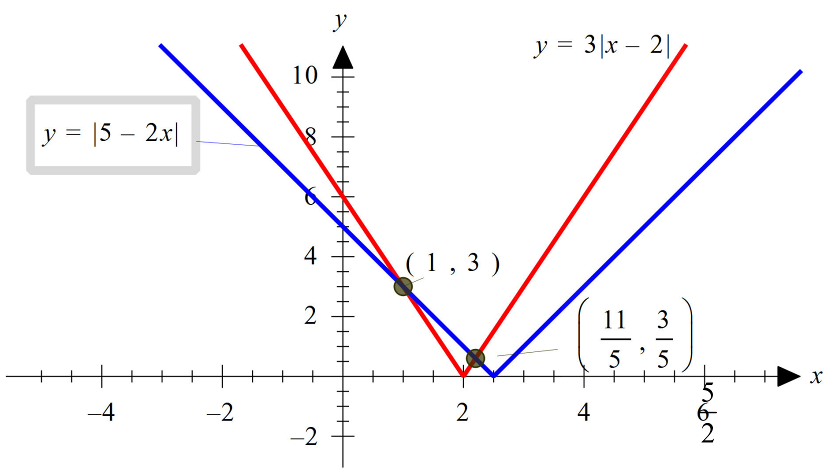
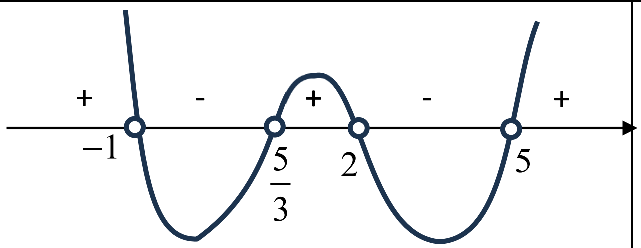
Raffles Institution
H2 Mathematics (9758)
Solution for 2023 A-Level Paper 2

Section A: Pure Mathematics

Question 1

No.	Suggested Solution	Remarks for Student
(a)	$3\lvert x-2\rvert < \lvert 5-2x\rvert$ $\Rightarrow 9(x-2)^2 < (5-2x)^2$ $\Rightarrow 9(x-2)^2-(5-2x)^2 < 0$ $\Rightarrow (3x-6+5-2x)(3x-6-5+2x) < 0$ $\Rightarrow (x-1)(5x-11) < 0$ $\Rightarrow 1 < x < \frac{11}{5}$ The required set is $\left\{x \in \mathbb{R} : 1 < x < \frac{11}{5}\right\}$ **Alternatively,**  From the graphs, $1 < x < \frac{11}{5}$ The required set is $\left\{x \in \mathbb{R} : 1 < x < \frac{11}{5}\right\}$	
(b)	$\frac{x+25}{x^2-4x-5}+3=\frac{x+25+3\left(x^2-4x-5\right)}{x^2-4x-5}=\frac{3x^2-11x+10}{x^2-4x-5}=\frac{(3x-5)(x-2)}{(x-5)(x+1)}$	

	$\dfrac{x+25}{x^2-4x-5} > -3$ $\Rightarrow \dfrac{x+25}{x^2-4x-5}+3>0$ $\Rightarrow \dfrac{(3x-5)(x-2)}{(x-5)(x+1)}>0$ $\Rightarrow (x+1)(3x-5)(x-2)(x-5)>0$ $\Rightarrow x<-1 \quad \text{or} \quad \dfrac{5}{3}<x<2 \quad \text{or} \quad x>5$  	

Question 2

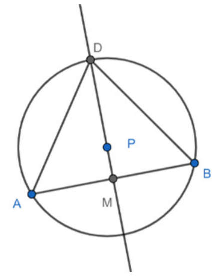
No.	Suggested Solution	Remarks for Student
(a)	$y=\ln(\sec x) \Rightarrow \dfrac{\mathrm{d}y}{\mathrm{d}x}=\dfrac{\sec x\tan x}{\sec x}=\tan x$ $\Rightarrow \dfrac{\mathrm{d}^2y}{\mathrm{d}x^2}=\sec^2 x$ $\Rightarrow \dfrac{\mathrm{d}^3y}{\mathrm{d}x^3}=2\sec x(\sec x\tan x)$ $=2\sec^2 x\tan x$ $=2\left(\dfrac{\mathrm{d}^2y}{\mathrm{d}x^2}\right)\left(\dfrac{\mathrm{d}y}{\mathrm{d}x}\right)$ (Shown)	
(b)	$\dfrac{\mathrm{d}^3y}{\mathrm{d}x^3}=2\left(\dfrac{\mathrm{d}^2y}{\mathrm{d}x^2}\right)\left(\dfrac{\mathrm{d}y}{\mathrm{d}x}\right) \Rightarrow \dfrac{\mathrm{d}^4y}{\mathrm{d}x^4}=2\left(\dfrac{\mathrm{d}^3y}{\mathrm{d}x^3}\right)\left(\dfrac{\mathrm{d}y}{\mathrm{d}x}\right)+2\left(\dfrac{\mathrm{d}^2y}{\mathrm{d}x^2}\right)^2$ When $x=0,\ y=\ln(\sec 0)=0,\ \dfrac{\mathrm{d}y}{\mathrm{d}x}=\tan 0=0,\ \dfrac{\mathrm{d}^2y}{\mathrm{d}x^2}=\sec^2 0=1,$ $\dfrac{\mathrm{d}^3y}{\mathrm{d}x^3}=2(1)(0)=0,\ \dfrac{\mathrm{d}^4y}{\mathrm{d}x^4}=2(0)(0)+2(1^2)=2$ Therefore, $y=0+0x+\dfrac{x^2}{2!}+0x^3+\dfrac{2x^4}{4!}+\ldots=\dfrac{x^2}{2}+\dfrac{x^4}{12}+\ldots$	
(c)	When $x=\dfrac{1}{4}\pi,\ y=\ln\left(\sec\dfrac{\pi}{4}\right)=\ln\left(\sqrt{2}\right)=\dfrac{1}{2}\ln(2)$ $\approx \dfrac{\left(\frac{\pi}{4}\right)^2}{2}+\dfrac{\left(\frac{\pi}{4}\right)^4}{12}+\ldots$ $\approx \dfrac{\pi^2}{32}+\dfrac{\pi^4}{3072}$ Therefore, $\ln(2)\approx \dfrac{\pi^2}{16}+\dfrac{\pi^4}{1536}$	
(d)	$\displaystyle\int_0^{\frac{1}{10}\pi}\ln(\sec x)\,\mathrm{d}x \approx \int_0^{\frac{1}{10}\pi}\frac{x^2}{2}+\frac{x^4}{12}\,\mathrm{d}x$ $=0.0052187161=0.005219$ (to 4 s.f.)	Use GC to evaluate

Question 3

No.	Suggested Solution	Remarks for Student
(a)	$\overrightarrow{BC} = 2\overrightarrow{AB}$ $\overrightarrow{OC} - \overrightarrow{OB} = 2\left(\overrightarrow{OB} - \overrightarrow{OA}\right)$ $\overrightarrow{OC} = 3\overrightarrow{OB} - 2\overrightarrow{OA}$ $= 3\begin{pmatrix} 1 \\ -2 \\ 8 \end{pmatrix} - 2\begin{pmatrix} -1 \\ 2 \\ 5 \end{pmatrix} = \begin{pmatrix} 5 \\ -10 \\ 14 \end{pmatrix}$	
(b)	$\overrightarrow{OD} = \begin{pmatrix} 1 \\ 2 \\ d \end{pmatrix}$, $\overrightarrow{AD} = \begin{pmatrix} 1 \\ 2 \\ d \end{pmatrix} - \begin{pmatrix} -1 \\ 2 \\ 5 \end{pmatrix} = \begin{pmatrix} 2 \\ 0 \\ d-5 \end{pmatrix}$, $\overrightarrow{BD} = \begin{pmatrix} 1 \\ 2 \\ d \end{pmatrix} - \begin{pmatrix} 1 \\ -2 \\ 8 \end{pmatrix} = \begin{pmatrix} 0 \\ 4 \\ d-8 \end{pmatrix}$ $\left\|\overrightarrow{AD}\right\| = \left\|\overrightarrow{BD}\right\| \Rightarrow \left\|\begin{pmatrix} 2 \\ 0 \\ d-5 \end{pmatrix}\right\| = \left\|\begin{pmatrix} 0 \\ 4 \\ d-8 \end{pmatrix}\right\|$ $\Rightarrow 2^2 + (d-5)^2 = 4^2 + (d-8)^2$ $\Rightarrow 4 + d^2 - 10d + 25 = 16 + d^2 - 16d + 64$ $\Rightarrow 6d = 51$ $\Rightarrow d = \frac{17}{2}$	
(c)	Let angle $ADB = \theta$ $\cos\theta = \frac{\overrightarrow{AD}.\overrightarrow{BD}}{\left\|\overrightarrow{AD}\right\|\left\|\overrightarrow{BD}\right\|} = \frac{\begin{pmatrix} 2 \\ 0 \\ \frac{7}{2} \end{pmatrix} . \begin{pmatrix} 0 \\ 4 \\ \frac{1}{2} \end{pmatrix}}{\sqrt{4+\frac{49}{4}}\sqrt{16+\frac{1}{4}}} = \frac{7}{65}$ $\Rightarrow \theta = \cos^{-1}\frac{7}{65} = 83.81769576^\circ = 83.8^\circ$ (to 1 d.p.) Angle $ADB = 83.8^\circ$ (to 1 d.p.)	

(d) Since $\left|\overrightarrow{AD}\right| = \left|\overrightarrow{BD}\right|$, so ΔABD is an isosceles triangle.

Let M be the midpoint of of AB.



Then, $\overrightarrow{OM} = \dfrac{\begin{pmatrix} 1 \\ -2 \\ 8 \end{pmatrix} + \begin{pmatrix} -1 \\ 2 \\ 5 \end{pmatrix}}{2} = \begin{pmatrix} 0 \\ 0 \\ 6.5 \end{pmatrix}$

Equation of the line of DM is $\mathbf{r} = \begin{pmatrix} 0 \\ 0 \\ 6.5 \end{pmatrix} + \lambda \left[\begin{pmatrix} 1 \\ 2 \\ 8.5 \end{pmatrix} - \begin{pmatrix} 0 \\ 0 \\ 6.5 \end{pmatrix} \right]$

$$= \begin{pmatrix} 0 \\ 0 \\ 6.5 \end{pmatrix} + \lambda \begin{pmatrix} 1 \\ 2 \\ 2 \end{pmatrix}, \lambda \in \mathbb{R}$$

Since P lies on line DM, then

$$\overrightarrow{OP} = \begin{pmatrix} 0 \\ 0 \\ 6.5 \end{pmatrix} + \lambda \begin{pmatrix} 1 \\ 2 \\ 2 \end{pmatrix} = \begin{pmatrix} \lambda \\ 2\lambda \\ 6.5 + 2\lambda \end{pmatrix}, \quad \text{for some } \lambda \in \mathbb{R}$$

$$\left|\overrightarrow{DP}\right| = \left|\overrightarrow{BP}\right|$$

Since P is the centre of the circle, then DP = BP = radius of the ticle

$$\left| \begin{pmatrix} \lambda \\ 2\lambda \\ 6.5+2\lambda \end{pmatrix} - \begin{pmatrix} 1 \\ 2 \\ 8.5 \end{pmatrix} \right| = \left| \begin{pmatrix} \lambda \\ 2\lambda \\ 6.5+2\lambda \end{pmatrix} - \begin{pmatrix} 1 \\ -2 \\ 8 \end{pmatrix} \right|$$

$$\left| \begin{pmatrix} -1+\lambda \\ -2+2\lambda \\ -2+2\lambda \end{pmatrix} \right|^2 = \left| \begin{pmatrix} -1+\lambda \\ 2+2\lambda \\ -1.5+2\lambda \end{pmatrix} \right|^2$$

$$(-1+\lambda)^2 + (-2+2\lambda)^2 + (-2+2\lambda)^2 = (-1+\lambda)^2 + (2+2\lambda)^2 + (-1.5+2\lambda)^2$$

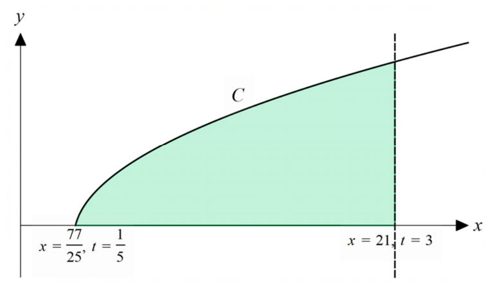
$$8 - 16\lambda + 8\lambda^2 = 4 + 8\lambda + 4\lambda^2 + 2.25 - 6\lambda + 4\lambda^2$$

$$18\lambda = \frac{7}{4}$$

$$\lambda = \frac{7}{72}$$

Therefore $\overrightarrow{OP} = \begin{pmatrix} 0 \\ 0 \\ 6.5 \end{pmatrix} + \dfrac{7}{72} \begin{pmatrix} 1 \\ 2 \\ 2 \end{pmatrix} = \begin{pmatrix} \frac{7}{72} \\ \frac{7}{36} \\ \frac{241}{36} \end{pmatrix}$

Question 4

No.	Suggested Solution	Remarks for Student
(a)	$x=2t^2+3 \Rightarrow \dfrac{\mathrm{d}x}{\mathrm{d}t}=4t$ When $t=\dfrac{1}{5}$, $\quad x=2\left(\dfrac{1}{5}\right)^2+3=\dfrac{77}{25}$ When $x=21$, $\quad 2t^2+3=21 \Rightarrow t=\sqrt{\dfrac{18}{2}}=3 \quad$ since $t \geq \dfrac{1}{5}$. Exact area between the curve C, the x-axis and the line $x=21$. $=\int_{\frac{77}{25}}^{21} y \, \mathrm{d}x$  $=\int_{\frac{1}{5}}^{3} (5t-1)(4t) \, \mathrm{d}t$ $=\int_{\frac{1}{5}}^{3} 20t^2-4t \, \mathrm{d}t$ $=\left[\dfrac{20t^3}{3}-2t^2\right]_{\frac{1}{5}}^{3}$ $=\dfrac{12152}{75}$ units^2	Note that the question asks for exact answer
(b)	Cartesian equation of D is $y=\dfrac{4}{\frac{x}{5}}=\dfrac{20}{x} \quad --(1)$ Substitute $x=2t^2+3$ and $y=5t-1$ into (1), $5t-1=\dfrac{20}{2t^2+3}$ $10t^3-2t^2+15t-3=20$ $10t^3-2t^2+15t-23=0$ $(t-1)(10t^2+8t+23)=0$ $t=1$ or $10t^2+8t+23=0$ Since discriminant$=8^2-4(10)(23)=-856<0$, the equation $10t^2+8t+23=0$, has no solution. Therefore, there is only one solution at $t=1$. When $t=1$, $\quad x=5$ and $y=4$ The curves C and D intersect at $A(5,\ 4)$ and there are no other points of intersections.	It is not enough to just say that there is just 1 answer for the value of t because this can be inferred from the question itself. You will need to show that t = 1 is the ONLY answer.
(c)	$x=2t^2+3 \Rightarrow \dfrac{\mathrm{d}x}{\mathrm{d}t}=4t$ and $y=5t-1 \Rightarrow \dfrac{\mathrm{d}y}{\mathrm{d}t}=5$ Therefore $\dfrac{\mathrm{d}y}{\mathrm{d}x}=\dfrac{5}{4t}$	

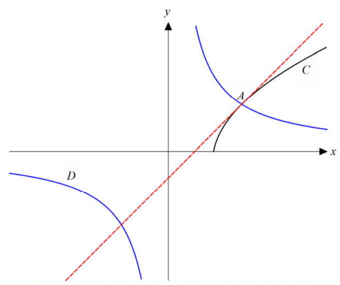
At A where $t=1$, $\dfrac{dy}{dx}=\dfrac{5}{4}$

Equation of the tangent of the curve at A is

$$y-4=\frac{5}{4}(x-5)\Rightarrow 4y-16=5x-25$$

$$\Rightarrow y=\frac{5}{4}x-\frac{9}{4}$$

When the tangent to the curve C at the point A meets the curve D,



$$\frac{20}{x}=\frac{5}{4}x-\frac{9}{4}$$

$$5x^2-9x-80=0$$

$$(5x+16)(x-5)=0$$

$$x=-\frac{16}{5} \quad \text{or} \quad x=5$$

When $x=-\dfrac{16}{5}$, $y=\dfrac{20}{-\frac{16}{5}}=-\dfrac{25}{4}$

The coordinates of the point where the tangent to the curve C at the point A meets the curve D for a second time is $\left(-\dfrac{16}{5},-\dfrac{25}{4}\right)$.

Section B: Probability and Statistics

Question 5

No.	Suggested Solution	Remarks for Student
(a)(i)	A and B are mutually exclusive. A and D are mutually exclusive.	
(ii)	$A=\{1, 3, 5, 7, ..., 35\} \quad \Rightarrow n(A)=18$ $C=\{3, 6, 9, 12, ..., 36\} \quad \Rightarrow n(C)=12$ $A\cap C=\{3, 9, 15, 21, 27, 33\} \Rightarrow n(A\cap C)=6$ $P(A)=\frac{18}{36}=\frac{1}{2}$ and $P(C)=\frac{12}{36}=\frac{1}{3}$ $\Rightarrow P(A)\times P(C)=\frac{1}{2}\times\frac{1}{3}=\frac{1}{6}$ $P(A\cap C)=\frac{6}{36}=\frac{1}{6}$ Since $P(A\cap C)=P(A)\times P(C)$, A and C are independent. Since A and B are complement, B and C are independent.	
(b)(i)	After the ball has become stuck in the slot labelled 36, $A=\{1, 3, 5, 7, ..., 35\}$ $B=\{2, 4, 6, 8, ..., 34\}$ $C=\{3, 6, 9, 12, ..., 33\}$ $D=\{6, 12, 18, 24, ..., 30\}$ A and B are still mutually exclusive. A and D are still mutually exclusive.	
(b)(ii)	$A=\{1, 3, 5, 7, ..., 35\} \quad \Rightarrow n(A)=18$ $C=\{3, 6, 9, 12, ..., 33\} \quad \Rightarrow n(C)=11$ $P(A)=\frac{18}{35}$ and $P(C)=\frac{11}{35}$ $\Rightarrow P(A)\times P(C)=\frac{18}{35}\times\frac{11}{35}=\frac{198}{1225}$ $A\cap C=\{3, 9, 15, 21, 27, 33\} \quad \Rightarrow n(A\cap C)=6$ $P(A\cap C)=\frac{6}{35}$ Since $P(A\cap C)\neq P(A)\times P(C)$, A and C are no longer independent.	

Question 6

No.	Suggested Solution	Remarks for Student
(a)	$P(\text{choosing 4 red counters}) = \dfrac{\binom{r}{4}\binom{b}{8}}{\binom{r+b}{12}}$ $P(\text{choosing 3 red counters}) = \dfrac{\binom{r}{3}\binom{b}{9}}{\binom{r+b}{12}}$ Since the above two probabilities are equal (given in the question), $\dfrac{\binom{r}{4}\binom{b}{8}}{\binom{r+b}{12}} = \dfrac{\binom{r}{3}\binom{b}{9}}{\binom{r+b}{12}}$ $\binom{r}{4}\binom{b}{8} = \binom{r}{3}\binom{b}{9}$ $\dfrac{r!}{4!(r-4)!} \times \dfrac{b!}{8!(b-8)!} = \dfrac{r!}{3!(r-3)!} \times \dfrac{b!}{9!(b-9)!}$ $4!(r-4)! \times 8!(b-8)! = 3!(r-3)! \times 9!(b-9)!$ $4(b-8) = 9(r-3)$ $4b - 32 = 9r - 27$ $9r + 5 = 4b \quad --(1) \quad \text{(Shown)}$	It is not surprising that the equation can be reduced to $\binom{r}{4}\binom{b}{8} = \binom{r}{3}\binom{b}{9}$ because the 2 probabilitie being equal is equivalent to saying the number of ways of choosing 4 red counters is equal to the number of ways of choosing 3 red counters
(b)	$P(\text{choosing 2 red counters}) = \dfrac{\binom{r}{2}\binom{b}{10}}{\binom{r+b}{12}}$ $P(\text{choosing 3 red counters}) = \dfrac{\binom{r}{3}\binom{b}{9}}{\binom{r+b}{12}}$ Therefore ,	

	$$\frac{\binom{r}{3}\binom{b}{9}}{\binom{r+b}{12}}=\frac{5}{3}\left[\frac{\binom{r}{2}\binom{b}{10}}{\binom{r+b}{12}}\right]$$ $$\binom{r}{3}\binom{b}{9}=\frac{5}{3}\binom{r}{2}\binom{b}{10}$$ $$\frac{r!}{3!(r-3)!}\times\frac{b!}{9!(b-9)!}=\frac{5}{3}\times\frac{r!}{2!(r-2)!}\times\frac{b!}{10!(b-10)!}$$ $$\frac{5}{3}\left[3!(r-3)!\times 9!(b-9)!\right]=2!(r-2)!\times 10!(b-10)!$$ $$\frac{5}{3}\times 3(b-9)=10(r-2)$$ $$5b-45=10r-20$$ $$10r+25=5b \quad --(2)$$ Solving (1) and (2), $r=15$ and $b=35$ The required probability $=P\left(1 \text{ red counter}\right)$ $=\frac{\binom{15}{1}\binom{35}{11}}{\binom{50}{12}}$ $=0.051551948=0.0516$ (to 3 s.f.)	Part **(b)** consists of two parts. Do **re-read the question** after completion to check whether all parts of the question are answered.

Question 7

No.	Suggested Solution	Remarks for Student
(a)(i)	The product moment correlation coefficient between x and y $= 0.9281271 = 0.9281$ (to 4 d.p.)	
(ii)	The product moment correlation coefficient between x and e^y $= 0.969724 = 0.9697$ (to 4 d.p.)	
(b)	Since the absolute value of the product moment correlation coefficient for $e^y = cx + d$ is closer to 1 than that of $y = ax + b$, $e^y = cx + d$ will gives a a better fit than . $y = ax + b$ The equation of the regression line for this model is $e^y = -1881.48 + 375.615x$ $e^y = -1880 + 376x$ (to 3 s.f.)	
(c)	For 2024, $x = 24$ and $y = \ln\left(-1881.481827 + 375.6152263(24)\right)$ $= 8.872526941 = 8.87$ (to 3 s.f.) The estimated number of mobile phone subscriptions is 8.87 million. This estimate is not reliable because $x = 24$ is not within the data range between 4 and 18 inclusive of 4 and 18.	

Question 8

No.	Suggested Solution	Remarks for Student
(a)(i)	Let μ (in kg) be the population mean mass of sugar in each bag. $H_0: \quad \mu = 1$ $H_1: \quad \mu \neq 1$	
(ii)	As the distribution of the mass of the bags of sugar is not known and the sample size of 10 bags is not large enough (>30) in order to use central limit theorem for the *z*-test, so it is not suitable. The sample as chosen in this way could be biased as it is not randomly chosen.	
(b)(i)	Let μ be the population mean mass of a bag of granulated sugar in kg, filled by the machine for 2 kg bag. $H_0: \mu = 2 \quad H_1: \mu < 2$ The sample mean, $\overline{x} = \frac{\sum x}{n} = \frac{78.88}{40} = \frac{493}{250} = 1.972$ The unbiased estimate for population variance, $s^2 = \frac{1}{n-1}\left(\sum x^2 - \frac{\left(\sum x\right)^2}{n}\right) = \frac{1}{39}\left(155.6746 - \frac{78.88^2}{40}\right) = 0.00316$ Perform a one-tailed test at 2.5% level of significance Under H_0, since $n = 40\ (> 30)$ is large, by Central Limit Theorem, $\overline{X} \sim \mathrm{N}\left(2, \frac{0.00316}{40}\right)$ approximately Using the *z*-test, $\mathrm{P}\left(\overline{X} < c\right) = 0.025 \Rightarrow c < 1.982579459$ The critical region is $\{c \in \mathbb{R} : 0 < c < 1.98\}$ (3 s.f.)	
(ii)	Since $\overline{x} = 1.972$ is inside the critical region, we reject H_0 and conclude that there is sufficient evidence at 2.5% level of significance that the mean mass of a bag of granulated sugar in kg, filled by the machine is less than 2 kg.	

Question 9

No.	Suggested Solution	Remarks for Student
(a)	Let L be the length, in metres, of a randomly chosen Long plank, therefore $L \sim \mathrm{N}\left(1.82,\ 0.2^2\right)$. Probability that the length of a randomly chosen Long plank is less than 1.79m $= P\left(L < 1.79\right) = 0.44038 \approx 0.440$ (3 s.f.)	
(b)	$L_1 + L_2 + L_3 + ... + L_8 \sim \mathrm{N}\left(8\left(1.82\right), 8\left(0.2\right)^2\right)$ $\Rightarrow L_1 + L_2 + L_3 + ... + L_8 \sim \mathrm{N}\left(14.56,\ 0.32\right)$ Probability that the total length of 8 randomly chosen Long planks is greater than 14.5 m $= P\left(L_1 + L_2 + L_3 + ... + L_8 > 14.5\right) = 0.542235 \approx 0.542$	
(c)	Let R be the length, in metres, of a randomly chosen Regular plank, therefore $R \sim \mathrm{N}\left(1.22,\ 0.3^2\right)$. Probability that the length of a randomly chosen Regular planks is longer than 1.25 m $= P\left(R > 1.25\right) \approx 0.460172$ Expected number of these 120 planks that are longer than 1.25 m $= 120 \times 0.460172 = 55.22064 \approx 55.2$ (correct to 3 s.f.)	
(d)	Consider $M = L_1 + L_2 + L_3 + ... + L_{10}$ and $V = R_1 + R_2 + R_3 + ... + R_{16}$ $W = L_1 + L_2 + L_3 + ... + L_{10} \sim \mathrm{N}\left(10\left(1.82\right), 10\left(0.2\right)^2\right)$ $\Rightarrow W \sim \mathrm{N}\left(18.2,\ 0.4\right)$ $V = R_1 + R_2 + R_3 + ... + R_{16} \sim \mathrm{N}\left(16\left(1.22\right), 16\left(0.3\right)^2\right)$ $\Rightarrow V \sim \mathrm{N}\left(19.52,\ 1.44\right)$ $V - W \sim \mathrm{N}\left(19.52 - 18.2,\ 1.44 + 0.4\right)$ $\Rightarrow V - W \sim \mathrm{N}\left(1.32,\ 1.84\right)$ Probability that the total length of 10 randomly chosen Long planks differs by less than 0.65m from the total length of 16 randomly chosen Regular planks $= P\left(\left\lvert V - W\right\rvert < 0.65\right) = 0.23746892 \approx 0.237$ (correct to 3 s.f.)	
(e)	Consider $K = \frac{1}{3}L - \frac{1}{2}R \sim \mathrm{N}\left(\frac{1}{3}\left(1.82\right) - \frac{1}{2}\left(1.2\right),\ \frac{1}{3^2}\left(0.2\right)^2 + \frac{1}{2^2}\left(0.3\right)^2\right)$ $\Rightarrow K \sim \mathrm{N}\left(-\frac{1}{300}, \frac{97}{3600}\right)$ Probability that the length of a randomly chosen Short plank made from a Long plank is greater than the length of one made from a Regular plank $= P\left(K > 0\right) = 0.491899 \approx 0.492$ (correct to 3 s.f.)	
(f)	The Long planks were cut twice and the Regular planks only once. There would be more wastage per Short plank produced from the Long planks than from the Regular planks. So, the answer to part **(e)** would be reduced.	

Question 10

No.	Suggested Solution	Remarks for Student
(a) (i)	• Any ornament made being faulty is independent of another ornament made being faulty. • The probability ornament made being faulty is constant throughout the day.	
(ii)	Let X be the number of ornaments out of 50 ornaments that are faulty. $X \sim \mathrm{B}(50,\ 0.04)$. $\mathrm{E}(X) = 50 \times 0.04 = 2; \quad \mathrm{Var}(X) = 50 \times 0.04 \times 0.96 = 1.92$ $\left\|\mathrm{E}(X) - \mathrm{Var}(X)\right\| = \left\|2 - 1.92\right\| = 0.08$ Therefore the numerical values of the mean and variance of this distribution differ by 0.08.	
(iii)	Probability that no more than 2 faulty ornaments are produced on a randomly chosen working day $= \mathrm{P}(X \le 2) = 0.67671 \approx 0.677$ (correct to 3 s.f.)	
(iv)	Let Y be the number of days, out of 5 days, that has no more than 2 faulty ornaments in each day. $Y \sim \mathrm{B}(5,\ 0.67671)$. Probability that no more than 2 faulty ornaments are produced on at least 3 days in a randomly chosen 5-day working week $= \mathrm{P}(Y \ge 3) = 1 - \mathrm{P}(Y \le 2)$ $= 0.804780 \approx 0.805 \quad (3\text{ s.f.})$	
(v)	Total number of ornaments produced in 5 days $5 \times 50 = 250$ Let K be the number of ornaments, out of 250 ornaments, that are faulty in 5 days $K \sim \mathrm{B}(250,\ 0.04)$. Probability that no more than 10 faulty items are produced in a randomly chosen 5-day working week. = Probability that no more than 10 faulty items are produced out of 250 ornaments $= P(Y \le 10) = 0.583057 \approx 0.583$ (correct to 3 s.f.)	
(b)	Probability that a pen **being faulty** is $1 - p$. Let L be the number of faulty pens, out of 6 pens. $L \sim \mathrm{B}(6, 1-p)$ P(Mr Lu's box is accepted) $= \mathrm{P}(L=0) + \mathrm{P}(L=1)$ $= p^6 + 6(1-p)p^5$ $= p^5(p + 6 - 6p)$ $= p^5(6 - 5p)$ Let M be the number of faulty pens, out of 3 pens.	

	$M \sim \mathrm{B}(3, 1-p)$ P(Mrs Ming's box is accepted) $= \mathrm{P}(M=0) + \mathrm{P}(M=1)\mathrm{P}(M=0)$ $= \mathrm{P}(M=0)\left[1+\mathrm{P}(M=1)\right]$ $= p^3\left[1+3(1-p)p^2\right]$ $= p^3\left(1+3p^2-3p^3\right)$ Consider $\mathrm{P}(\text{Mrs Ming's box is accepted}) - \mathrm{P}(\text{Mr Lu's box is accepted})$ $= p^3\left(1+3p^2-3p^3\right) - p^5(6-5p)$ $= p^3\left[\left(1+3p^2-3p^3\right) - p^2(6-5p)\right]$ $= p^3\left[1+3p^2-3p^3-6p^2+5p^3\right]$ $= p^3\left[1-3p^2+2p^3\right]$ $= p^3(p-1)^2(2p+1)$ Since $0<p<1$, then $p^3, (p-1)^2$ and $2p+1$ are positive, and hence $p^3(p-1)^2(2p+1)>0$. Thus, Mrs Ming accepts a greater proportion of boxes than Mr Lu does.	As this question asks for algebraic method, you **cannot** draw the graph of $y=x^3\left(1+3x^2-3x^3\right)$ and $y=x^5(6-5x)$, and then say that one graph is above the other for $0<x<1$.