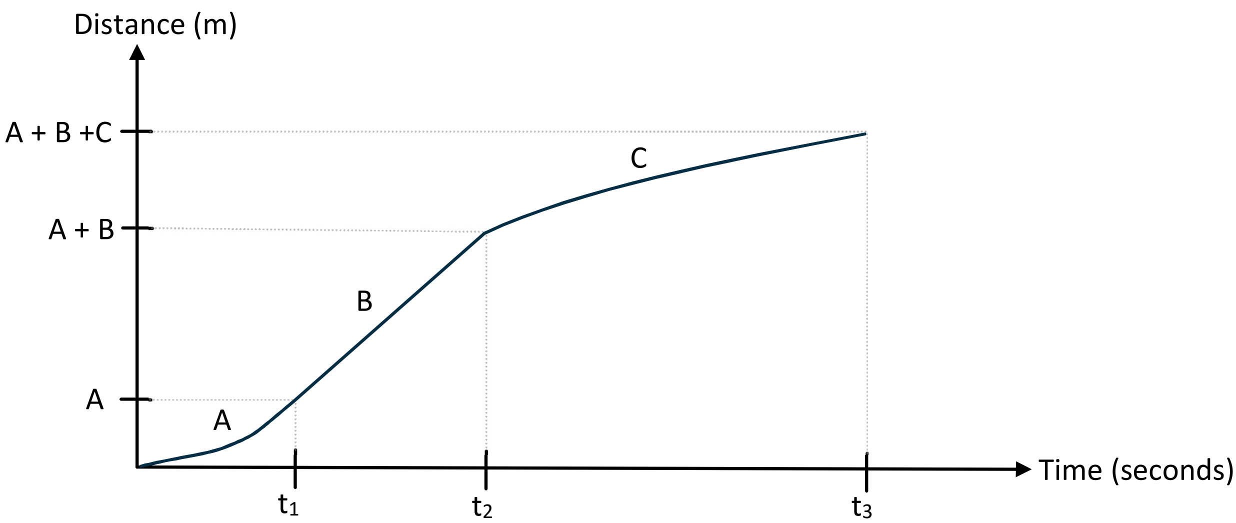
Speed-Time Graph

In a speed-time graph, the acceleration of the graph can be determined by finding the gradient of the curve. The distance can be determined by finding the area under the curve.

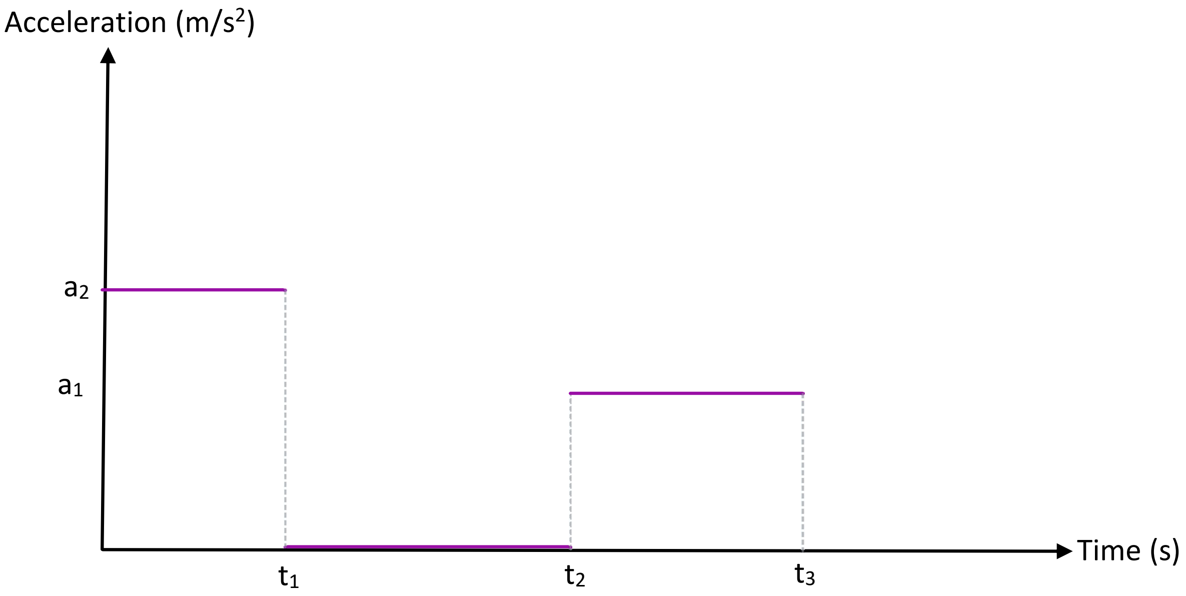


Conversion of Speed-Time Graph to Distance-Time Graph

Since part A of the speed-time graph shows constant acceleration, it will be a slow to fast curve on the distance-time graph. Since part B of the speed-time graph shows constant speed, it will be a linear line on the distance-time graph. Since part C of the speed-time graph shows constant retardation, it will be a fast to slow curve on the distance-time graph.



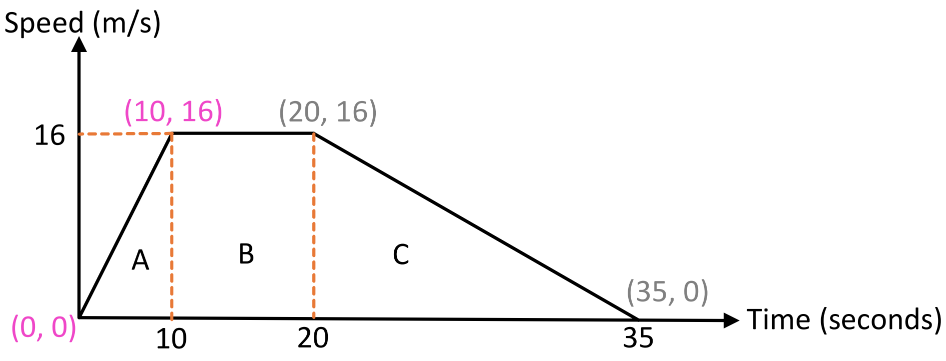
Conversion of Speed-Time Graph to Acceleration-Time Graph



Word Problem Involving Speed-Time Graph

Example: Modified from O Level 2013 Paper 1 Question 24

The diagram shows the speed-time graph for a car journey between two road junctions.



(a) Calculate the acceleration of the car after 5 seconds. Shown in purple.

Acceleration = $\frac{16-0}{10-0}$

= 1.6 m/s^2

(b) Calculate the acceleration of the car from t = 10 and t = 20

Note: Since it is a horizontal line, it means that the car is travelling at constant speed, hence there is no acceleration.

Acceleration = 0 m/s^2

(c) Calculate the retardation when t = 30. Shown in grey.

Gradient = $\frac{16-0}{20-35}$

= -1.07 m/s^2 (3 s.f.)

∴ Retardation = 1.07 m/s^2

Note: If retardation is written as -1.07 m/s^2, the answer will be wrong as retardation means negative gradient.

(d) Calculate the total distance travelled between the two road junctions.

Total distance = $\frac{1}{2}$ x [(20 - 10) + 35] x 16

= 360 m

(e) Calculate the average speed for the whole journey.

Average speed = $\frac{360}{35}$

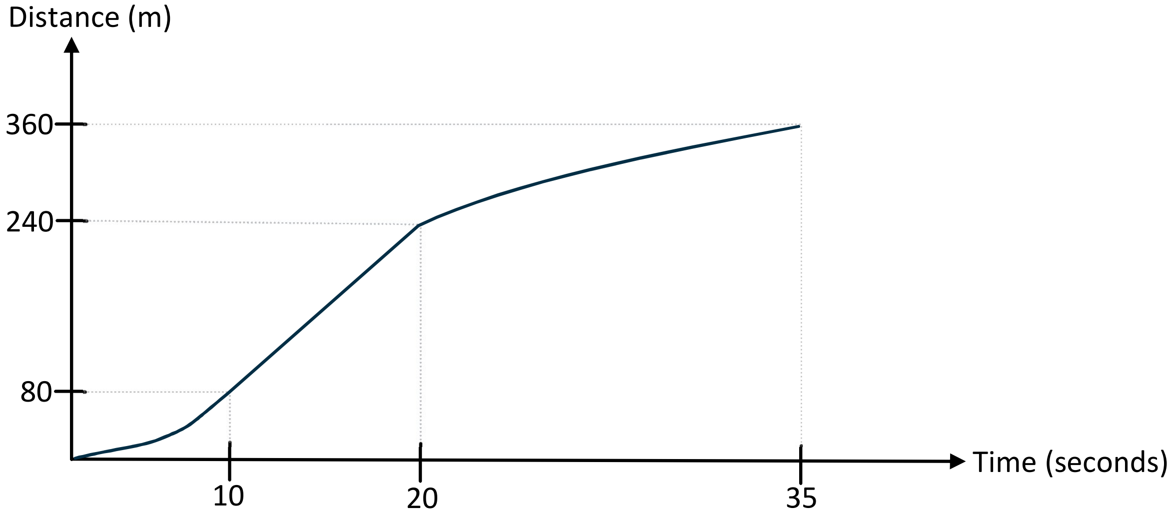
= 10.3 m/s

(f) Sketch the distance-time graph for the journey.

Distance of A = $\frac{1}{2}$ x 10 x 16
= 80 m

Distance of B = (20 – 10) x 16
= 160 m

Distance of C = $\frac{1}{2}$ x (35 – 20) x 16
= 120 m

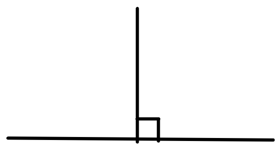


Since part A of the speed-time graph shows constant acceleration, it will be a slow to fast curve on the distance-time graph. Since part B of the speed-time graph shows constant speed, it will be a linear line on the distance-time graph. Since part C of the speed-time graph shows constant retardation, it will be a fast to slow curve on the distance-time graph.

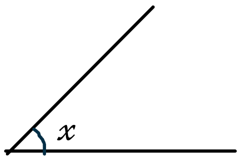
Note: Distance is a cumulative curve. A distance-time graph never have a negative gradient!

G1. Angles, Triangles and Polygons

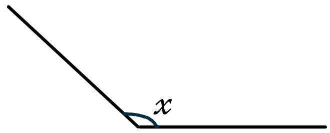
G1.1 Common Characteristics of Angles, Triangles and Polygons



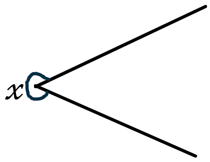
Right angle (90°)



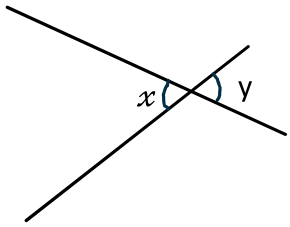
Acute angle ($< 90^\circ$)



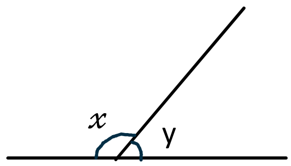
Obtuse angle ($90^\circ < x < 180^\circ$)



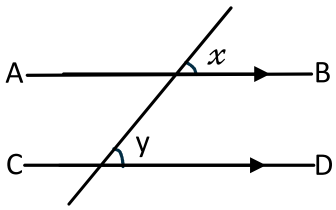
Reflex angle
($180^\circ < x < 360^\circ$)



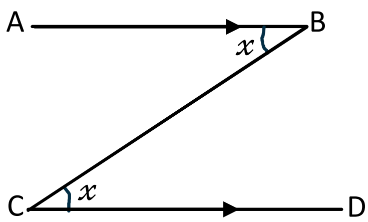
Vertically opposite angles
$\angle x = \angle y$



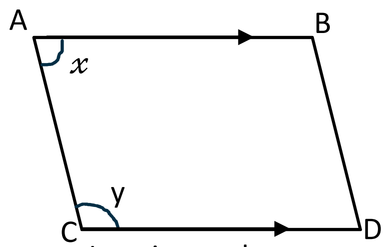
$\angle x = 180^\circ - y^\circ$
(angle on a straight line)



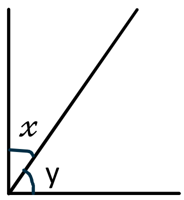
Corresponding angles
$\angle x = \angle y$
(Corr $\angle$, AB // CD)



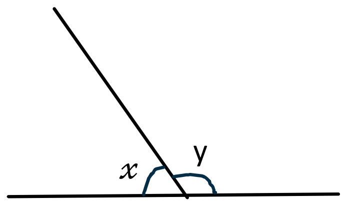
Alternate angles
$\angle x = \angle y$
(alt $\angle$, AB//CD)



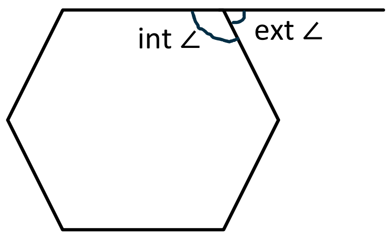
Interior angles
$\angle x + \angle y = 180^\circ$
(int $\angle$, AB//CD)



Complementary angle
$\angle x + \angle y = 90^\circ$

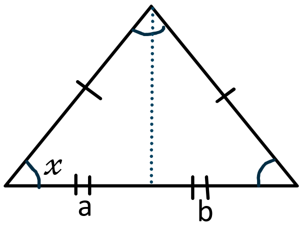


Supplementary angle
$\angle x + \angle y = 180^\circ$

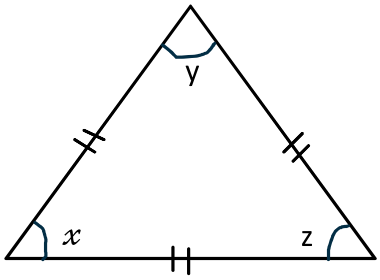


Interior and exterior
angles of a polygon
int $\angle$ + ext $\angle$ = 180°

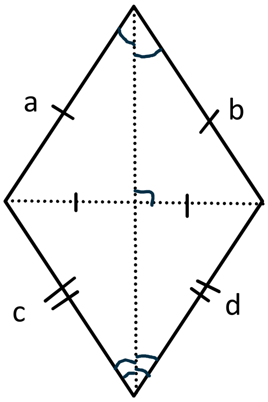
G1.2 Properties of Triangles and Quadrilaterals



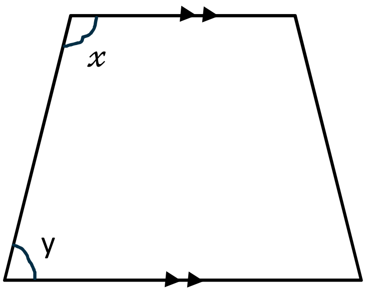
Isosceles triangle
$\angle x = \angle y$
(base angle of isosceles triangle)
length of a = length of b



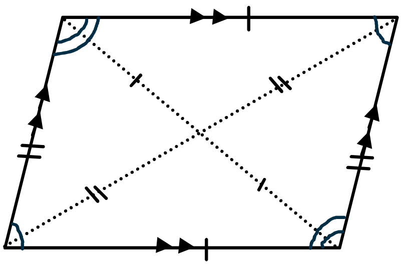
Equilateral triangle
$\angle x = \angle y = \angle z = 60^o$



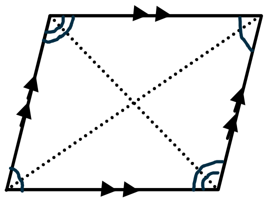
Kite
Quadrilateral with no parallel sides. It has two pairs of equal adjacent sides and one pair of equal opposite angles.



Trapezium
Quadrilateral with one pair of parallel opposite sides



Parallelogram
Quadrilateral with two pairs of parallel opposite sides. Opposite sides are equal in length. Opposite angles are equal.

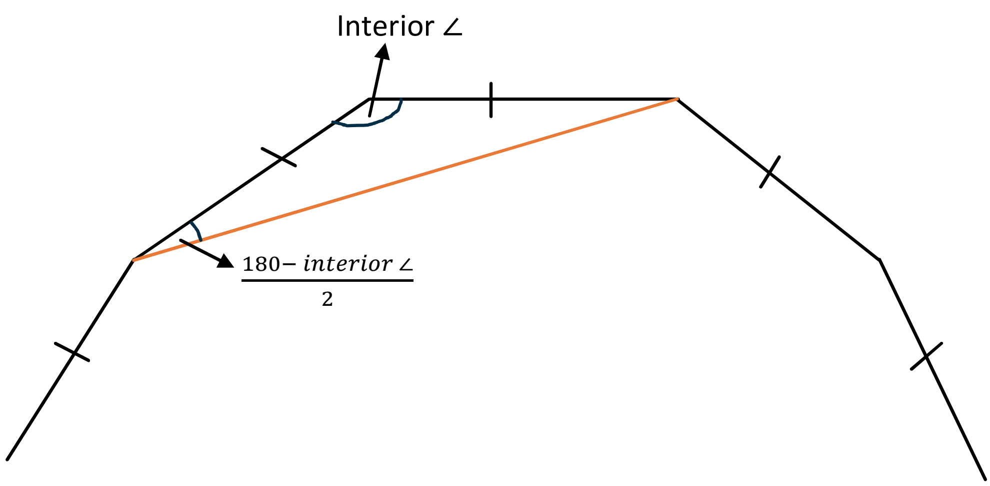


Rhombus (slanted square)
Quadrilateral with two pairs of parallel opposite sides. It has four equal sides. Opposite angles are equal. The diagonals bisect each other at right angles. Diagonals bisect the interior angles.

G1.3 Polygons

Number of Sides	Name of Polygon
3	Triangle
4	Quadrilateral
5	Pentagon
6	Hexagon
7	Heptagon
8	Octagon (Remember: octopus)
9	Nonagon
10	Decagon (Remember: decade)

- ☺ Total sum of interior angles = (n – 2)(180)
- ☺ Sum of one interior angle = $\frac{(n-2)(180)}{n}$
- ☺ Sum of one exterior angle = $\frac{360}{n}$, where n = number of sides
- ☺ Number of sides = $\frac{360}{\text{exterior angle}}$
- ☺ Exterior angle = 180° – interior angle (angle on a straight line)
- ☺ For n – sided regular polygons, always look out for isosceles triangle.



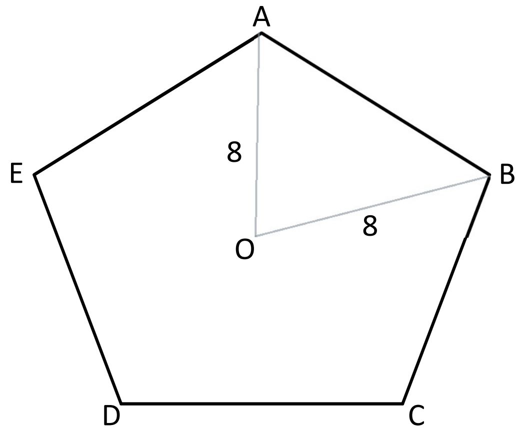
Example: N Level 2012 Paper 1 Question 21

(a) The interior angle of a regular polygon is 156°. How many sides has the polygon?

Exterior angle = 180° – interior angle
= 180° – 156°
= 24° (∠ on a str line)

Number of sides = $\frac{360}{exterior\ angle}$
= $\frac{360}{24}$
= 15

(b) ABCDE is a regular pentagon, centre O. The distance from the centre of the pentagon to each vertex is 8 cm. Calculate the area of the pentagon



∠AOB = 360 ÷ 5
= 72°

Note: Pentagon can be divided into 5 triangles

Area of the pentagon
= 5 x Area of ΔAOB
= 5 x $\frac{1}{2}$ x 8 x 8 x sin72
= 152 cm^2 (3 s.f.)

Example: O Level 2013 Paper 2 Question 7 (a)

(a) Find the interior angle of a regular 20-sided polygon.

Interior angle = $\frac{(n-2) x\ 180}{n}$

= $\frac{(20-2)\ x\ 180}{20}$

= 162°

(b) An n-sided polygon has 4 interior angles measuring 120° each. The remaining interior angles all measure p° each. Find an expression for p in terms of n.

(n – 2) x 180 = 4 (120) + (n – 4)(p)

180n – 360 = 480 + (n – 4)(p)

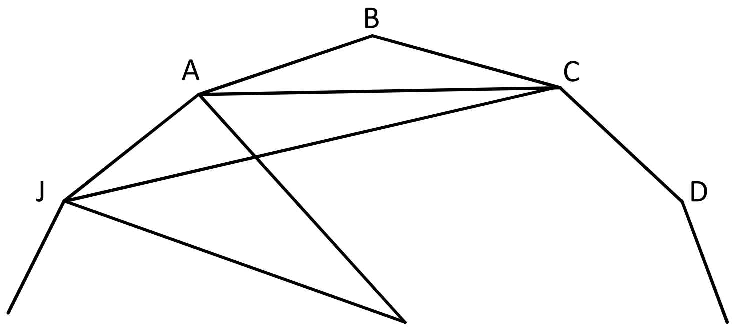
180n – 360 – 480 = (n-4)p

(n – 4)p = 180n – 840

p = $\frac{180n - 840}{n-40}$

Example: N Level 2010 Paper 1 Question 24

The diagram shows part of a regular decagon ABCDEFGHUIJ. O is the centre of the circle that passes through A, B, C, D, E, F, G, H, I and J.



Note: Since it is a regular decagon, JA = AB = BC = CD

(a) Find angle ABC.

$\angle ABC = \frac{(n-2)\ x\ 180}{n}$

$= \frac{(10-2)\ x\ 180}{10}$

$= 144^o$

(b) Find angle AOJ.

$\angle AOJ = \frac{360}{10}$

$= 36^o$ ($\angle$ at a point)

Note: Decagon can be split into 10 triangles.

(c) Find angle CAJ.

$\angle ABC = 144^o$

$\angle BAC = \frac{180-144}{2}$

$= 18^o$ (base $\angle$ of isos Δ)

$\angle CAJ = 144^o - 18^o$

$= 126^o$

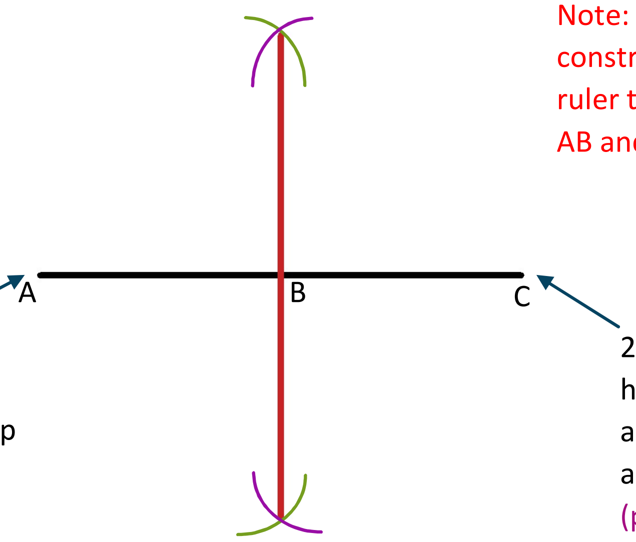
(d) Give the name of the special quadrilateral ABCJ.

Trapezium

Note: AB is parallel to CJ

G1.4. Construction of Perpendicular Bisector

Note: To know if the construction is correct, use a ruler to check that the length of AB and BC is the same.



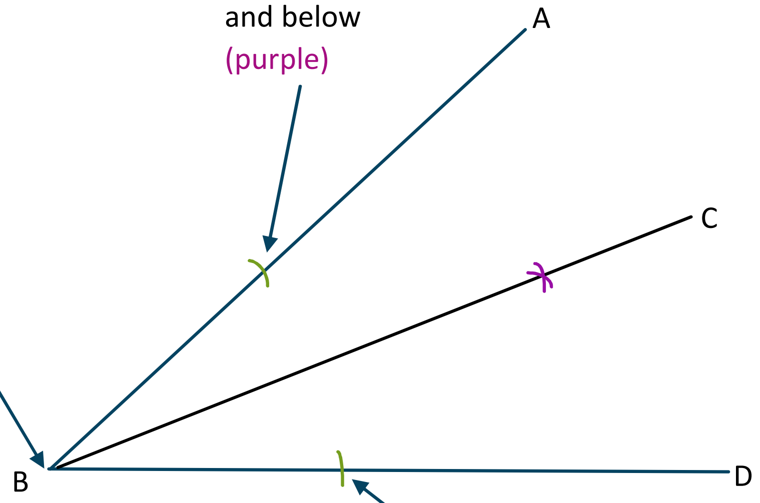
1. Needle here. Mark an arc on top and below (green)

2. Needle here. Mark an arc on top and below (purple)

1.5 Construction of Angle Bisector

Note: To check that construction is correct, use a protractor to check that the angle of ABC and CBD is the same.

2. Needle here. Mark an arc on top and below (purple)



1. Needle here. Mark an as shown (green)

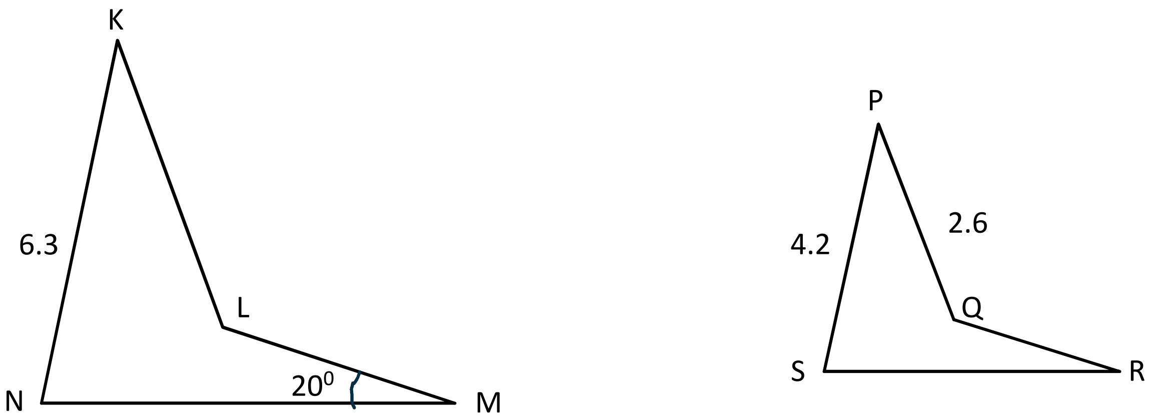
2. Needle here. Mark an arc as shown. (purple)

G2. Congruency and Similarity

G2.1 Enlargement and Reduction of Plane Figure by Scale Factor

Enlargement and reduction of plane figure by scale factor always involve multiplication and division of an integer or fraction. When multiplied by an integer, figure will become bigger. When multiplied by a fraction whereby the numerator is smaller than the denominator, the figure will become smaller. When multiplied by a fraction whereby the denominator is smaller than the numerator, the figure will become bigger. Scale factor is never negative.

Example: N Level 2012 Paper 1 Question 14



(a) Determine the scale factor

Scale factor = $\frac{New\ measurement}{Old\ measurement}$

$= \frac{4.2}{6.3}$

$= \frac{2}{3}$

(b) Find length of KL

KL x $\frac{2}{3}$ = 2.6

KL = 2.6 ÷ $\frac{2}{3}$

= 3.9 cm

(c) Determine the reflex angle QRS

∠QRS = ∠LMN

= 20°

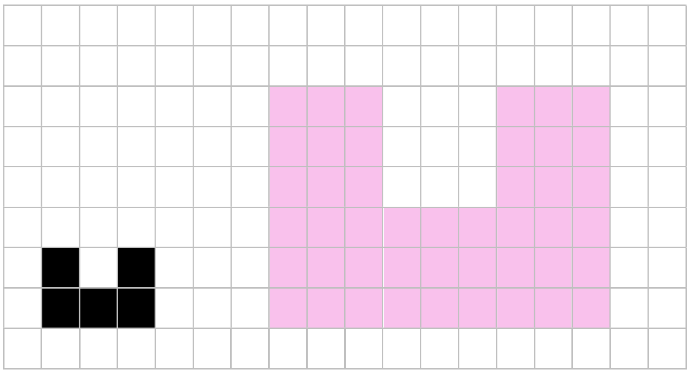
Reflex ∠QRS = 360° – 20°

= 340° (∠ at a point)

Note: Angle does not change even when the size of the figure is changed

Example: N Level 2013 Paper 1 Question 6

Enlarge the shaded figure in the grid below by scale factor 3 and draw the enlarged figure within the grid. Answer is shown in pink.



Note: All sides will be multiplied by 3

G2.2 Determining if Two Triangles are Congruent

For two triangles to be congruent, they need to be of the same size and shape. It also means that their corresponding angles are equal and their corresponding sides are equal. The order on how the triangles are written is very important. For example when ΔABC is congruent to ΔXYZ, ∠A must correspond to ∠X, ∠B must correspond to ∠Y and ∠C must correspond to ∠Z.

Test for Congruency Between Two Triangles

- ☺ SSS: Two triangles are congruent if the three sides of one triangle are equal to each other.
- ☺ SAS: Two triangles are congruent if two sides and the included angle of one triangle are equal to each other
- ☺ AAS: Two triangles are congruent if two angles and a side of one triangle are equal to each other
- ☺ ASA: Two triangles are congruent if two angles and the included side of one triangle are equal to one another
- ☺ RHS: Two triangles are congruent if the hypotenuse and one side of one right angled triangle are equal to one another.

Note: AAA cannot be used to prove for congruent triangles as it is possible for similar triangles to be proven by AAA similarity.

G2.3 Determining if Two Triangles are Similar

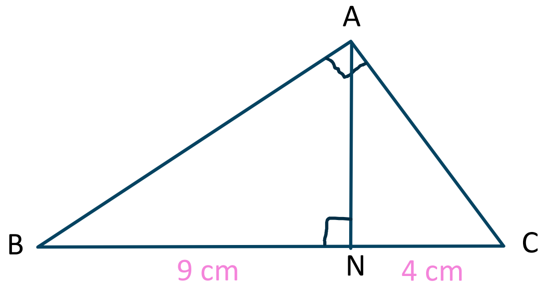
For two triangles to be similar, they need to be of the same shape, but they do not need to be of the same size. If the two triangles are similar, their corresponding angles are equal and their corresponding sides are in the same ratio. The order on how the triangles are written is very important. For example when ΔABC is similar to ΔXYZ, ∠A must correspond to ∠X, ∠B must correspond to ∠Y and ∠C must correspond to ∠Z.

Test for Similarity Between Two Triangles

- ☺ AA: Two triangles are similar if two of the angles are equal
- ☺ SSS: Two triangle are similar if all three corresponding sides has the same ratio
- ☺ SAS: Two triangles are similar if two of their sides are proportional and the included angle is equal.

Note: Two reasons are usually sufficient to prove that both triangles are similar. However, if three reasons can be derived, it is better to state all 3 reasons.

Example: N level 2011 Paper 2 Question 2



(a) Explain why angle BAN = angle CAN

$\angle BAC = \angle ANB = 90^o$ (Given)
$\angle BAN + \angle CAN = 90^o$
$\angle ACN + \angle CAN = 90^o$
Therefore, $\angle BAN$ is equal to $\angle CAN$.

(b) Triangles BAN and CAN are similar. BN = 9 cm and NC = 4 cm. Calculate the length of AN. Show your working.

By similar triangles,

$$\frac{BN}{AN} = \frac{AN}{CN}$$

$$\frac{9}{AN} = \frac{AN}{4}$$

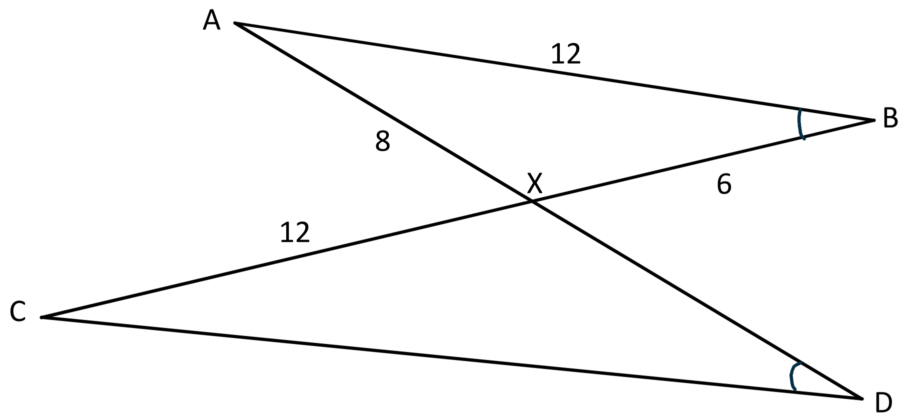
$AN^2 = 36$
AN = 6 cm or AN = - 6 (rejected since length cannot be negative)

Note: It is possible to have the same naming even when two different triangles are involved. Check order if not convinced.

G2.4 Questions on Congruency and Similarity

Example: O Level 2012 Paper 1 Question 23

In the diagram, triangle ABX is similar to triangle CDX. Angle ABX = angle CDX. All measurements are in centimetres.



(a) Calculate CD.

By similar triangles,

$\frac{AB}{CD} = \frac{AX}{CX}$

$\frac{12}{CD} = \frac{8}{12}$

8 CD = 144

CD = 18 cm

(b) Calculate AD.

By similar triangles,

$\frac{XB}{XD} = \frac{AX}{CX}$

$\frac{6}{XD} = \frac{8}{12}$

72 = 8 XD

XD = 9 cm

AD = 8 + 9

= 17 cm

(c) The area of triangle ABX is 21.3 cm^2. Calculate the area of triangle CDX.

By similar triangles,

$\frac{Area\ of\ \Delta ABX}{Area\ of\ \Delta CDX} = (\frac{8}{12})^2$

$\frac{21.3}{Area\ of\ \Delta CDX} = \frac{4}{9}$

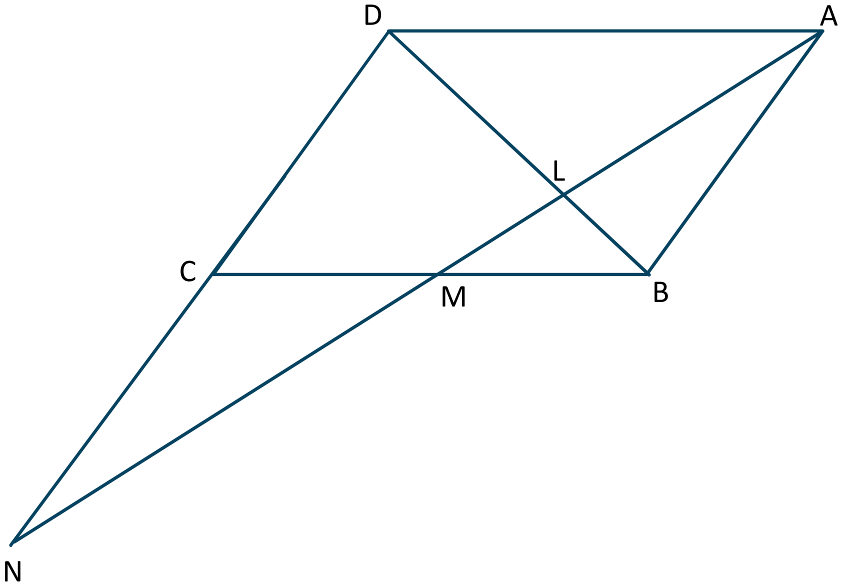
4 x Area of ΔCDX = 21.3 x 9

Area of ΔCDX = $\frac{191.7}{4}$

= 47.9 cm^2 (3 s.f.)

Example: O Level 2010 Paper 1 Question 23

ABCD is a parallelogram and L is a point on DB. The line AL produced meets BC at M and DC produced at N.



(a) Show that triangles ALB and NLD are similar

∠ALB = ∠NLD (vertically opposite angles)
∠LAB = ∠LND (alternate angles, AB// ND)
∠LBA = ∠LDN (alternate angles, AB// ND)
Therefore, ΔALB is similar to ΔNLD (AAA similarity)
Note: Since it is a parallelogram, AD // BC and DC // AB

(b) Name a triangle similar to triangle NCM.

ΔABM is similar to ΔNCM.

(c) Name two triangles that are congruent.

ΔBDC is congruent to ΔDBA.

(d) Given that DL = 3LB, find $\frac{AB}{CN}$

Since ΔALB is similar to ΔNLD,

$\frac{AB}{ND} = \frac{LB}{LD}$

$= \frac{1}{3}$

Therefore, ND = 3AB
Since ND = CN + DC
and DC = AB (opposite sides of parallelogram)
CN + AB = 3AB
CN = 2AB

$\frac{AB}{CN} = \frac{1}{2}$

(e) $\frac{Area\ of\ \Delta ABL}{Area\ of\ \Delta ADL} = \frac{\cancel{\frac{1}{2}} x\ b\ x\ \cancel{h}}{\cancel{\frac{1}{2}} x\ b\ x\ \cancel{h}}$

$= \frac{LB}{DL}$

$= \frac{1}{3}$

Note: The triangles are related to each other by common height

(f) $\frac{Area\ of\ \Delta MLB}{Area\ of\ \Delta ALD}$

Since ΔMLB is similar to ΔALD

$\frac{Area\ of\ \Delta MLB}{Area\ of\ \Delta ALD} = (\frac{LB}{LD})^2$

$= (\frac{1}{3})^2$

$= \frac{1}{9}$

Note: The triangles are related to each other by similar triangles

G2.5 Ratio of Area and Volumes of Similar Plane Figures

Take the ratio of masses to be the same as the ratio of volumes since the mass of the solid is proportional to its volume unless otherwise stated.

Ratio of area of similar solids = $(\frac{l_1}{l_2})^2$

Ratio of volume of similar solids = $(\frac{l_1}{l_2})^3$

Density = $\frac{\text{Mass}}{\text{Volume}}$

Example: O level 2008 Paper 1 Question 19

Two similar jugs have base areas of 45 cm^2 and 125 cm^2.

(a) Find, in its simplest integer form, the ratio of the height of the smaller jug to the height of the larger jug.

By similar solids,

$\frac{A_S}{A_L} = (\frac{H_S}{H_L})^2$

$\frac{45}{125} = (\frac{H_S}{H_L})^2$

$\frac{H_S}{H_L} = \sqrt{\frac{45}{125}}$

$= \frac{3}{5}$

Therefore, the ratio of the height of smaller jug to that of larger jug is 3:5.

(b) The surface area of the top of the smaller jug is 63 cm^2. Find the surface area of the top of the larger jug.

By similar solids,

$\frac{A_S}{A_L} = \frac{45}{125}$

$\frac{A_S}{63} = \frac{45}{125}$

$A_L = \frac{45}{125} \times 63$

$= 175$ cm^2

Therefore, the surface area of the top of the larger jug is 175 cm^2.

(c) The capacity of the larger jug is 2.5 litres. Find the capacity of the smaller jug. Give your answer in cubic centimetres.

By similar solids,

$\frac{V_S}{V_L} = (\frac{H_S}{H_L})^3$

$\frac{V_S}{2.5} = (\frac{3}{5})^3$

$V_s = \frac{27}{125} \times 2.5$

$= 0.54\ l$

$= 540$ cm^3

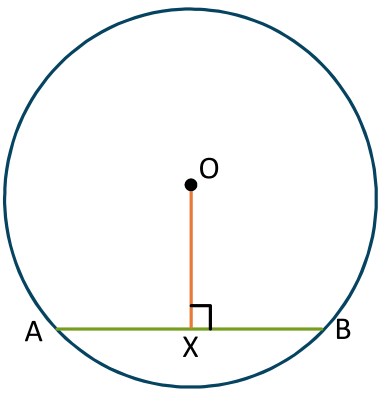
Therefore, the capacity of the smaller jug is 540 cm^3.

G3. Properties of Circles

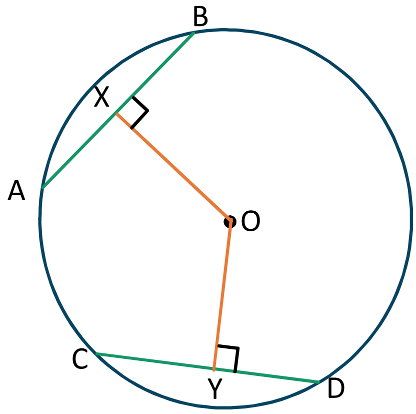
G3.1 Common Terminologies Related to Circles

- Diameter: Longest chord. Passes through the centre
- Radius: Half of a diameter
- Chord: Lies within the circle and touches 2 points of the circle
- Tangent: Lies outside the circle and touches the circle at one point only

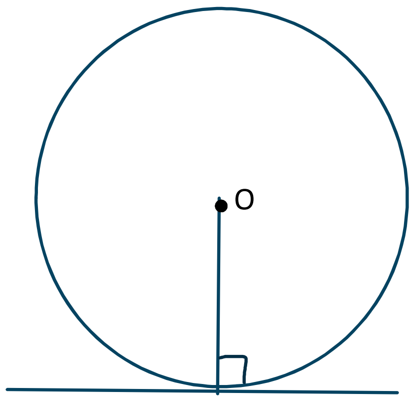
G3.2 Symmetrical Properties of Circle



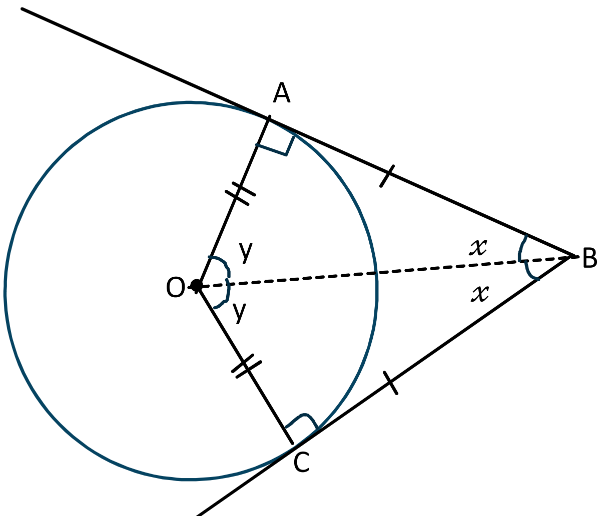
AX = XB
(⊥ from centre bisects chord)



Equal chords are equidistant from the centre



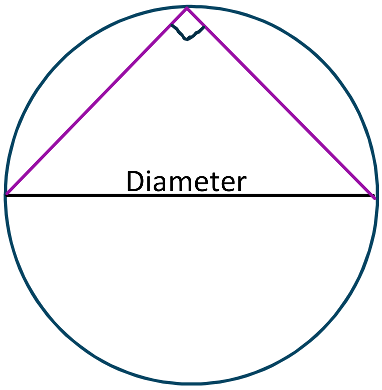
Tangent is perpendicular to radius
(tan ⊥ rad)



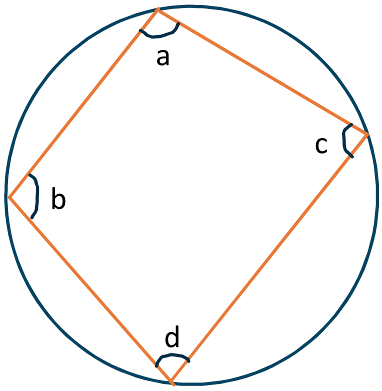
AB = BC
(Tangent from exterior point)

When the two tangents meet, 2 congruent triangles are formed.

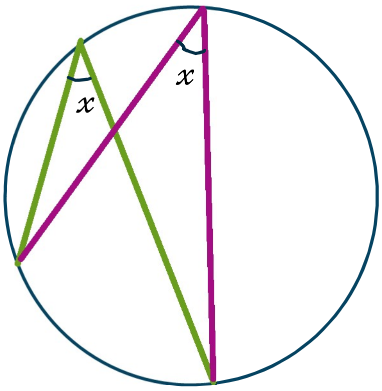
G3.3 Angle Properties of Circle



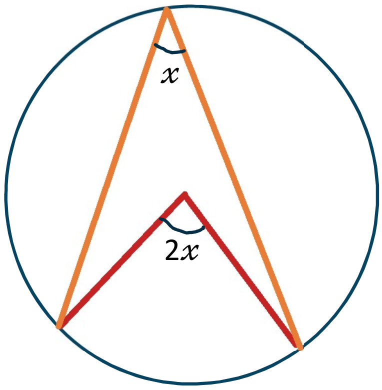
Right angle in semicircle
(Right ∠ in semicircle)



∠a + ∠d = 180°; ∠b + ∠c = 180°
Opposite angles in cyclic quadrilateral are supplementary
(Opp ∠ in cyclic quad are supp)



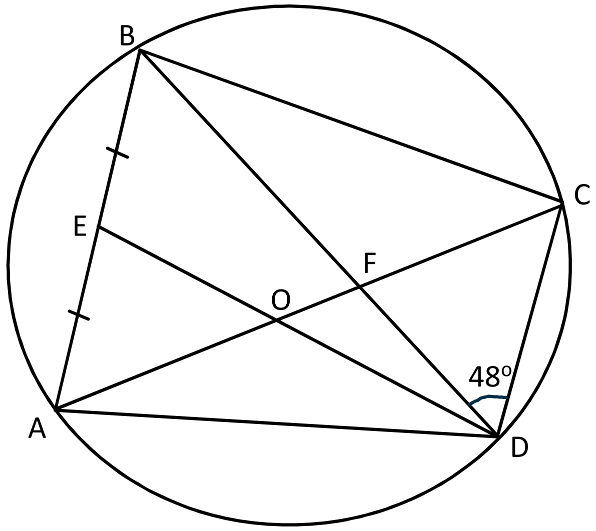
Angle in the same segment
(∠ in the same seg)



Angle at centre is twice the angle at the circumference
(∠ at centre is twice ∠ at circumference)

G3.4 Problems Involving Properties of Circle

Example: O Level 2014 Paper 2 Question 7 (a)

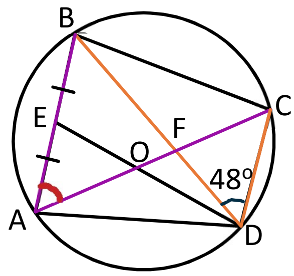


The diagram shows a circle ABCD, centre O. E is the midpoint of the chord AB. ED passes through O. F is the point of intersection of BD and diameter AC. Angle BDC = 48°. Find, giving reasons for each answer.

i) Angle BAC

$\angle BAC = \angle BDC$

$= 48^o$ (angle in the same segment)

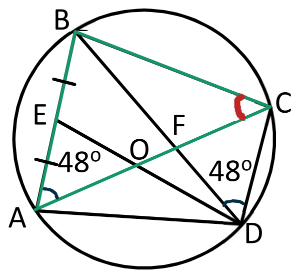


ii) Angle BCA

$\angle ABC = 90^o$ (right angle in semicircle)

$\angle BCA = 180^o - 90^o - 48^o$

$= 42^o$ ($\angle$ sum of Δ)



iii) Angle AOD

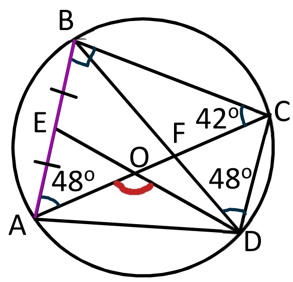
$\angle AEO = 90^o$ ($\perp$ from centre bisect chord)

$\angle EOA = 180^o - 90^o - 48^o$

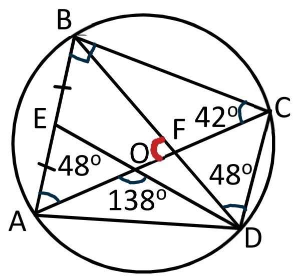
$= 42^o$ ($\angle$ sum of Δ)

$\angle AOD = 180^o - 42^o$

$= 138^o$ ($\angle$ on str line)



iv) Angle AFB



$\angle OAD = (180^\circ - 138^\circ) \div 2$

$= 21^\circ$ (base $\angle$ of isos Δ)

$\angle FBC = 21^\circ$ ($\angle$ in the same seg)

$\angle BFC = 180^\circ - 21^\circ - 42^\circ$

$= 117^\circ$ (sum of Δ)

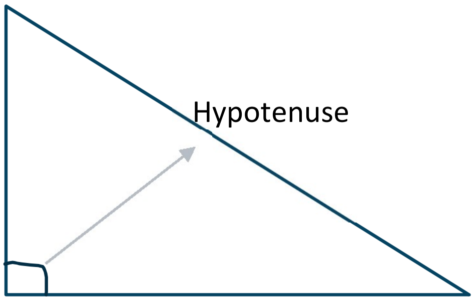
$\angle AFB = 180^\circ - 117^\circ$

$= 63^\circ$ ($\angle$ on str line)

G4. Pythagoras' Theorem and Trigonometry

G4.1 Pythagoras Theorem

Pythagoras theorem is used to determine the length of one side of the triangle when given 2 other sides. It is used for right angled triangle only. The hypotenuse of the triangle is the longest side of the triangle. It is also the side of the triangle that is opposite the right angle.

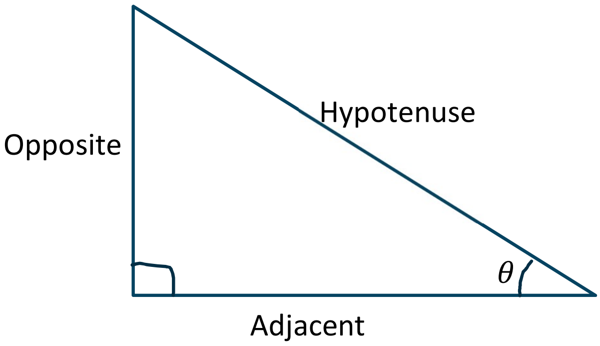


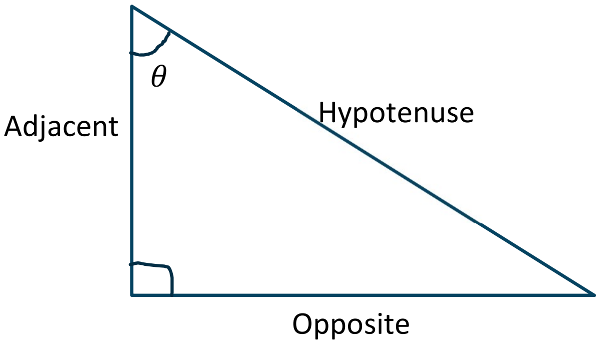
$a^2 + b^2 = c^2$, where c^2 is the hypotenuse of the triangle

Note: Should the question ask you to prove that the triangle is right angled, start off with: by the CONVERSE of Pythagoras theorem. You cannot start off with by Pythagoras Theorem as the triangle has not been proven to be right angled.

G4.2 TOA CAH SOH

This method is used for right angled triangle only. The hypotenuse of the triangle will always be the one opposite the right angle of the triangle and thus it will never change. The adjacent and opposite sides of the triangle must always be done with reference to the angle of interest. TOA CAH SOH is always required for questions on angles of elevation and depression.





G4.3 Trigonometric Ratios

	Acute angle	**Obtuse Angle**
Sin θ	Positive	Positive
Cos θ	Positive	Negative
Tan θ	Positive	Negative

Note: If question state to find sin ∠ABC, do not find the actual angle. Leave the answer in fraction form.

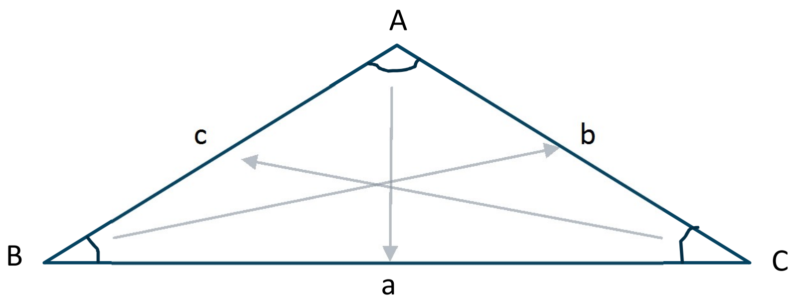
G4.4 Sine Rule and Cosine Rule

Sine Rule

$\frac{a}{\sin A} = \frac{b}{\sin B}$

The small letters represents the length and the capital letters represent the angles.

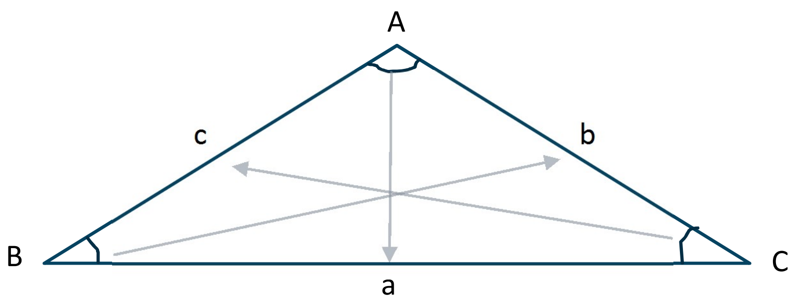
The length of the non-right angled triangle must always be opposite to the angle. For sine rule, it is used to determine the side if given 2 sides and 1 angle or if you are given 2 sides and tasked to find an angle. Total minimum number of sides and angles including unknown required to use this rule is 2 sides and 2 angles.



Cosine Rule

$a^2 = b^2 + c^2 – 2bc \cos A$

The length of the non-right angled triangle must always be opposite to the angle. For cosine rule, it is used to determine the side when given 3 angles or it is used when you are tasked to find one angle, given 2 sides. Total minimum number of sides and angles including unknown required to use this rule is 3 sides and 1 angle.



When doing trigonometric questions, note that the shortest distance is the perpendicular distance. Other common rules to note is that the angle of elevation is equal to the angle of depression. If a triangle is isosceles or equilateral, the perpendicular from the centre bisects the chord and angle. If the triangle is neither isosceles nor equilateral, do not assume.

Example: O level 2014 Paper 1 Question 4

The sine of an angle is 0.720. Give two possible values for the angle

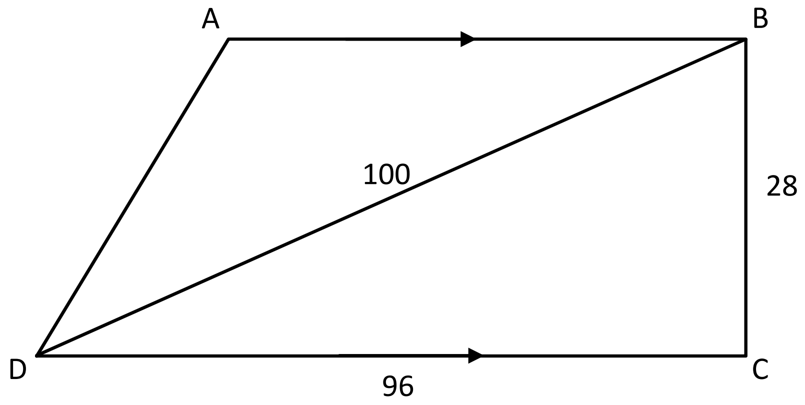
$\sin\theta = 0.720$

$\theta = \sin^{-1} 0.720$

$= 46.05^{\circ}$ (4 s.f.) or $\theta = 180^{\circ} - 46.05^{\circ}$

$= 46.1^{\circ}$ (1 d.p) $= 133.9^{\circ}$ (1 d.p.)

Example: N Level 2012 Paper 1 Question 24



(a) Show that BCD is a right angled triangle.

By the converse of Pythagoras theorem,

$CD^2 + BC^2$

$= 96^2 + 28^2$

$= 10\ 000$

$= 100^2$

$= BD^2$

Therefore, BCD is a right angled triangle.

(b) Find $\angle CDB$.

$\sin \angle CDB = \frac{28}{100}$

$\angle CDB = \sin^{-1} \frac{28}{100}$

$= 16.3^{\circ}$ (1 d.p)

(c) The area of the trapezium is 2394 m^2. Find AB.

Area of trapezium = $\frac{1}{2}$ x sum of parallel sides x height

Area of trapezium = $\frac{1}{2}$ x (AB + 96) x 28

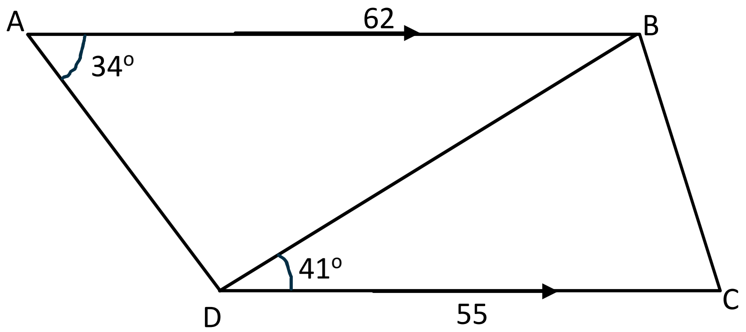
2394 = 14 (AB + 96)

171 = AB + 96

AB = 75 m

Example: O Level 2013 Paper 2 Question 3

The diagram shows a company logo, ABCD, in the shape of a trapezium with AB parallel to DC. AB = 62 mm, CD = 55 mm, angle BAD = 34° and angle BDC = 41°.



(a) Calculate angle ADB

∠ABD = 41° (alt ∠, AB //DC)
∠ADB = 180° – 34° – 41°
= 105° (∠ sum of Δ)

(b) Calculate length BD

By sine rule,

$$\frac{AB}{\sin\angle ADB} = \frac{BD}{\sin\angle BAD}$$

$$\frac{62}{\sin 105} = \frac{BD}{\sin 34}$$

62sin34 = BD sin105

$$BD = \frac{62\sin 34}{\sin 105}$$

= 35.89 mm (4 s.f.)
= 35.9 mm (3 s.f.)

(c) Calculate the area of trapezium ABCD

Area of trapezium = ($\frac{1}{2}$ x 62 x 35.89 x sin 41) + ($\frac{1}{2}$ x 55 x 35.89 x sin 41)
= 1377.44 mm^2 (6 s.f.)
= 1380 mm^2 (6 s.f.)

Note: Since height of the trapezium cannot be determined in this question, the area of the trapezium can be found by finding the area of the two triangles and totalling them together.

(d) An enlarged copy of the logo is made. In the enlargement, CD = 88 mm. Find the area of the enlarged logo.

Let the area of the enlarged logo be x mm^2.

By similar triangles,

$\frac{A_1}{A_2} = (\frac{l_1}{l_2})^2$

$\frac{1377.82}{x} = (\frac{55}{88})^2$

$\frac{1377.82}{x} = \frac{25}{64}$

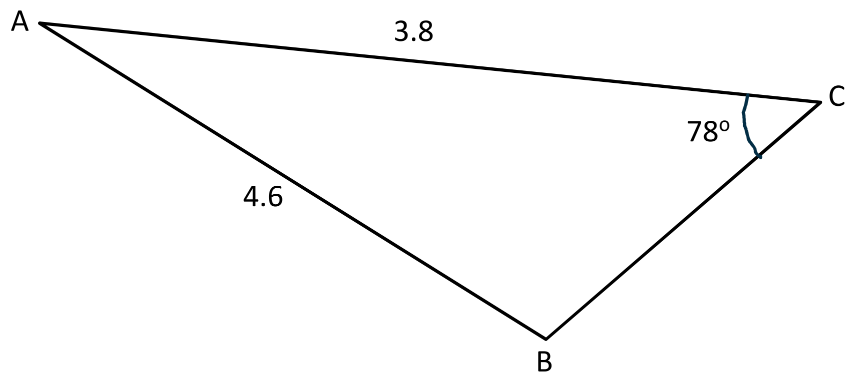
$1377.82 \times 64 = 25x$

$25x = \frac{1377.82 \times 64}{25}$

$= 3530$ mm^2 (3 s.f.)

Example: O Level 2014 Paper 2 Question 9

The diagram shows a triangular flower bed ABC on horizontal ground. AB = 4.6m, AC = 3.8m and angle ACB = 78°.



(a) Calculate angle ABC

By sine rule,

$\frac{AC}{\sin \angle ABC} = \frac{AB}{\sin \angle ACB}$

$\frac{3.8}{\sin \angle ABC} = \frac{4.6}{\sin 78}$

sin ∠ABC = $\frac{3.8 \sin 78}{4.6}$

= 53.90° (4 s.f.)

= 53.9° (1 d.p.)

(b) Calculate the area of the flower bed.

∠CAB = 180° – 78° – 53.90°

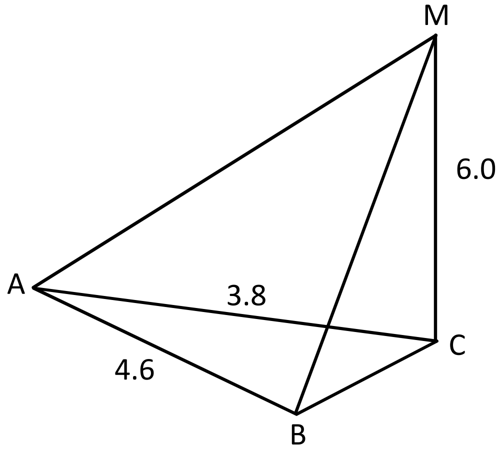
= 48.10° (∠ sum of Δ)

Area of flower bed = $\frac{1}{2}$ x 4.6 x 3.8 x sin 48.10°

= 6.5048 m^2 (5 s.f.)

= 6.50 m^2 (3 s.f.)

The base of a vertical flagpole, CM is at vertex C of the flower bed. The flagpole is held in place by two cables, AM and BM. CM = 6.0 m.



(c) Show that AM = 7.10 m, correct to 2 decimal places.

By Pythagoras theorem,

AM = $\sqrt{6^2 + 3.8^2}$

= 7.102 m (4 s.f.)

= 7.10 m (2 d.p.) [Shown]

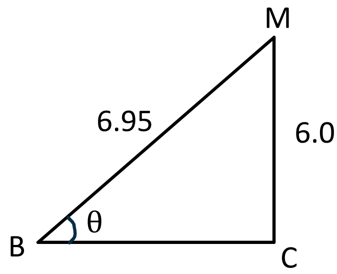
(d) Given that BM = 6.95 m, find the angle of elevation of M from B.

sin ∠MBC = $\frac{6}{6.95}$

∠MBC = $\sin^{-1} \frac{6}{6.95}$

= 59.69° (4 s.f.)

= 59.7° (1 d.p.)



(e) Find angle AMB, the angle between the two angles.

By cosine rule,

$AB^2 = AM^2 + BM^2 – 2(AM)(BM)\cos \angle AMB$

$4.6^2 = 7.102^2 + 6.95^2 – 2(7.102)(6.95) \cos \angle AMB$

21.16 = 50.44 + 48.3025 – 98.72 cos ∠AMB

-77.5825 = – 98.69 cos ∠AMB

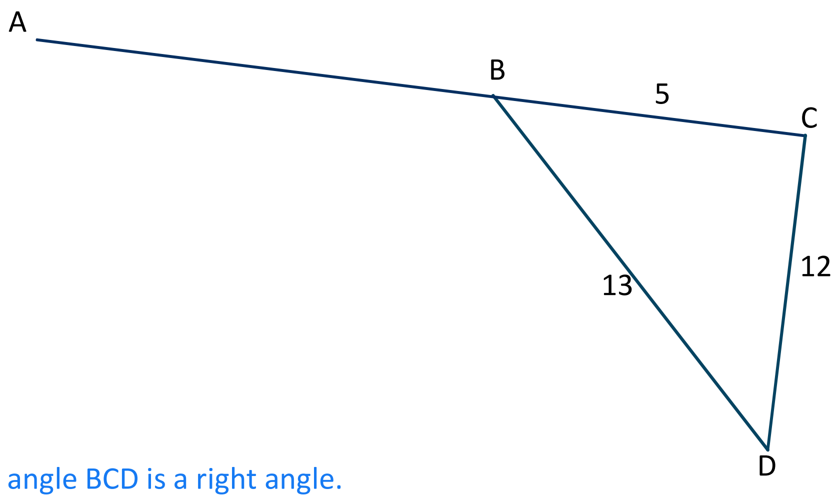
cos ∠AMB = $\frac{-77.5825}{-98.72}$

∠AMB = $\cos^{-1} \frac{-77.5825}{-9\ .72}$

= 38.2° (1 d.p.)

Example: N Level 2008 Paper 1 Question 16

In the diagram, BC = 5 cm, CD = 12 cm, BD = 13 cm and ABC is a straight line.



(a) Explain why angle BCD is a right angle.

By the converse of Pythagoras theorem,
$BC^2 + CD^2$
$= 5^2 + 12^2$
$= 25 + 144$
$= 169$
$= 13^2$
$= BD^2$
Therefore, angle BCD is a right angle.

(b) Express tan ∠BDC as a fraction

$\tan \angle BDC = \frac{BC}{CD}$
$= \frac{5}{12}$

Note: Differentiate between tan ∠BDC and ∠BDC. If the question requires tan ∠BDC, the answer is usually in fraction. If the question requires ∠BDC, the actual angle corrected to 1 decimal place is required

(c) Express cos ∠ABD as a fraction

$\cos \angle ABD = -\cos \angle CBD$
$= -\frac{BC}{BD}$
$= -\frac{5}{13}$

G4.5 Flow chart to determine formula to be used

Right angled triangle

← **Pythagoras Theorem**

To determine length not involving angle

$a^2 + b^2 = c^2$

where c is the hypotenuse of the triangle

→ **Trigonometry**

To determine length involving angle

$\sin\theta = \frac{opposite}{hypotenu}$

$\tan\theta = \frac{opposite}{adjacent}$

$\cos\theta = \frac{adjacent}{hypotenus}$

Non- Right angled triangle

← **Cosine rule**

Used to determine angle when given 3 sides

OR

Used to determine side when given 2 sides and one angle

→ **Sine Rule**

Used to determine angle when give 2 sides and 1 angle

OR

Used to determine side when given 2 angles and one side

Area of triangle

← **Non-right angled triangle**

Area = $\frac{1}{2}$ab sin C

Given that angle C must be found between two lengths

→ **Right angled triangle**

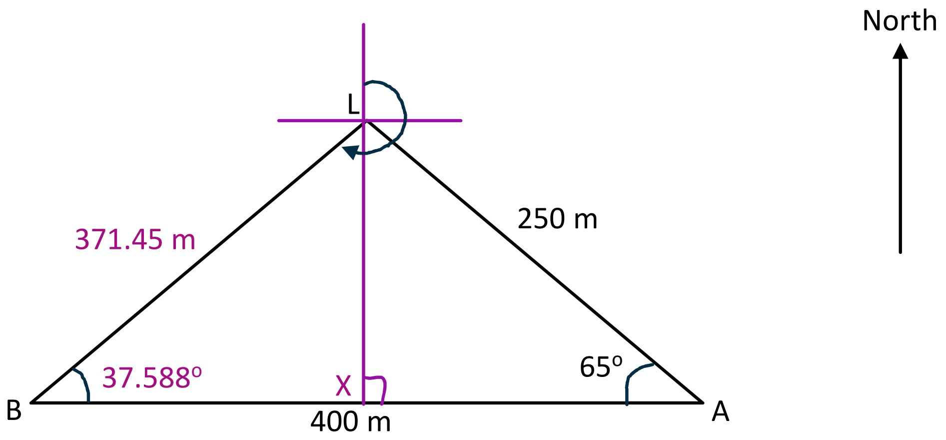
Area = $\frac{1}{2}$ x base x height

G4.6 Bearings

Example: O Level 2011 Paper 2 Question 6

The base, L, of a lighthouse is at sea level. Yatch A is 250 m from L. Yatch B is 400 m due west of Yatch A. Angle LAB = 65°.

Note: If Yatch B is due west of Yatch A, it means that A and B lies on a straight line.



(a) Calculate the length of LB

By cosine rule,
$LB^2 = AB^2 + AL^2 - 2(AB)(AL)\cos\angle BAL$
$LB^2 = 400^2 + 250^2 - 2(400)(250)(\cos 65)$
LB = 371.45 m (5 s.f.)
LB = 371 m (3 s.f.)

(b) Calculate the area of triangle LAB

Area of $\Delta LAB = \frac{1}{2} \times 400 \times 250 \times \sin 65$
$= 45315.39\ m^2$ (7 s.f.)
$= 45300\ m^2$ (3 s.f.)

(c) Calculate the angle LBA.

By sine rule,
$$\frac{LA}{\sin\angle LBA} = \frac{LB}{\sin\angle LAB}$$
$$\frac{250}{\sin\angle LBA} = \frac{371.45}{\sin 65}$$
$250 \sin 65 = 371.45 \times \sin\angle LBA$
$\sin\angle LBA = \frac{250 \sin 65}{371.45}$
$\sin\angle LBA = 0.60998$
$\angle LBA = \sin^{-1} 0.60998$
$= 37.588$ (5 s.f.)
$= 37.6°$ (1 d.p.)

(d) Calculate the bearing of B from L. Shown in purple.

∠BLX = 180° – 90° – 37.588°
= 52.412° (∠ sum of Δ)

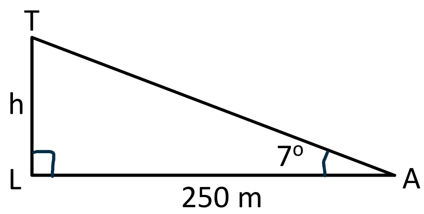
Bearing of B from L = 52.412° + 180°
= 232.4° (1 d.p.)

(e) The angle of elevation of the top of the lighthouse seen from A is 7°. Calculate the angle of elevation of the top of the lighthouse from B.

Let h be the height of the lighthouse.

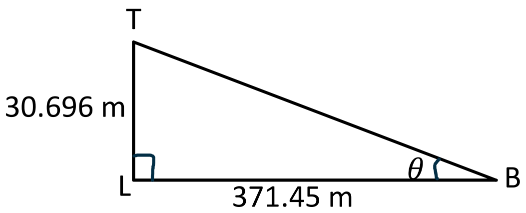
tan 7 = $\frac{h}{250}$

h = 250 tan7
= 30.696 m (5 s.f.)



Let θ be the angle of elevation of the top of the lighthouse from B.

tan θ = $\frac{h}{BL}$
= $\frac{30.696}{371}$
= 4.728° (4 s.f.)
= 4.7° (1 d.p.)

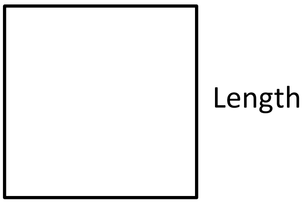


Note: Angle of elevation and angle of depression questions always involve right angle triangle.

G5. Mensuration

G5.1 Formulas of Common Figures and Solids

<u>Square</u>



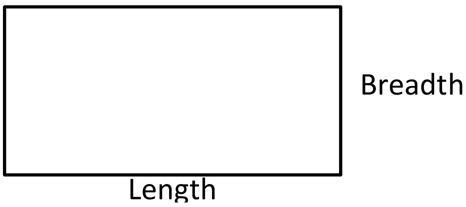
Perimeter of square = 4 x length

= $4l$

Area of square = length x length

= l^2

<u>Rectangle</u>



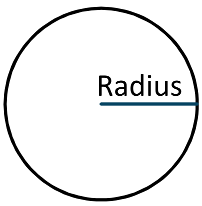
Perimeter of rectangle = (2 x length) + (2 x breadth)

= $2l + 2b$

Area of rectangle = length x breadth

= $l \times b$

<u>Circle</u>

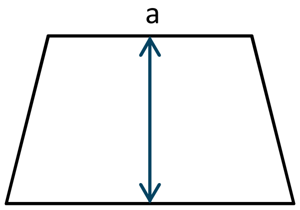


Circumference of circle = $2\pi r$

= πd

Area of circle = πr^2

Parallelogram



Area of parallelogram = base x height

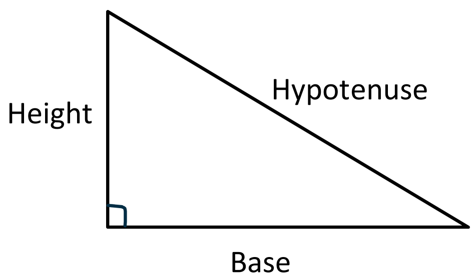
= b x h

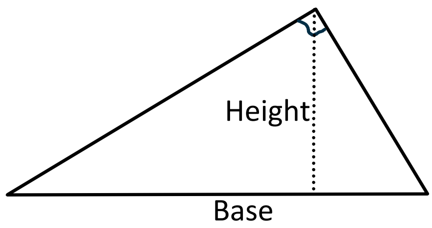
Note: Base must be perpendicular to height

Area of trapezium = $\frac{1}{2}$ x sum of parallel sides x height

= $\frac{1}{2}$ x (a + b) x h

Right-angled Triangle



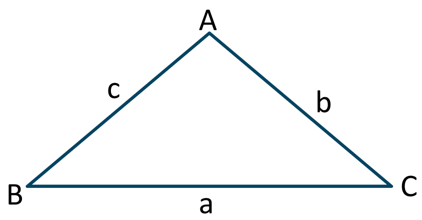


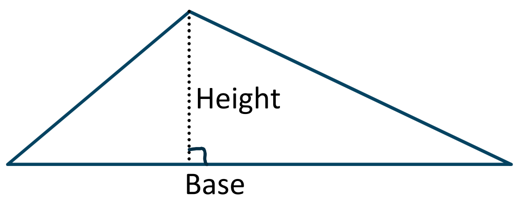
Area of right angled triangle = $\frac{1}{2}$ x base x height

= $\frac{1}{2}$ x b x h

Note: Base must be perpendicular to height

Non-right angled Triangle





Area of non-right angled triangle = $\frac{1}{2}$ x a x b x sin C

Note: Angle used must be between the two lengths

Area of non – right angled triangle = $\frac{1}{2}$ x base x height

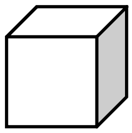
Note: Base must be perpendicular to height

Volume of Solids

General formula for volume of solid figures = base area x height

General formula for volume of solid figures with pointed end = $\frac{1}{3}$ x base area x height

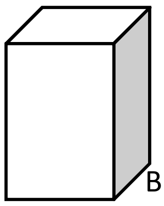
Cube



Volume of cube = l x l x l

Surface area of cube = 6 x l x l

Cuboid



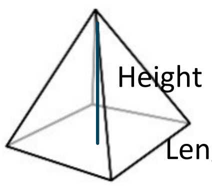
Volume of cuboid = length x breadth x height

= l x b x h

Surface area of cuboid = 2(length x base) + 2(length x height) + 2(height x base)

= 2 lb + 2 lh + 2 hb

Pyramid



Volume of pyramid = $\frac{1}{3}$ x base area x height

Surface area of pyramid with square base

= Area of square + 4 (Area of triangle)

= l^2 + 4 ($\frac{1}{2}$ x b x h)

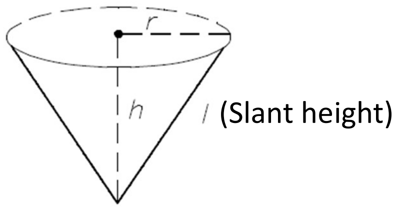
Surface area of pyramid with rectangle base

= Area of rectangle + 2 (Area of triangle) + 2 (Area of triangle)

= (l x b) + 2 ($\frac{1}{2}$ x b x h) + 2 ($\frac{1}{2}$ x b x h)

Note: The triangles have different dimensions due to the shape of the base used

Cone



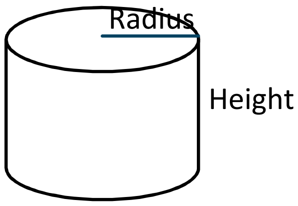
Volume of cone = $\frac{1}{3}\pi r^2 h$

Total surface area of cone = $\pi r l + \pi r^2$

Curved surface area = $\pi r l$

Note: The length, radius and slant height of the cone are linked by Pythagoras theorem.

Cylinder

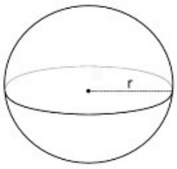


Volume of cylinder = $\pi r^2 h$

Total surface area of cylinder = $2\pi rh + 2\pi r^2$

Curved surface area of cylinder =$2\pi rh$

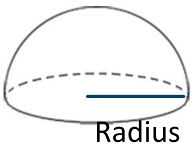
Sphere



Volume of sphere = $\frac{4}{3}\pi r^3$

Surface area of sphere = $4\pi r^2$

Hemisphere



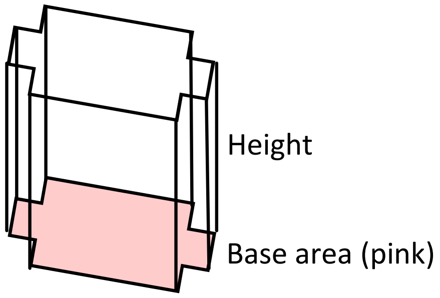
Volume of hemisphere = $\frac{2}{3}\pi r^3$

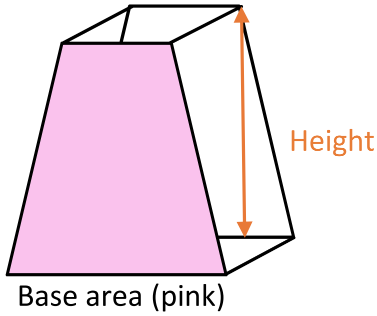
Curved surface area of hemisphere = $2\pi r^2$

Total surface area of hemisphere = $3\pi r^2$

Prism

Prism is any solid that have a uniform cross section throughout the whole figure. Example:





Volume of prism
= Base area (varies according to shape) x height
Surface area of prism
= Perimeter of cross section x length + 2 x area of cross section

Conversion Involving Degree and Radian

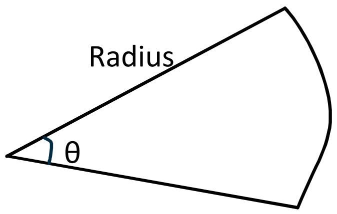
180 degrees = π radians

1 degree = $\frac{\pi}{180}$

π radians = 180 degrees

1 radian = $\frac{180}{\pi}$

Arc Sector



Arc length (radian) = rθ

Perimeter of sector (radian) = rθ + 2r

Arc length (degrees) = $\frac{\theta}{360}$ x 2πr

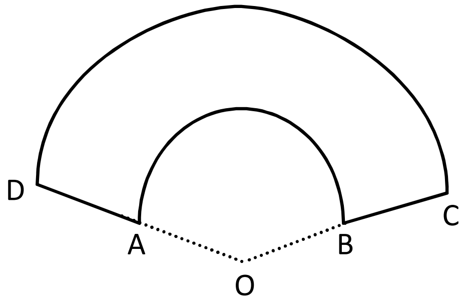
Perimeter of sector (degrees) = $\frac{\theta}{360}$ x 2πr + 2r

Area of sector (radian) = $\frac{1}{2}r^2\theta$

Area of sector (degrees) = $\frac{\theta}{360}$ x πr^2

G5.2 Questions on Mensuration

Example: O Level 2014 Paper 2 Question 7 (b)



The diagram shows a mirror ABCD. AB and DC are arcs of circles centre O with radii 20 cm and 50 cm respectively. The perimeter of the mirror is 235 cm.

(a) Calculate the angle AOB in radians.

Let the angle AOB be θ

Perimeter = $50\theta + 20\theta + 30 + 30$

$235 = 70\,\theta + 60$

$70\theta = 135$

$\theta = 2.5$ rad

(b) Calculate the area of the mirror.

Area of big sector DAOBC = $\frac{1}{2}r^2\theta$

$= \frac{1}{2} \times 50^2 \times 2.5$

$= 3125$

Area of small sector AOB = $\frac{1}{2}r^2\theta$

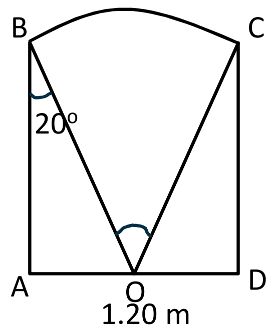
$= \frac{1}{2} \times 20^2 \times 2.5$

$= 500$

Area of mirror = 3125 – 500

= 2625 cm^2

Example: O Level 2013 Paper 2 Question 8



The diagram shows a symmetrical window frame. OBC is a sector of the circle, centre O. AD = 1.20m, angle ABO = 20° and AB is perpendicular to AD.

(a) Calculate the radius of the sector OBC

Note: Since it is symmetrical, AO = OD

AO = 1.20 ÷ 2

= 0.6 m

sin 20° = $\frac{0.6}{BO}$

BO = $\frac{0.6}{sin20}$

= 1.754 m (4 s.f.)

= 1.75 m (3 s.f.)

(b) Calculate the angle BOC in radians

∠BOA = 180° – 20° – 90°

= 70° (sum of Δ)

∠BOC = 180° – 70° – 70°

= 40° (∠ on str line)

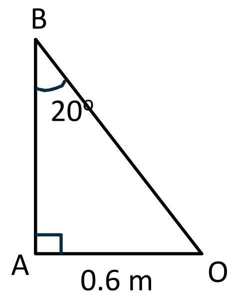
180° = π rad

1° = $\frac{\pi}{180}$

40° = $\frac{\pi}{180}$ x 40

= $\frac{2}{9}\pi$ radian

(c) Calculate the total perimeter of the window frame.



tan20 = $\frac{0.6}{AB}$
AB = 0.6 tan20
= 1.648 m (4 s.f.)

Arc length = 1.75 ($\frac{2}{9}$ π)
= 1.2217 m (5 s.f.)

Total perimeter = 1.648 + 1.20 + 1.648 + 1.2217
= 5.72 m (3 s.f.)

(d) A company manufactures windows. The cost of manufacture is $78.50 per square metre of window. Work out the cost of manufacturing this window.

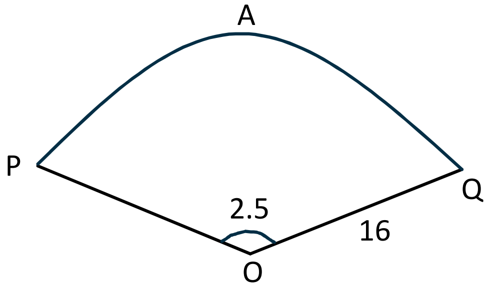
Area of triangle = 2 x $\frac{1}{2}$ x 0.6 x 1.648
= 0.9888 m^2

Area of sector = $\frac{1}{2}r^2\theta$
= $\frac{1}{2}$ x 1.75^2 x $\frac{2}{9}\pi$
= 1.069 m^2 (4 s.f.)

Area of the window
= 0.9888 + 1.069
= 2.0578 m^2

Cost of manufacturing the window
= 2.0578 x $78.50
= $162 (3 s.f.)

Example: O Level 2012 Paper 1 Question 21



OPAQ is a sector of a circle, centre O of radius 16 cm. The angle at the centre is 2.5 radians.

(a) Calculate the length of the arc PAQ.

Length of arc PAQ = $r\theta$

$= 16 \times 2.5$

$= 40$ cm

(b) The sector is formed into a cone by joining the two radii, OP and OQ together. Calculate the radius of the base of the cone.

Perimeter = $2\pi r$

$40 = 2\pi r$

$r = \frac{40}{2\pi}$

$= 6.37$ cm (3 s.f.)

(c) Change 2.5 radians to degrees.

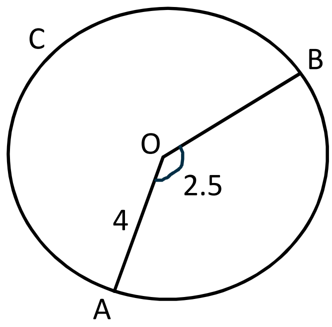
π radians = 180°

1 radian = $(\frac{180}{\pi})^{\circ}$

2.5 radians = $\frac{180}{\pi} \times 2.5$

$= 143.2^{\circ}$ (1 d.p.)

Example: O level 2011 Paper 1 Question 15



A, B and C lie on a circle with centre O and radius 4 cm. ∠AOB = 2.5 radians.

(a) Find the area of the minor sector AOB

Area of the minor sector AOB = $\frac{1}{2}$ x 4^2 x 2.5
= 20 cm^2

(b) Write down an expression, in terms of π, for the reflex angle AOB.

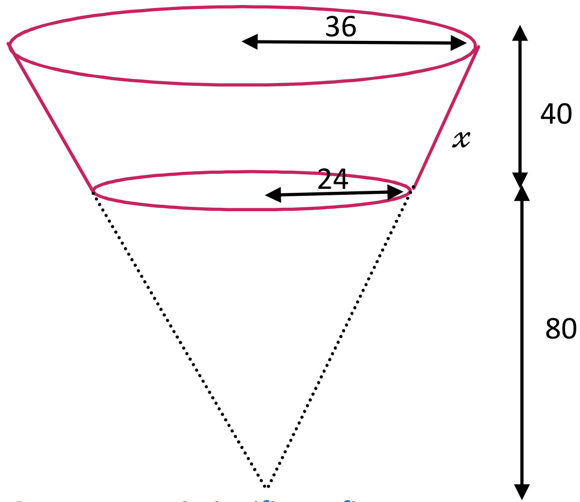
Reflex ∠AOB = (2π – 2.5) radians (∠ at a point)
Note: 360^o is 2π radians

(c) Find an expression, in terms of π, for the length of the arc ACB.
Length of arc ACB = 4 (2π - 2.5)
= (8π – 10) cm

Calculation of Frustum

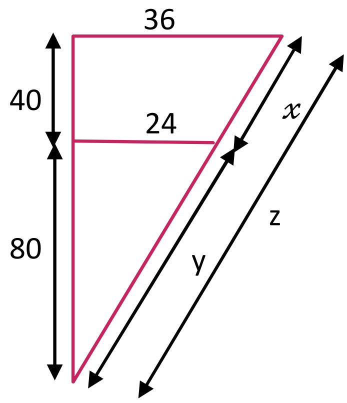
Example: O Level 2011 Paper 2 Q7 (b)

The diagram shows a pot which is part of a circular cone of height 120 cm. The open end of the pot is a circle of radius 36 cm. The base of the pot is a circle of radius 24 cm. The height of the pot is 40 cm. The slant height of the pot is x cm. [You may ignore any holes in the base of the pot.]



(a) Show that x = 41.8, correct to 3 significant figures.

By Pythagoras theorem,



$y^2 = 24^2 + 80^2$

$= \sqrt{24^2 + 80^2}$

$= 83.52$ cm (4 s.f.)

$z^2 = 36^2 + (40 + 80)^2$

$z^2 = 15696$

$z = \sqrt{15969}$

$z = 125.28$ cm (5 s.f.)

$x = 125.28 - 83.52$

$= 41.8$ cm (3 s.f.) (Shown)

(b) Calculate the total surface area of the outside of the pot.

Curved surface area = $\pi(36)(125.28) - \pi(24)(83.52)$

$= 7871.6 \text{ cm}^2$ (5 s.f.)

Area of circle = $\pi(24)^2$

$= 1809.6 \text{ cm}^2$ (5 s.f.)

Total surface area = 7871.6 + 1809.6

$= 9680 \text{ cm}^2$ (3 s.f.)

Note: In order to determine the curve surface area of frustum, take curved surface area of big cone – curve surface area of small cone. There is no formula for direct determination of frustum.

(c) Determine the volume of pot

Volume of big cone = $\frac{1}{3}\pi r^3$

= $\frac{1}{3}\pi (36)^3$

= 15552π cm^3

Volume of small cone = $\frac{1}{3}\pi r^3$

= $\frac{1}{3}\pi (24)^3$

= 4608π cm^3

Volume of pot = $15552\pi - 4608\pi$

= 34400 cm^3

(d) Another pot is to be made with a volume twice the volume of this pot. Given that the two pots are geometrically similar, find the height of the larger pot.

By similar solids,

$(\frac{V_S}{V_L}) = (\frac{h_S}{h_L})^3$

$(\frac{1}{2}) = (\frac{40}{h_L})^3$

$\frac{40}{h_L} = \sqrt[3]{\frac{1}{2}}$

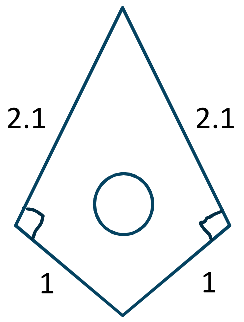
$h_L = \frac{40}{\sqrt[3]{\frac{1}{2}}}$

= 50.4 cm (3 s.f.)

Therefore, the height of the larger pot is 50.4 cm.

Example: O level 2014 Paper 1 Question 22.

The cross section of a gold pendant is a quadrilateral with two right angles and a circular hole, as shown. All measurements are in centimetres.



The diameter of the circular hole is 0.8 cm. The uniform thickness of the pendant is 0.3 cm. The mass of 1 cubic centimetre of gold is 19.3 grams. The price of 1 gram of the gold is $69.65. Calculate the value of the gold in the pendant. Give your answer to the nearest cent.

Note: Cut the diagram in half to get 2 congruent triangles

Area of the pendant = $(2 \times \frac{1}{2} \times \text{base} \times \text{height}) - \pi r^2$

$= (2 \times \frac{1}{2} \times 2.1 \times 1) - \pi(\frac{0.8}{2})^2$

$= (2.1 - 0.16\pi)$

$= 1.597 \text{ cm}^2$ (4 s.f.)

Volume of the pendant = base area x height

= 1.597 x 0.3

= 0.4792 cm^3

$1 \text{ cm}^3 = 19.3$ g

$0.4792 \text{ cm}^3 = 19.3 \times 0.4792$

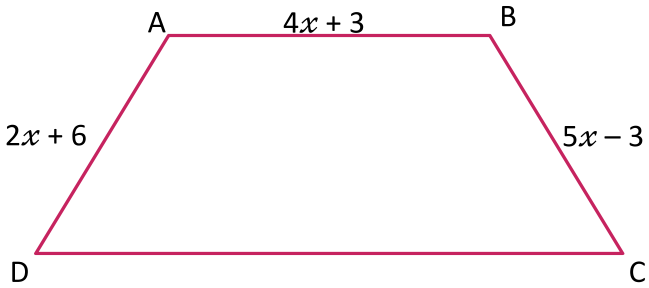
= 9.249 g (4 s.f)

Value of the gold = 9.249 x $69.65

= $644.17 (correct to the nearest cents)

Example: O Level 2014 Paper 1 Question 24

Expressions for the lengths of three sides of a trapezium are shown on the diagram below. All lengths are in centimetres.



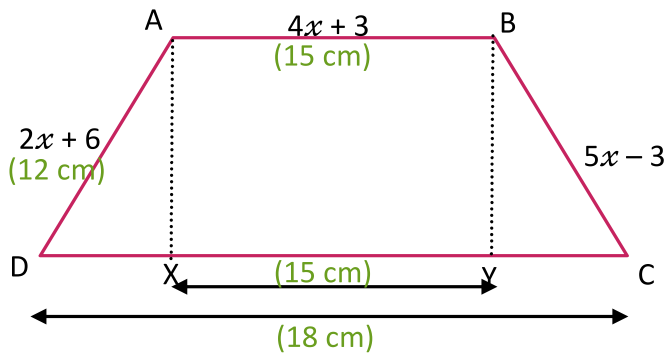
(a) The perimeter of this trapezium is given by the expression $(20x - 3)$ cm. Find an expression, in terms of x, for the length of DC. Give your expression in its simplest form.

Length of DC
$= (20x - 3) - [(2x + 6) + (4x + 3) + (5x - 3)]$
$= 20x - 3 - (11x + 6)$
$= 20x - 3 - 11x - 6$
$= 9x - 9$
$= 9(x - 1)$ cm

(b) Given that AD = BC, calculate the perimeter of the trapezium.

Since AD = BC,
$2x + 6 = 5x - 3$
$9 = 3x$
$x = 3$
Perimeter of the trapezium $= 20x - 3$
$= 20(3) - 3$
$= 57$ cm

(c) The perpendicular from A to DC meets DC at x. The perpendicular from B to DC at Y. DX = CY. Calculate the area of the trapezium.



Length of AB = $4x + 3$
= $4(3) + 3$
= 15 cm

Length of DC = $9(x - 1)$
= $9(3 - 1)$
= 18cm

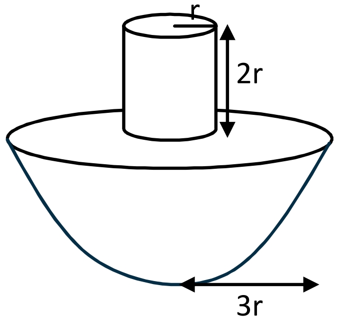
Length of AD = $2x + 6$
= 12 cm

DX = CY
= $\frac{(18 - 15)}{2}$
= 1.5 cm (sides of an isosceles trapezium)

By Pythagoras theorem,
AX = BY
= $\sqrt{12^2 - 1.5^2}$
= 11.906 cm (5 s.f.)

Area of trapezium = $\frac{1}{2}$ x (15 + 18) x 11.906
= 196 cm^2 (3 s.f.)

Example: O Level 2013 Paper 1 Question 6



This solid is made from a cylinder and a hemisphere. The cylinder has radius r and height 2r. The hemisphere has radius 3r. Find an expression, in terms of π and r, for the total surface area of the solid.

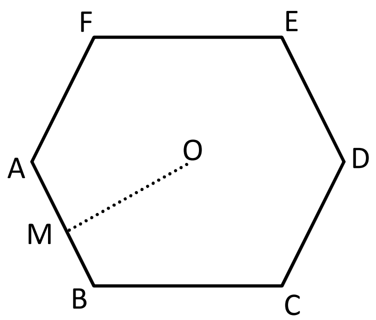
Curved surface area of hemisphere $= 2\pi r^2$
$= 2\pi(3r)^2$
$= 18\pi r^2$ units^2

Curved surface area of cylinder $= 2\pi rh$
$= 2\pi r(2r)$
$= 4\pi r^2$ units^2

Area of circle $= \pi r^2$
$= \pi(3r)^2$
$= 9\pi r^2$

Total surface area $= 18\pi r^2 + 4\pi r^2 + 9\pi r^2$
$= 31\pi r^2$ units^2

Example: O Level 2012 Paper 2 Question 9

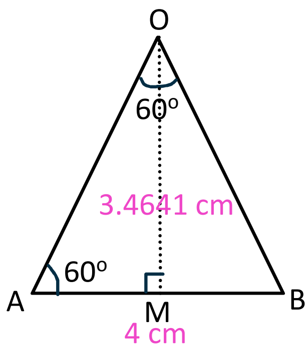


A regular hexagon, ABCDEF, has sides of length 4 cm. M is the midpoint of AB and O is the centre of the hexagon.

(a) Show that the area of the hexagon is 41.6 cm^2, correct to 3 significant figures.

$\angle AOB = 360^o \div 6$
$= 60^o$ ($\angle$ at a point)

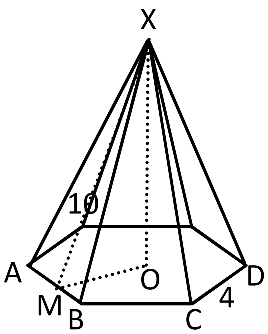
$\angle AOB = \frac{180-60}{2}$
$= 60^o$ (base $\angle$ of isos. Δ)



MB = 4 ÷ 2
= 2 cm

$\tan 60^o = \frac{OM}{2}$
$OM = 2 \tan 60^o$
= 3.4641 cm (5 s.f.)

Area of the hexagon ABCDEF
$= 6 \times (\frac{1}{2} \times 4 \times 3.4641)$
= 41.5692 cm
= 41.6 cm^2 (3 s.f.)



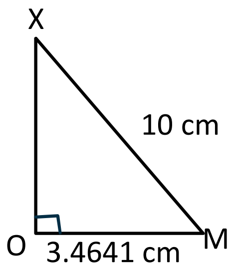
(b) Hexagon ABCDEF forms the base of a pyramid. The vertex, X, is directly above O. The slant height, MX, of the pyramid is 10 cm. Calculate the total surface area of the pyramid.

Total surface area = base area + 6 x area of Δ

$= 41.5692 + 6\,(\frac{1}{2} \times 4 \times 10)$

$= 162$ cm^2

(c) Calculate the height, OX, of the pyramid.



By Pythagoras theorem,

$OX^2 + 3.4641^2 = 10^2$

$OX^2 + 12.00 = 100$

$OX^2 = 88$

OX = 9.3808 (5 s.f.)

OX = 9.38 cm (3 s.f.)

(d) Calculate the volume of the pyramid.

Volume of the pyramid $= \frac{1}{3} \times$ base area x height

$= \frac{1}{3} \times 41.5692 \times 9.3808$

$= 129.98$ cm^3 (5 s.f.)

$= 130$ cm^3

(e) Another similar pyramid is made, with a hexagonal base of side 9 cm. Find the volume of the pyramid.

Let the volume of the new pyramid be x.

By similar solids,

$\frac{129.98}{x} = (\frac{4}{9})^3$

$\frac{129.98}{x} = (\frac{64}{729})$

$64x = 94755.42$

$x = 1480$ cm^3 (3 s.f.)

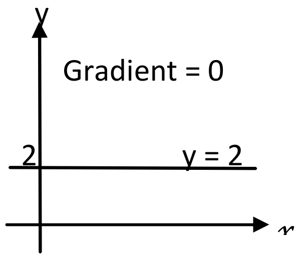
G6. Coordinate Geometry

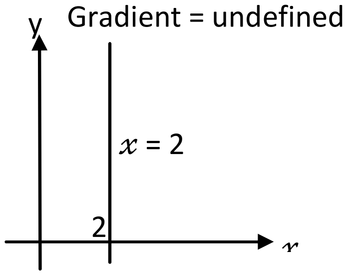
G6.1 General Equation:

$y = mx + c$,
where, m = gradient of the linear equation
c = y – intercept

G6.2 Gradient (m) of a Linear Graph:

Gradient = $\frac{y_2 - y_1}{x_2 - x_1}$





G6.3 Length of Line Segment

Length = $\sqrt{(x_2-x_1)^2+(y_2-y_1)^2}$

Example: Determine the equation of the line given the co-ordinates A (-2, 1) and B (3, -7).

$m = \frac{y_2 - y_1}{x_2 - x_1}$ Note: Determine gradient first
$= \frac{-7 - 1}{3 - (-2)}$
$= -1.6$

$y = -1.6x + c$
when $x = -2$, $y = 1$, Note: Take from **either** A or B NOT one each.
$1 = -1.6(-2) + c$
$1 = 3.2 + c$
$c = -2.2$
Therefore, $y = -1.6x - 2.2$

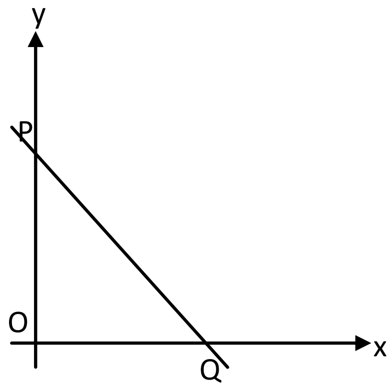
Example: Find the length of AB given the coordinates A (-2, 1) and B (3, -7).

Length of AB = $\sqrt{((-2) - 3)^2 + (1 - (-7))^2}$
= 9.43 units (3 s.f.)

G6.4 Questions on Coordinate Geometry

Example: O Level 2012 Paper 1 Question 20

The diagram shows a sketch of the graph y = 10 – 2x. The line crosses the axes at P and Q.



(a) Find the coordinates of P and Q.

When x = 0,
y = 10 – 2(0)
y = 10

P (0, 10)

When y = 0,
0 = 10 - 2x
2x = 10
x = 5

Q (5, 0)

Note:

When line cuts the y axis, x = 0

When line cuts the x axis, y = 0

(b) Calculate the length of the line joining P to Q.

Length of PQ
$= \sqrt{(0-5)^2 + (10-0)^2}$
$= \sqrt{25 + 100}$
= 11.2 units (3 s.f.)

Example: O level 2012 Paper 2 Question 3 (b)

The line with equation $4y - 3x + 8 = 0$

(a) Find the gradient of line *l*.

$4y = 3x - 8$
$y = \frac{3}{4}x - 2$

Note: Make y the subject of the formula and take the term in front of x.

(b) Find the y – intercept of line *l*.

y – intercept = -2.

(c) The line with equation $3x + 2y = 5$ intersects the line *l* at point C. Find the coordinates of C.

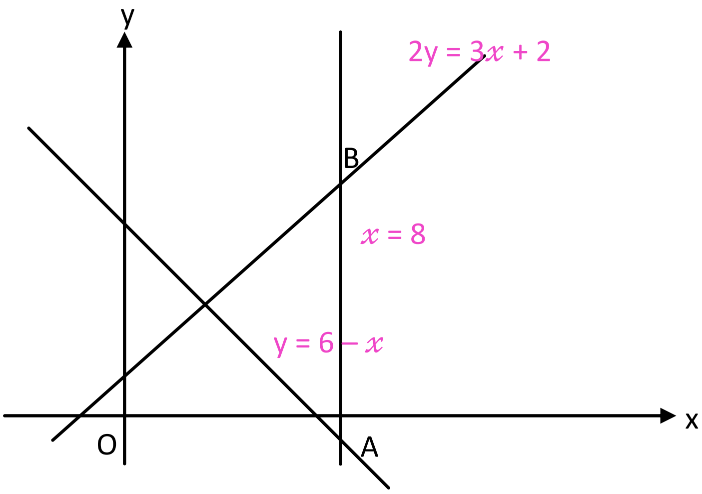
$y = \frac{3}{4}x - 2$ --------(1)

$3x + 2y = 5$
$2y = -3x + 5$
$y = \frac{-3x+5}{2}$ ---------(2)

Sub (2) into (1),
$\frac{3}{4}x - 2 = \frac{-3x+5}{2}$
$1.5x - 4 = -3x + 5$
$4.5x = 9$
$x = 2$

When $x = 2$,
$y = \frac{-3(2)+5}{2}$
$= -0.5$
C $(2, -\frac{1}{2})$

Example: O Level 2011 Paper 1 Question 19



The diagram, which is not drawn accurately, shows the three lines $x = 8$, $y = 6 - x$ and $2y = 3x + 2$. Note: You need to be able to identify the equations of the curve. It is not provided in the question. The gradient of the lines will be the clue.

(a) Find the coordinates of A and B

Substitute $x = 8$ into $y = 6 - x$,
$y = 6 - 8$
$= -2$
$\therefore$ A (8, -2)
Substitute $x = 8$ into $2y = 3x + 2$,
$2y = 3(8) + 2$
$2y = 26$
$y = 13$
$\therefore$ B (8, 13)

(b) Find the gradient of the line $y = 6 - x$.
$y = mx + c$
$y = -x + c$
Therefore, gradient of the line is -1.

(c) The point (0, k) is the same distance from A as it is from B. Find the value of k.
Length of A from point = Length of B from point
$\sqrt{(0-8)^2 + (k-(-2))^2} = \sqrt{(0-8)^2 + (k-13)^2}$
$64 + (k+2)^2 = 64 + (k-13)^2$
$(k+2)^2 = (k-13)^2$
$k^2 + 4k + 4 = k^2 - 26k + 169$
$30k = 165$
$k = 5.5$
Note: $(k + 2)^2$ to follow the expansion rules of $(a + b)^2$

G7. Vectors in Two Dimensions

G7.1 Converting Coordinates into Column Vector

When the question gives the coordinates of A as (1, -2), it is written in column vector as $\begin{pmatrix} 1 \\ -2 \end{pmatrix}$.

$\underset{OA}{\rightarrow} = \begin{pmatrix} 1 \\ -2 \end{pmatrix}$

$\underset{AO}{\rightarrow} = -\begin{pmatrix} 1 \\ -2 \end{pmatrix}$

$= \begin{pmatrix} -1 \\ 2 \end{pmatrix}$

G7.2 Addition of Vectors

$\underset{AB}{\rightarrow} = \underset{AO}{\rightarrow} + \underset{OB}{\rightarrow}$

The alphabets circled in orange must follow the desired vector and that the alphabet in the middle must be the same in order to apply the triangle law for addition of vectors.

G7.3 Parallel Vectors

Two vectors are the same when they are in the same direction.

Proving of Parallel Vectors

Example:

$\underset{AB}{\rightarrow} = 2\mathbf{a} + 3\mathbf{b}$ and $\underset{CD}{\rightarrow} = 6\mathbf{a} + 9\mathbf{b}$

$= 3\,(2\mathbf{a} + 3\mathbf{b})$

$= 3\underset{AB}{\rightarrow}$

Since $\underset{CD}{\rightarrow} = 3\underset{AB}{\rightarrow}$, therefore, CD is parallel to AB.

Determining one of the coordinates when given that AB is parallel to CD.

Example: Given that $\underset{AB}{\rightarrow} = \begin{pmatrix} 2 \\ -2 \end{pmatrix}$ and $\underset{CD}{\rightarrow} = \begin{pmatrix} 5 \\ b \end{pmatrix}$. Determine the value of b.

$\underset{AB}{\rightarrow} // \underset{CD}{\rightarrow}$

$\underset{AB}{\rightarrow} = k\,\underset{CD}{\rightarrow}$

$\begin{pmatrix} 2 \\ -2 \end{pmatrix} = k\begin{pmatrix} 5 \\ b \end{pmatrix}$

2 = 5k

k = 0.4

When k = 0.4,

-2 = 0.4b

b = -5

G7.4 Collinear Vectors

Example: Given that $\underset{AB}{\rightarrow}$ = 2**a** + 3**b** and $\underset{BD}{\rightarrow}$ = 6**a** + 9**b**. Determine the value of b.

$\underset{AB}{\rightarrow}$ = 2**a** + 3**b** and $\underset{BD}{\rightarrow}$ = 6**a** + 9**b**

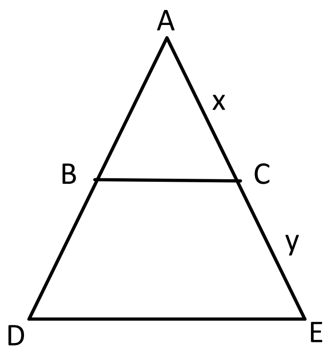
= 3 (2**a** + 3**b**)

= 3$\underset{AB}{\rightarrow}$

Since $\underset{AB}{\rightarrow}$ = 3$\underset{BD}{\rightarrow}$ and B is the common point, therefore, A, B and D lies on at straight line/ are collinear.

G7.5 Determining the Ratios of the Area of the Triangles in Vectors

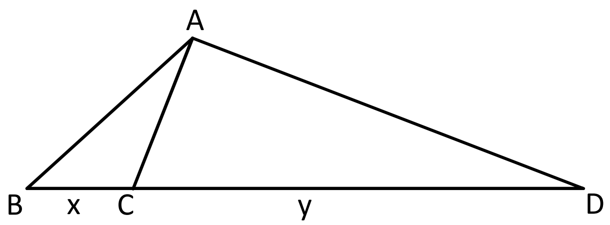
Similar triangles



$$\frac{A_1}{A_2} = (\frac{l_1}{l_2})^2$$

$$\frac{Area\ of\ \Delta ABC}{Area\ of\ \Delta ADE} = (\frac{x}{x+y})^2$$

Triangles with Common Height



$$\frac{Area\ of\ \Delta ABC}{Area\ of\ \Delta ABD} = \frac{\cancel{\frac{1}{2}} x\ b\ x \cancel{h}}{\cancel{\frac{1}{2}} x\ b\ x \cancel{h}}$$

$$= \frac{x}{x+y}$$

G7.6 Question on Vectors

Example: O Level 2011 Paper 2 Question 8 (a)

a) P is the point (3, 4). Q is the point (-1, 2). Write down the column vector $\overrightarrow{PQ}$

$\overrightarrow{PQ} = \overrightarrow{PO} + \overrightarrow{OQ}$

$= \begin{pmatrix} -3 \\ -4 \end{pmatrix} + \begin{pmatrix} -1 \\ 2 \end{pmatrix}$

$= \begin{pmatrix} -4 \\ -2 \end{pmatrix}$

Note: When given coordinates of P, it means OP.

b) Find $|\overrightarrow{PQ}|$

$|\overrightarrow{PQ}| = \sqrt{(-4)^2 + (-2)^2}$

= 4.47 units (3 s.f.)

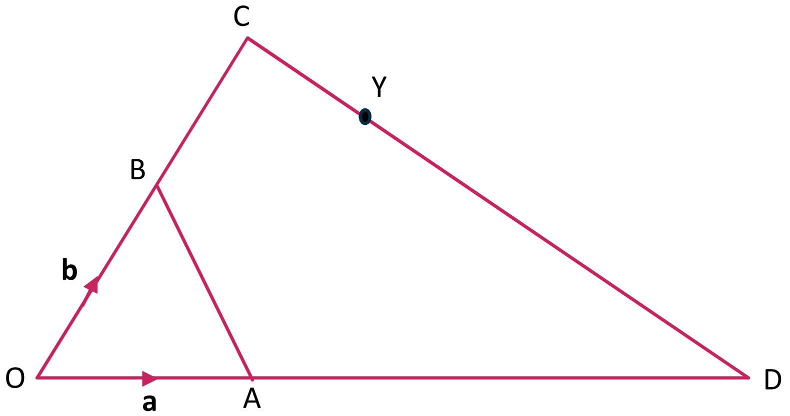
c) Given that $\overrightarrow{PL} = \frac{1}{2}|\overrightarrow{PQ}|$, find $\overrightarrow{PL}$

$|\overrightarrow{PL}| = \frac{1}{2}|\overrightarrow{PQ}|$

$= \frac{1}{2}\begin{pmatrix} -4 \\ -2 \end{pmatrix}$

$= \begin{pmatrix} -2 \\ -1 \end{pmatrix}$

Example: O Level 2011 Paper 2 Question 8 (b)



In the diagram, $\overrightarrow{OA}$ = **a**,

$\overrightarrow{OB}$ = **b**

$\overrightarrow{OB} = 2\overrightarrow{BC}$

$\overrightarrow{AD} = 2\overrightarrow{OA}$

$\overrightarrow{AB} = 2\overrightarrow{AX}$.

Express, as simply as possible, in terms of **a** and **b**. Note: Label as much information on the diagram as possible especially the ratios!

(a) $\overrightarrow{AB}$

$\overrightarrow{AB} = \overrightarrow{AO} + \overrightarrow{AB}$

$= -\mathbf{a} + \mathbf{b}$

(b) $\overrightarrow{OX}$

$\overrightarrow{AX} = \frac{1}{2}\overrightarrow{AB}$

$= \frac{1}{2}(-\mathbf{a} + \mathbf{b})$

$\overrightarrow{OX} = \overrightarrow{OA} + \overrightarrow{AX}$

$= a + \frac{1}{2}(-\mathbf{a} + \mathbf{b})$

$= \mathbf{a} - \frac{1}{2}\mathbf{a} + \frac{1}{2}\mathbf{b}$

$= \frac{1}{2}\mathbf{a} + \frac{1}{2}\mathbf{b}$

$= \frac{1}{2}(\mathbf{a} + \mathbf{b})$

(c) $\overrightarrow{CD}$

$\overrightarrow{OC} = \frac{3}{2}\overrightarrow{OB}$

$= \frac{3}{2}\mathbf{b}$

$\overrightarrow{OD} = 3\overrightarrow{OA}$

$= 3\mathbf{a}$

$\overrightarrow{CD} = \overrightarrow{CO} + \overrightarrow{OD}$

$= -\frac{3}{2}\mathbf{b} + 3\mathbf{a}$

$= 3\mathbf{a} - \frac{3}{2}\mathbf{b}$

(d) Y is the point on CD such that CY: YD = 1:2. Express $\overrightarrow{OY}$ in terms of **a** and **b**.

$\overrightarrow{CY} = \frac{1}{3}\overrightarrow{CD}$

$= \frac{1}{3}(3\mathbf{a} - \frac{3}{2}\mathbf{b})$

$= \mathbf{a} - \frac{1}{2}\mathbf{b}$

$\overrightarrow{OY} = \overrightarrow{OC} + \overrightarrow{CY}$

$= \frac{3}{2}\mathbf{b} + \mathbf{a} - \frac{1}{2}\mathbf{b}$

$= \mathbf{a} + \mathbf{b}$

(e) Hence, write down two facts about O, and Y.

$\overrightarrow{OX} = \frac{1}{2}(\mathbf{a} + \mathbf{b})$

$= \frac{1}{2}\overrightarrow{OY}$

Since $\overrightarrow{OX} = \frac{1}{2}\overrightarrow{OY}$ and O is the common point, therefore, O, X and Y are collinear.

G8. Problems in Real World Context

To be updated in year 2017.

STATISTICS AND PROBAILITY

S1. Data Analysis

S1.1 Construction and Interpretation of Tables

Example: O Level 2009 Paper 2 Question 10 (a)

The thirty pupils in a class each took a Mathematics test. Their scores are shown in the table below.

6	7	4	8	5	6	6	7	5	6	7	6	8	6	7
7	10	6	7	6	5	8	6	6	7	5	6	7	4	5

(a) Construct a frequency table from this information.

Score (x)	**Tally**	**Frequency**
4	//	2
5	~~////~~	5
6	~~////~~ ~~////~~	11
7	~~////~~ ///	8
8	///	3
9		0
10	/	1

(b) Calculate the mean score.

Mean score = $\frac{(2\ x\ 4)+(5\ x\ 5\ )+(6\ x\ 11)+(7\ x\ 8\ )+(8\ x\ 3\ )+(9\ x\ 0)+(10\ x\ 1)}{30}$

= 6.3

(c) Calculate the standard deviation.

$fx^2 = (2 \times 4^2) + (5 \times 5^2) + (11 \times 6^2) + (8 \times 7^2) + (3 \times 8^2) + (0 \times 9^2) + (1 \times 10^2)$

= 1237

Standard deviation = $\sqrt{\frac{1237}{30} - 6.3^2}$

= 1.24 (3 s.f.)

Example: O Level 2014 Paper 1 Question 5

Alyssa wants to find out how much time students spend on the Internet. She uses this question as a questionnaire.

How many hours do you spend on the Internet? Tick one box			
☐ 1 – 2	☐ 3 – 4	☐ 5 – 7	☐ Over 7

List two things wrong with this question.

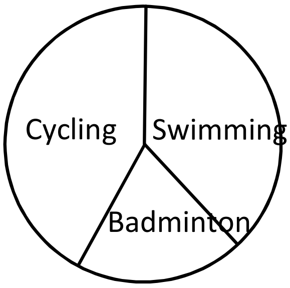
1. Time limit such as per day, per week or per month is not set
2. The intervals for the number of hours spent are not continuous.
3. Duration of each option is not constant.

S1.2 Pie Chart

Formula to determine angle representing sector = $\frac{sector\ required}{total}$ x 360°

Formula to calculate number of subjects given angle = $\frac{angle\ given}{360}$ x total number of subjects

Example: N Level 2012 Paper 1 Question 15



The pie chart represents the time Brian spent swimming, cycling and playing badminton one week. The total time he spent on these three sports that week was 15 hours. The angle representing the time spent swimming is 140°.

(a) That week Brian spent 7 hours cycling. Calculate the angle of the sector representing cycling.

Angle of the sector = $\frac{7}{15}$ x 360°

= 168°

(b) On each of the seven days of the week, Brian spent the same amount of time swimming. How many minutes did he spend swimming each day?

Amount of time spent swimming = $\frac{140}{360}$ x 15

= $5\frac{5}{6}$ hours

= 5 hours 50 min

= 350 min

Amount of time spent swimming each day = 350 ÷ 7

= 50 min

S1.3 Pictogram

When doing question on pictogram, take note how many objects does each shape represents

Example: N Level 2013 Paper 1 Question 4

The pictogram represents the survey of the colour of each of the cars in a carpark

Red	● ● ● ◖
Blue	● ● ● ● ●
Silver	● ● ● ● ● ●
White	● ● ● ◖
Black	● ● ● ●
Others	● ● ● ● ◖

● represents 2 cars

(a) How many cars are there in the car park?

Total number of cars = (3.5 x 2) + (5 x 2) + (6 x 2) + (3.5 x 2) + (4 x 2) + (4.5 x 2)
= 52

(b) How many more silver cars are there than white cars?

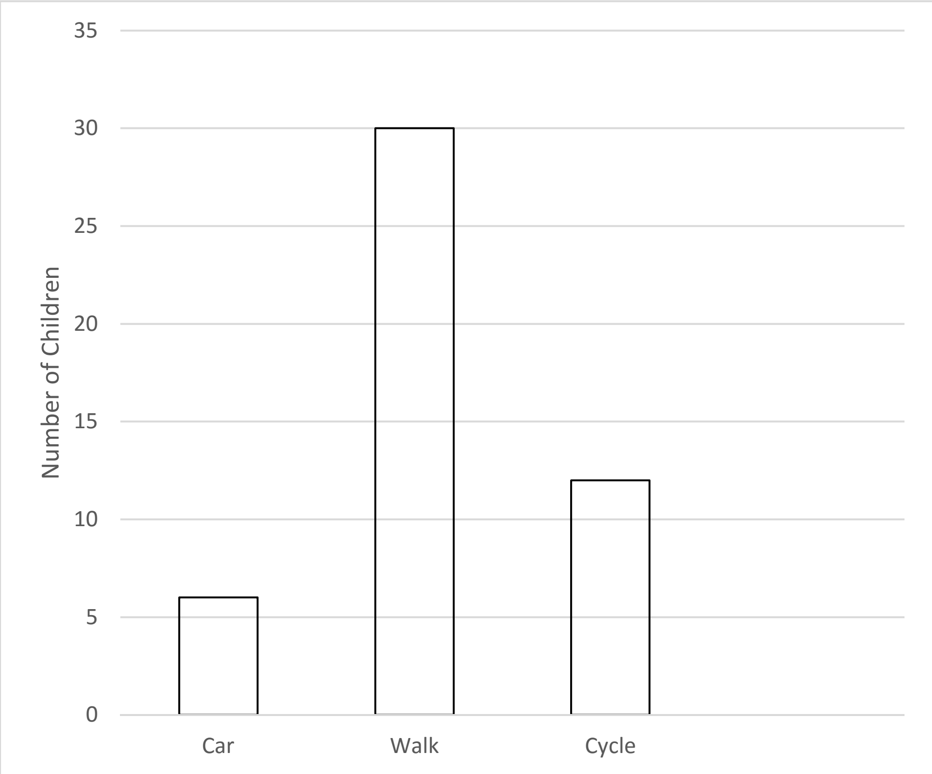
Number of silver cars = 6 x 2
= 12

Number of white cars = 3.5 x2
= 7

Difference = 12 – 7
= 5

S1.4 Bar Graph

Example: O Level 2008 Paper 1 Question 2



In a survey, 48 children were asked how they travelled to school. The results of the survey are shown in the bar chart.

(a) Express the total number of children who walked or cycled as a fraction of the total number of children. Give your answer in its lowest terms.

Fraction = $\frac{30 + 12}{6+30+}$

$= \frac{7}{8}$

(b) The same information is to be shown in a pie chart. Find the angle which represents the children who travelled by car.

Angle representing the children who travelled by car = $\frac{6}{48} \times 360^{o}$

$= 45^{o}$

S1.5 Histogram

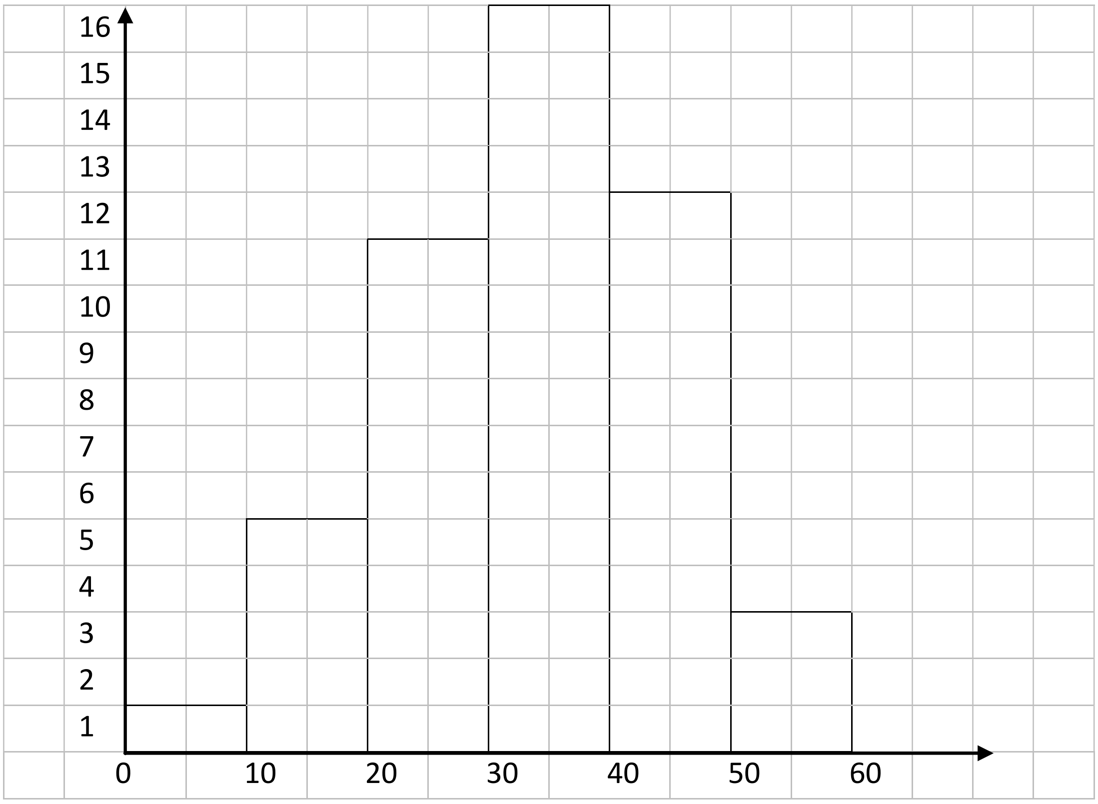
Example: N Level 2012 Paper 1 Question 20

The table shows the distribution of the times taken by 48 mathematicians to solve a mathematical puzzle.

Time (t seconds)	Frequency
$0 < t \le 10$	1
$10 < t \le 20$	5
$20 < t \le 30$	11
$30 < t \le 40$	16
$40 < t \le 50$	12
$50 < t \le 60$	3

(a) On the grid, draw a histogram to represent this information.

Note: There are no gaps in a histogram



(b) Find the probability that one of these mathematicians, chosen at random, took more than 40 seconds to solve the puzzle. Give your answer as a fraction in its simplest form.

P (more than 40 seconds to solve the puzzle) $= \frac{12+3}{48}$

$= \frac{5}{16}$

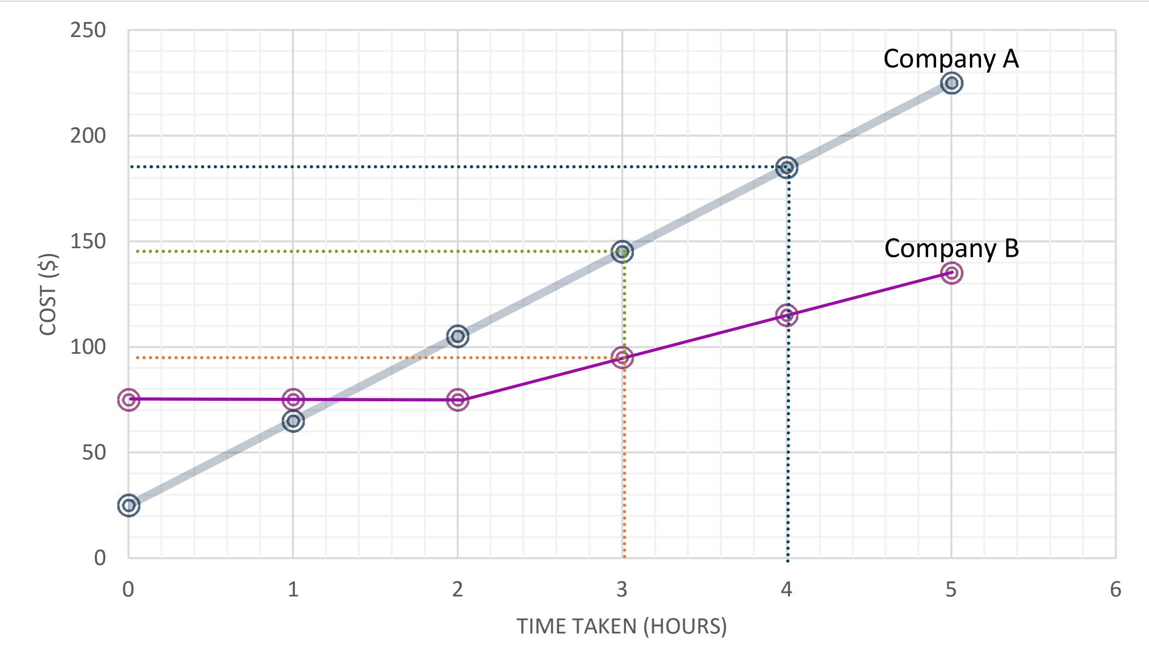
(c) A mathematician solve the puzzle in 20 seconds. In which time interval from the frequency table would this time be placed?

$10 < t \le 20$

S1.6 Line Graph

Example: N Level 2014 Paper 1 Question 9

Two companies, A and B carry out household plumbing repairs. The graph below is used by company A to work out the cost of a job.



(a) Find the cost for a job by company A that takes 4 hours. Shown in blue.
Cost for the job = \$185

(b) Company B charges a basic cost of \$75 for the first two hours plus a fixed rate of \$20 per hour thereafter. On the grid, draw a graph to show the charges made by company B for jobs of up to 5 hours. Shown in purple on the graph.

Note: Basic cost is read from the y-intercept. Fixed rate per hour is determined by the gradient of the line.

(c) Which company is cheaper for a job taking 3 hours and by how much? Company A is shown in green while Company B is shown in orange on the graph.
Difference = \$145 - \$95
= \$50
∴ Company B is cheaper for a job taking 3 hours by \$50.

S1.7 Mean, Mode and Median

Mean/ median

Mean is the average of a set of data. It can be determined by taking the total divided by the number of subjects/ objects in the question.

Median is an estimate of mean. Median lies at the $(\frac{n+1}{2})^{\text{th}}$ position and is the middle number of the series. It is found on the 50$^{\text{th}}$ percentile in a cumulative frequency curve.

Examples of cases where a higher mean or median is equal to a better performance are number of students passing a test, number of goals etc. Example of cases where a lower mean or median does not equate to a better performance is number of failures for a class test, waiting time of a restaurant, number of fine tickets issued etc.

Mode

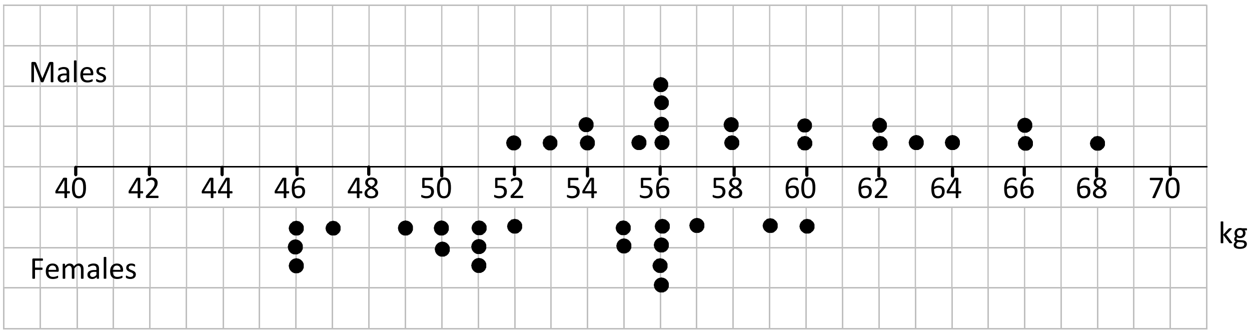
Mode is the value that occur the most. Note: students often write the frequency as the answer but it is the x value that is correct.

Standard deviation/ Interquartile range/ Range

Both standard deviation and interquartile range measures the consistency of the group of data. When the standard deviation or the interquartile range of A is higher than B, it means that A is less consistent than B. The interquartile range can be determined by determining the value of the lower quartile (25$^{\text{th}}$ percentile) and the value of the upper quartile (75$^{\text{th}}$ percentile). The difference of both values is found.

S1.8 Dot Diagram

Example: N level 2014 Paper 1 Question 18



Note: When counting the dots for the position of the median, be careful of the direction.

(a) Find the median mass for the males.

Position of the median = $\frac{20+1}{2}$

= 10.5^{th} position

Median mass for the males = $\frac{58+58}{2}$

= 58 kg

(b) Find the median mass for the females.

Position of the median = $\frac{20+1}{2}$

= 10.5^{th} position

Median mass for the females = $\frac{51+52}{2}$

= 51.5 kg

(c) Write down two comments comparing the masses of the male students and the female students.

1. Majority of the male students are heavier than the female students.
2. The median mass of 59.3 kg for the male students is heavier than the median mass for the females of 52.5 kg.

S1.9 Stem and Leaf Diagram

Example: O Level 2010 Paper 1 Question 18

		Boys							Girls			
					9	3	8					
			5	4	2	4						
		8	4	2	0	5	8	9	9			
	9	9	5	1	1	6	4	7	8	9		
7	7	5	5	5	2	7	1	3	5	6	8	9
					0	8	0					
		Key (Boys)							Key (Girls)			
		9|3	means 39						3|8	means 38		

(a) Write down the mode of the boy's marks

Mode = 75 marks

(b) Write down the median of the girl's marks.

Median position of the girl's mark = $\frac{15+1}{2}$

= 8^{th} position

Median = 69

(c) Explain briefly whether the boys or the girls performed better in the test.

Median position of the boy's mark = $\frac{20+1}{2}$

= 10.5^{th} position

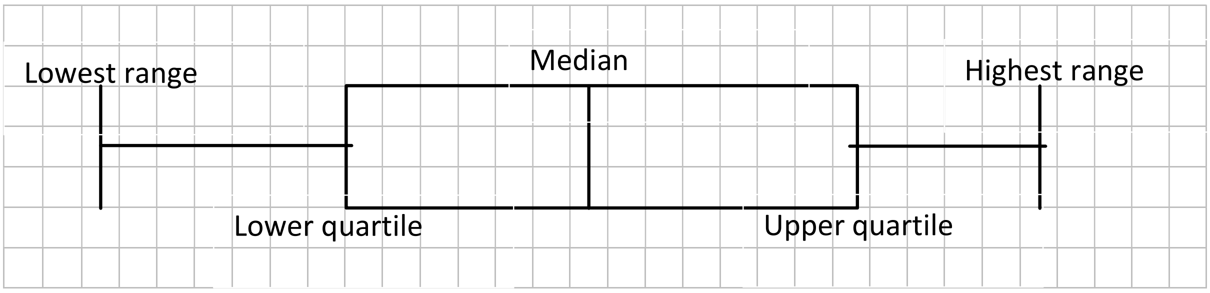
Median of the boy's mark = $\frac{61+65}{2}$

= 63

Since the median of the girls' marks is higher than the boys, the girls performed better in the test.

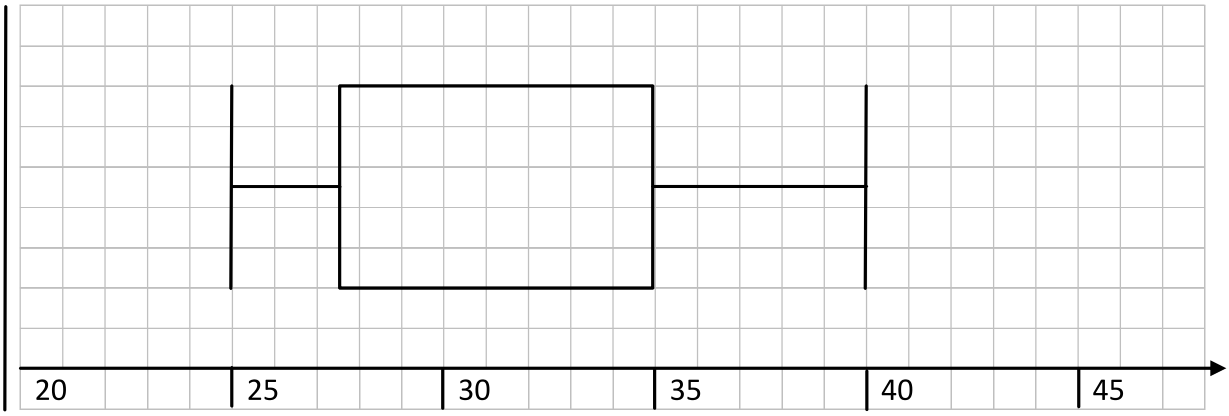
S1.10 Box and Whiskers

Interpretation of Box and Whiskers



Example: O level 2014 Paper 1 Question 10

A group of students were asked to estimate a time interval of 30 seconds. The results are represented in the box-and-whisker plot.



(a) Find the median of the estimates.

Median = 32 seconds

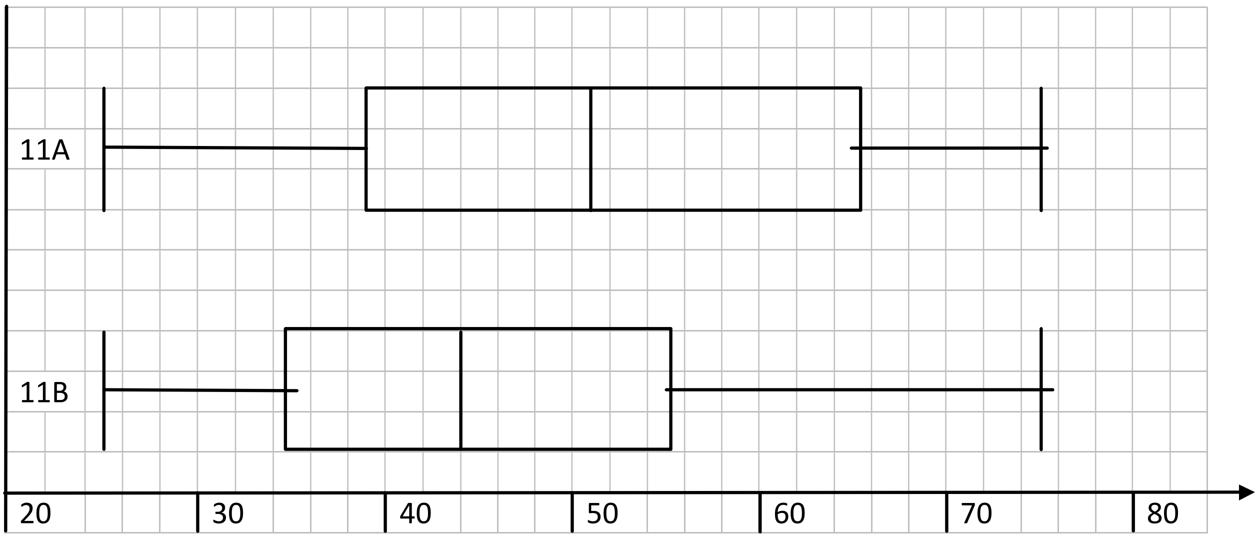
(b) Find the interquartile range of the estimates.

Interquartile range = 35 – 27.5

= 7.5 seconds

Example: N Level 2008 Paper 2 Question 11

Two classes, 11A and 11B, took a Mathematics test marked out of 80. The distribution of results is shown in the box and whisker plots below.



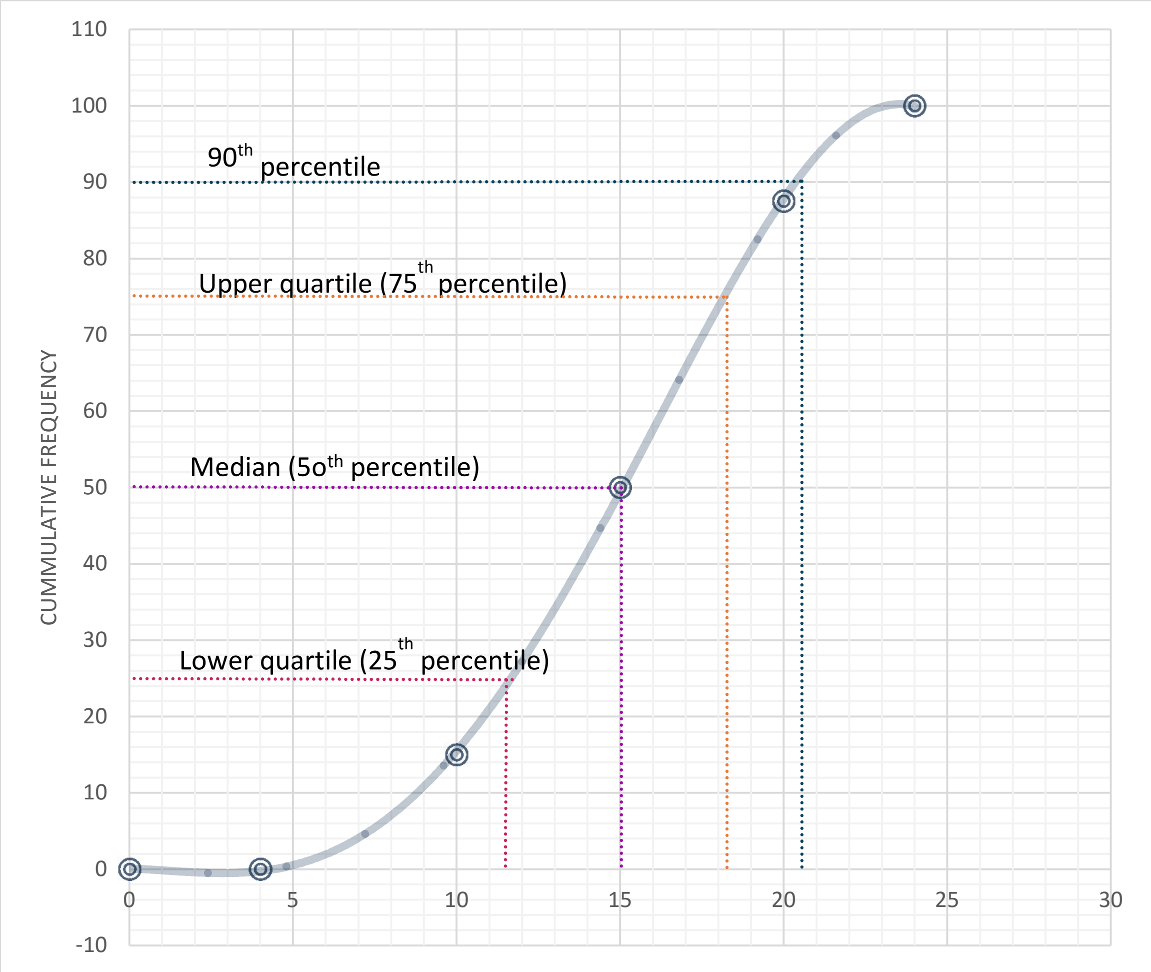
(a) Which class scored the better marks? Give a reason for your answer.
Class 11A scored better marks as the median mark of class 11A is higher than the median mark of class 11B.

(b) Find the interquartile range for 11A.
Interquartile range = 65 – 39
= 26 marks

(c) Which class had the wider spread? Give a reason for your answer.
Interquartile range for Class 11B = 55 – 35
= 20
Since the interquartile range for Class 11A is greater than Class 11B, Class 11A is said to be more widely spread/ less consistent in their results.

S1.11 Cumulative Frequency Curve

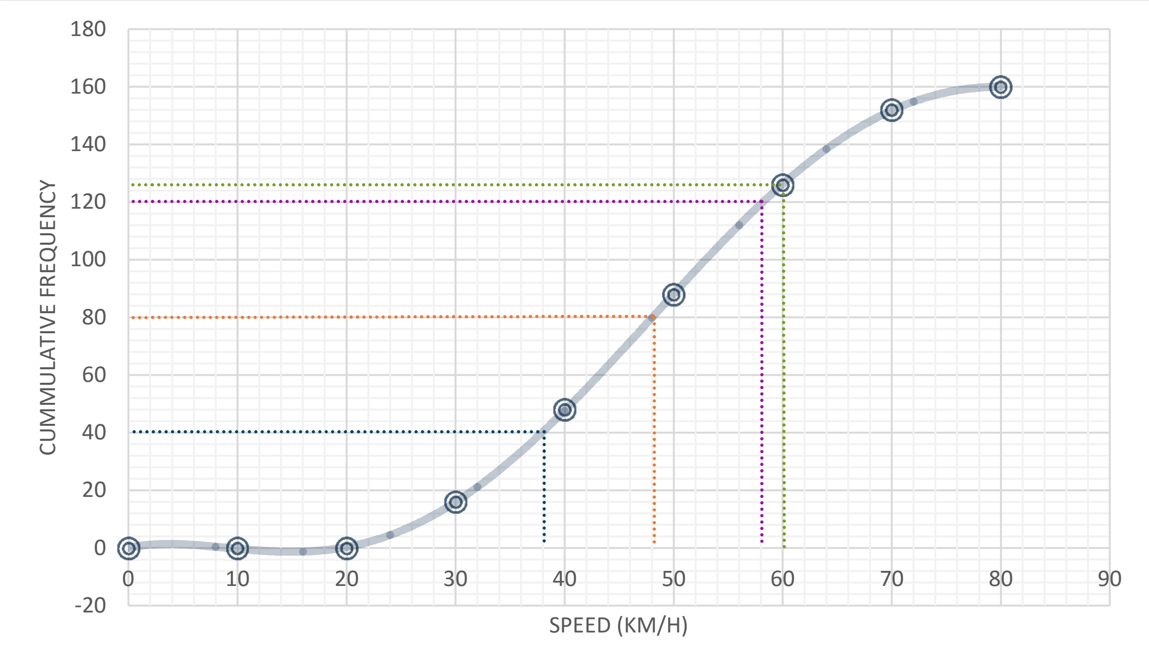
Interpretation of the Cumulative Frequency Curve



The values of the respective percentile are dependent on the total frequency of the set of the data. For example, if the total number of students is 240, to find the position of the median, the working will be $\frac{50}{100}$ x 240 = 120th position.

Example: N level 2013 Paper 2 Question 12 (a)

The speeds in km/h, of 160 cars travelling along a road were measured. The cumulative frequency graph summaries the results.



(a) Find the median speed. Shown in orange on the graph.

Position of median = $\frac{50}{100}$ x 160

= 80^{th} position

Median speed = 48 km/h

(b) Find the interquartile range.

Position of lower quartile = $\frac{25}{100}$ x 160

= 40^{th} position

Lower quartile = 38 km/h (Shown in dark blue on the graph)

Position of upper quartile = $\frac{75}{100}$ x 160

= 120^{th} position

Upper quartile = 58 km/h (Shown in purple on the graph)

Interquartile range = 58 – 38

= 20 km/h

(c) Find the percentage of the cars that were travelling faster than 60 km/h. Shown in green on the graph.

Number of cars that were travelling at less than 60 km/h = 126

Number of cars that were travelling at more than 60 km/h = 160 – 126

= 34

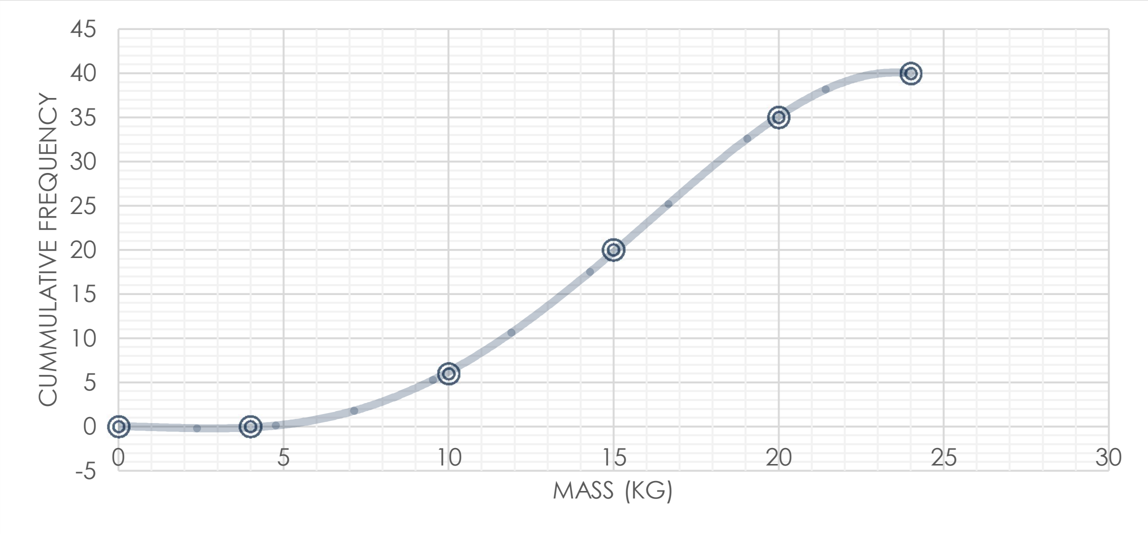
Percentage of cars that were travelling faster than 60 km/h = $\frac{34}{160}$ x 100%

= 21.25%

Note: Any numbers that is read off the graph is 'less than'. To find 'more than', take the total number from the cumulative frequency to minus off what was found.

Example: O Level 2008 Paper 2 Question 10

The total mass of the tomatoes produced by each of 40 tomato plants was measured. The cumulative frequency curve below shows the distribution of the masses.



(a) Copy and complete the grouped frequency table of the mass of the tomatoes on each plant. Answer is shown in purple.

Mass (kg)	$4 \le x < 8$	$8 \le x < 12$	$12 \le x < 16$	$16 \le x < 20$	$20 \le x < 24$
Frequency	3 – 0 = 3	10 – 3 = 7	24 – 10 = 14	35 – 24 = 11	40 – 35 = 5

Note: To determine the frequency, example, $4 \le x < 8$, determine the frequency at x = 4 and x = 8. Find the difference in both the values.

(b) Using your grouped frequency table, calculate an estimate of the mean mass of tomatoes produced by each plant

Estimated mass (kg)	6	10	14	18	22
Frequency	3	7	14	11	5

Estimated mean mass = $\frac{(6 \times 3)+(10 \times 7)+(14 \times 14)+(18 \times 11)+(22 \times 5)}{40}$

= 14.8 kg

(c) Calculate the standard deviation

$\Sigma f x^2 = 3(6)^2 + 7(10)^2 + 14(14)^2 + 11(18)^2 + 5(22)^2$

= 9536

Standard deviation = $\sqrt{\frac{9536}{40} - 14.8^2}$

= 4.4 kg

Note: Standard deviation requires units too

(d) The tomatoes produced by another group of 40 plants have the same median but a larger standard deviation. Describe how the cumulative frequency curve will differ from the given curve.
The cumulative frequency curve of the second group of plants will have a wider spread since it has a larger standard deviation. As a result, the gradient of the curve around the gradient will be less steep than the original curve.

S1.12 Calculations for Grouped Data

Example: N Level 2010 Paper 1 Question 25

Height (cm)	$135 < h \leq 145$	$145 < h \leq 155$	$155 < h \leq 165$
Frequency	9	18	3

(a) Calculate an estimate of the mean height of the 30 children.

Estimated height (cm)	140	150	160
Frequency	9	18	3

$$\text{Estimated mean} = \frac{(140 \times 9)+(150 \times 18) + (160 \times 3)}{30}$$
$$= 148 \text{ cm}$$

(b) Explain why this is only an estimate of the mean.
Since the frequency table does not show the heights of the individual children, this is only an estimate of the mean.

(c) Calculate the greatest possible mean height of the 30 children.
$\Sigma f x^2 = 9(140)^2 + 18(150)^2 + 3(160)^2$
$= 658200$

$$\text{Standard deviation} = \sqrt{\frac{658200}{30} - 148^2}$$
$$= 6 \text{ cm}$$

Greatest possible mean height = mean + standard deviation
= 148 + 6
= 154 cm

(d) Explain if the range of the data is 30 cm.
No. the table does not show if the minimum height is 135 cm and the maximum height is 165 cm. therefore, it cannot be concluded that the range of the data is 30 cm.

Example: O Level 2014 Paper 2 Question 10 (a)

The table below summarises the times taken by 100 males to complete a 10km race.

Time (t minutes)	$30 \le t < 40$	$40 \le t < 50$	$50 \le t < 60$	$60 \le t < 70$	$70 \le t < 80$
Frequency	15	32	30	16	7

(a) What percentage of the males ran faster than 10 km/h?

For males to be faster than 10 km/h, they must take less than 60 min to finish the race since speed = $\frac{10\ km}{1\ hour}$ = 10km/h and 1 h = 60 minutes.

Percentage of the males that ran faster than 10 km/h

= $\frac{15+32+30}{100}$ x 100%

= 77%

(b) Calculate an estimate of the mean time

Estimated Time (t minutes)	35	45	55	65	75
Frequency	15	32	30	16	7

Mean = $\frac{(35\ x\ 15)+(45\ x\ 32)+(55\ x\ 30)+(65\ x\ 16)+(75\ x\ 7)}{100}$

= 51.8 min

(c) Calculate the standard deviation

$fx^2 = 35^2 \times 15 + 45^2 \times 32 + 55^2 \times 30 + 65^2 \times 16 + 75^2 \times 7$

= 280900

Standard deviation = $\sqrt{\frac{280900}{100} - 51.8^2}$

= 11.2 min (3 s.f.)

(d) The mean time for females to complete the race was 58.3 minutes and the standard deviation was 9.6 minutes. Make two comparisons between the times for males and the times for females.

Since the mean time for females to complete the race is higher than the mean time taken for the males to complete the race, the females is said to be slower since they take a longer time to finish the race.

Since the standard deviation of the times taken by females is more than the times taken by males, the females are said to be less consistent in their timings than the males.

S2. Probability

S2.1 Introduction of Probability

Probability is the chance that an event will occur. A probability of 0 means that the event will never occur while the probability of 1 means that the event will definitively occur, therefore the ranges of P is $0 \leq P \leq 1$.

Tree diagram and possibility diagrams are visual aids that helps to list out the possibilities of the events that may occur.

When the word 'or' is used in the question, the probabilities are added together. When the word 'and' is used in the question, the probabilities are multiplied together.

S2.2 Problems Involving Probability

Example: O Level 2012 Paper 1 Question 3

A bag contains 10 red marbles, 5 blue marbles and 3 yellow marbles.

(a) A marble is chosen at random and then replaced. What is the probability that it is a red marble?

$$P(\text{red marble}) = \frac{Number\ of\ red\ marbles}{Total\ number\ of\ marbles} = \frac{10}{18} = \frac{5}{9}$$

(b) How many more blue marbles must be placed in the bag on that the probability of choosing a blue marble would be $\frac{1}{2}$?

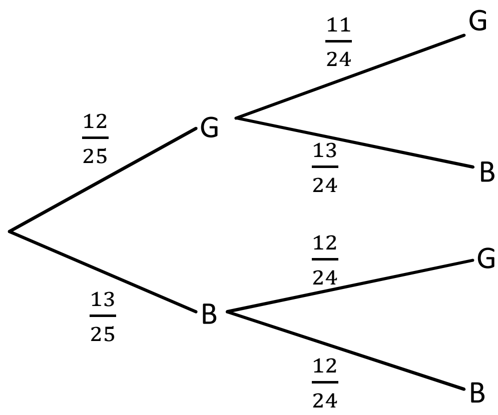
Let x be the number of blue marbles

$$\frac{5+x}{18+x} = \frac{1}{2}$$
$$2(5 + x) = 18 + x$$
$$10 + 2x = 18 + x$$
$$2x - x = 18 - 10$$
$$x = 8$$

Note: When blue marbles are added to the bag, the total number of marbles will increase as well.

Example: O Level 2014 Paper 2 Question 10 (b)

In a class of 25 children, there are 13 boys and 12 girls. Two of the children are selected at random to represent the class at a conference. Draw a tree diagram to show the probabilities of the possible outcomes.



(a) Find as a fraction in its simplest form, the probability that two girls are selected

P (two girls are selected) = $\frac{12}{25} \times \frac{11}{24}$

$= \frac{11}{50}$

(b) Find the probability that one boy and one girl are selected

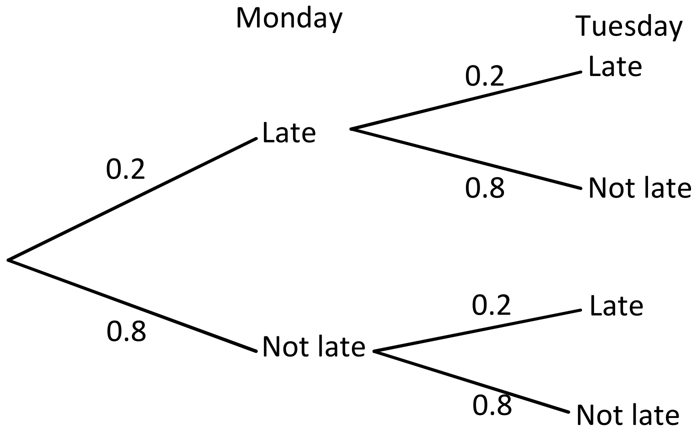
P (one girl and one boy are selected) = $(\frac{13}{25} \times \frac{12}{24}) + (\frac{12}{25} \times \frac{13}{24})$

$= \frac{13}{25}$

Example: N Level 2008 Paper 2 Question 11 (b)

Kim catches a bus to school. On any given day the probability that the bus is late is 0.2.

(a) Copy and complete the tree diagram showing the probabilities that the bus is late or not on the first two days of the week. Shown in purple.



(b) Calculate the probability that the bus is late on both Monday and Tuesday.

P (late on Monday and Tuesday) = 0.2 x 0.2
= 0.04

(c) Calculate the probability that the bus is late on one of these two days but not the other.

P (late on one of the days) = (0.2 x 0.8) + (0.8 x 0.2)
= 0.32

Example: N Level 2014 Paper 1 Question 2

A bag contains only green, blue and yellow counters. The probability of taking a green counter from the bag is 0.45. The probability of taking a blue counter from the bag is 0.3. A counter is taken from the bag at random.

(a) Find the probability that the counter is blue or green.

P (counter is blue or green) = 0.3 + 0.45
= 0.75

(b) Find the probability that the counter is yellow.

P (counter is yellow) = 1 – 0.3 – 0.45
= 0.25

Example: O Level 2013 Paper 2 Question 10 (a)

A bag contains five counters, numbered 1, 2, 3, 4, 5. Two counters are taken from the bag at random, one after the other, without replacement.

(a) Draw a possibility diagram to represent the outcomes.

	1	**2**	**3**	**4**	**5**
1		(1, 2)	(1, 3)	(1, 4)	(1, 5)
2	(2, 1)		(2, 3)	(2, 4)	(2, 5)
3	(3, 1)	(3, 2)		(3, 4)	(3, 5)
4	(4, 1)	(4, 2)	(4, 3)		(4, 5)
5	(5, 1)	(5, 2)	(5, 3)	(5, 4)	

Note: Since the counters are removed without replacement, no two counters of the same number can be removed from the bag. Do not count the grey squares as part of the total.

(b) Find in its simplest form, the probability that both counters have numbers less than 3

P (both counters with numbers less than 3) = $\frac{2}{10}$

= $\frac{1}{10}$

(c) Find the probability that neither counter has an even number

P (neither counter has an even number) = $\frac{3}{10}$

(d) Find the probability that the sum of the numbers is 10.

P (sum of the number is 10) = 0

(e) Find the probability that the product of the numbers is less than 6.

P (product of the numbers is less than 6) = $\frac{8}{20}$

= $\frac{2}{5}$