Chapter 5 WORK, ENERGY AND POWER

Content

- Work
- Energy conversion and conservation
- Potential energy and kinetic energy
- Power

Learning Outcomes

Candidates should be able to:

(a) define and use work done by a force as the product of the force and displacement in the direction of the force.

(b) calculate the work done in a number of situations, *including the work done by a gas which is expanding against a constant external pressure:* $W = p\Delta V$ [1].

(c) give examples of energy in different forms, its conversion and conservation, and apply the principle of energy conservation.

(d) show an appreciation for the implications of energy losses in practical devices and use the concept of efficiency to solve problems.

(e) derive, from the equations for uniformly accelerated motion in a straight line, the equation $E_K = \frac{1}{2}mv^2$.

(f) recall and use the formula $E_K = \frac{1}{2}mv^2$.

(g) distinguish between gravitational potential energy, electric potential energy and elastic potential energy.

(h) deduce that the elastic potential energy in a deformed material is related to the area under the force extension graph.

(i) show an understanding of and use the relationship between force and potential energy in a uniform field to solve problems.

(j) derive, from the definition of work done by a force, the formula $E_P = mgh$ for gravitational potential energy changes near the Earth's surface.

(k) recall and use the formula $E_P = mgh$ for potential energy changes near the Earth's surface.

(l) define power as work done per unit time and derive power as the product of a force and velocity in the direction of the force.

[1] Italicized part, i.e. *work done by an expanding gas* is not in the H1 syllabus.

5.1 Introduction

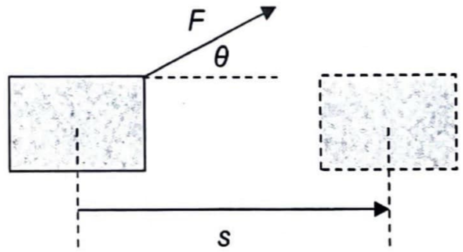
Energy is one of the most fundamental concepts in science, and is discussed in the context of Newtonian mechanics. Energy is present in various forms, with endless conversion from one form to another. The conservation of energy is an essential principle in Physics. The concept of work links energy and force, as work is a means of energy conversion through the application of a force. In certain situations, the concepts of work and energy can be applied to solve the dynamics of a mechanical system without directly resorting to Newton's laws. Beyond mechanics, this problem-solving approach focusing on energy can be applied to a wide range of phenomena in electromagnetism, and thermal and nuclear physics. The work-energy approach often provides a much simpler analysis than that obtained from the direct application of Newton's laws, since the former deals with scalar rather than vector quantities. In this section, we will discuss the concept of work.

5.1.1 Work Done by a Constant Force on a System

Work done by a Constant Force on a System

In scientific terms, work is done by a force on an object when the object is moved in the direction of the force.

A constant force F acts on a box below at an angle θ to the horizontal and displaces it horizontally to the right over a displacement s.



The work done on a given system by a **constant force** F is given by

Formula

$$W = Fs\cos\theta$$

Definition

Work done by a constant force is the product of the force and the displacement in the direction of the force.

Note

- When the component of F ($F\cos\theta$) is in the same direction as the displacement, the work done by the force is positive as $\cos\theta > 0$.
- When the component of F ($F\cos\theta$) is in the opposite direction to the displacement, the work done by the force is negative as $\cos\theta < 0$.
- The component $F\sin\theta$ of the force is perpendicular to the displacement, hence no work is done by this component.
- Work is a scalar quantity and its S.I. unit is the joule (J).

Definition

1 J is the work done on an object by a force of 1 N when the object is displaced by 1 m in the direction of the force.

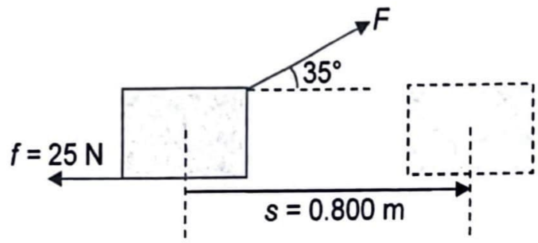
Example 1

A force F of 50 N is applied on a box of mass 4.0 kg at an angle of 35° to the horizontal for a displacement s of 0.800 m horizontally. A constant frictional force f of 25 N acts on the box. Calculate the work done on the box, by the

a) applied force F,

b) frictional force f, and

c) normal force N from the ground.



a) $W = (F\cos\theta)\,s = (50\cos35°)(0.800) = 32.8\text{ J}$

b) $W = (f\cos180°)\,s = (25)(-1)(0.800) = -20.0\text{ J}$

c) $W = (N\cos90°)\,s = 0\text{ J}$

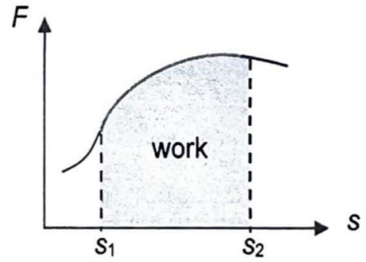
5.1.2 Work Done by a Variable Force on a System

Work Done by a Variable Force on a System

If the force F acting on the body varies with displacement s, then the work done by the variable force F over a displacement s_1 to s_2 is calculated by applying:

$$W = \int_{s_1}^{s_2} F\,ds$$

(Note: F and s are parallel)



Note that the work done by a variable force is equal to the **area under the *force – displacement* graph**.

5.1.3 Work Done by an External Force on a Spring

Work Done on a Spring

The external force F needed to produce an extension or compression x in a spring that obeys Hooke's Law is $F = kx$, where k is the spring constant. This force is a variable force as its magnitude depends on the extension or compression of the spring.

From the graph,

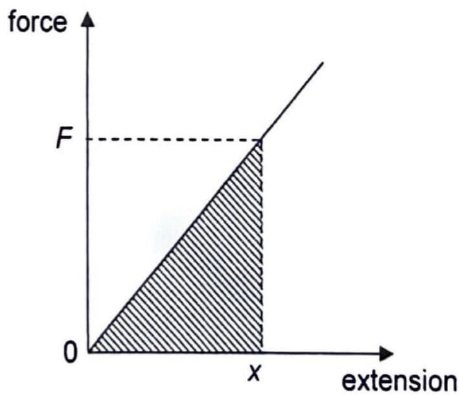
work done in stretching unextended spring by x

= area under force-extension graph = $\frac{1}{2}Fx$

Since $F = kx$,

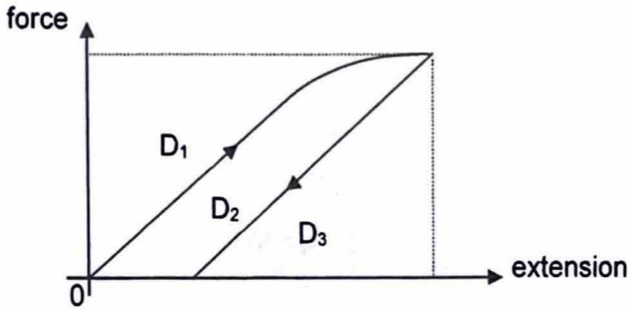
Formula

$$W = \frac{1}{2}kx^2$$



The above expression can also be used when the spring is being compressed.

Example 2 The loading of a spring and the unloading process take different paths in a force-extension graph as shown below.



Which area represents the work done to extend the spring?

$D_2 + D_3$

The spring is permanently deformed, as it remains extended even when there is no force applied.
D_3 is the 'recoverable' elastic potential energy,
while D_2 is energy used to change the interatomic structure/ arrangement such that the shape is changed permanently.

5.1.4 Work Done by a Gas

Work Done by a Gas at Constant Pressure

Consider a system of gas in a cylinder with a frictionless, movable piston. Suppose the gas is gently heated such that it expands slowly at constant pressure.

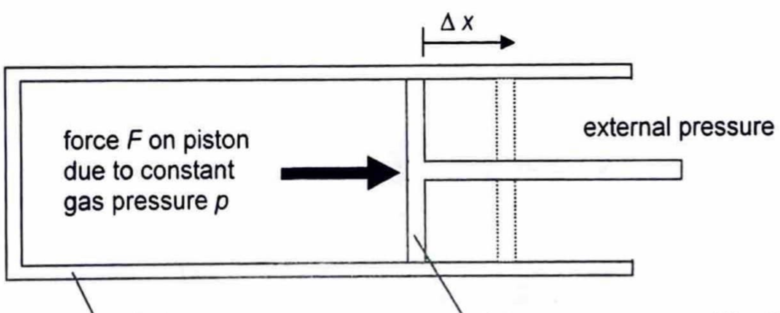


Fig. 2

As the gas expands, a force F is applied on the piston by the gas molecules to move the piston against the external pressure (also constant). The work done by force F in displacing the piston of cross-sectional area A through a small distance Δx is the work done **by** the gas[2], W_{gas}.

$$W_{gas} = F\Delta x = (pA)\Delta x$$

Formula

$$\boxed{W_{gas} = p\,\Delta V}$$

where $A\Delta x = \Delta V$, the change in volume.

Note

- When the gas expands, work done by the gas is positive.
- If the gas contracts, work done by the gas is negative.

[2] will be covered in greater detail in H2 Chap 9: First Law of Thermodynamics.

Example 3 The gas in a cylinder is expanding against an external pressure of 1.01×10^5 Pa. The piston sealing one side of the cylinder has a cross-sectional area of $0.050\ m^2$. Find the work done by the gas as the piston is pushed out by 1.0 cm when it is slowly heated.

The gas here expands under equilibrium condition,
i.e. its pressure is kept the same as external pressure at all times.

Work done by gas = $p\Delta V$
$= (1.01\times10^5)(0.050\times0.010)$
$= 50.5$ J

5.2 Energy

5.2.1 Different Forms of Energy

Forms of Energy Energy is the capacity to do work. It exists around us in many different forms. Some of the common forms are shown in the table below.

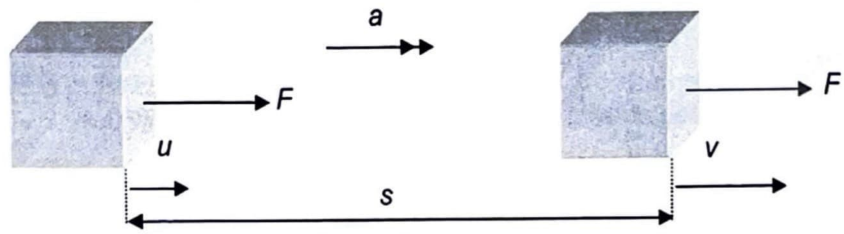
Form of Energy	Examples
Chemical potential energy – a form of potential energy related to the structural arrangement of atoms or molecules in a substance	• fuels such as oil, wood, coal • electric cells, food and explosives
Nuclear energy – energy released from atomic nuclei	• radioactive decays • nuclear reactions (fusion, fission)
Electrical energy – energy possessed by charge carriers moving under the influence of a potential difference	• the energy associated with current in circuits and electrical appliances • power stations • charged capacitors
Electric potential energy – energy due to position of a charge in an electric field of another charge/ system of charges	• protons in a nucleus • charged particles in particle accelerators
Internal energy – the sum of microscopic kinetic energies (associated with random motion of atoms or molecules) and potential energies (due to interatomic or intermolecular forces)	• energy of *any* system, such as a gas in a container, hot water in a beaker, a deformed spring, a car that crashed into a wall
Light energy (or radiant energy)	• energy stored in electromagnetic waves
Gravitational potential energy – energy due to position of a mass in a gravitational field	• waterfalls; water in a dam • raised objects • satellites and planets
Elastic potential energy – energy stored due to stretching or compressing an object	• compressed or stretched springs • bouncing basketball
Kinetic energy – energy due to motion of a body	• objects in motion

In this chapter, the focus is on some forms of mechanical energies, namely, kinetic energy, elastic and gravitational potential energies. Electric potential energy is another form of mechanical energy that will be studied later in **H2 Electric Fields / H1 Electromagnetism**. They are called mechanical energies because they enable a system to do mechanical work. Note that all forms of energies are scalars.

5.2.2 Kinetic Energy

Derivation of Kinetic Energy

Consider a body of mass m that is moving with an initial velocity u. It is acted on by a **constant resultant force** F which is parallel to u. The body accelerates with a uniform acceleration a to a final velocity v over a displacement s.



Derivation

Since the displacement of the body is in the same direction as the applied force,

$W = Fs = (ma)s$ ..... (1)

From $v^2 = u^2 + 2as$

$s = \dfrac{v^2 - u^2}{2a}$ ..... (2)

Substituting equation (2) into (1),

$W = (ma)\left(\dfrac{v^2 - u^2}{2a}\right)$

$= \boxed{\dfrac{1}{2}mv^2 - \dfrac{1}{2}mu^2}$ ..... (3)

Thus, the work done W is equal to the *change* in the quantity $\frac{1}{2}m \times (\text{velocity})^2$, which is termed the kinetic energy, E_K.

In general, the kinetic energy E_K of a body of mass m moving with speed v can be expressed as

Formula

$$\boxed{E_K = \frac{1}{2}mv^2}$$

- This is the <u>translational</u> kinetic energy as it is the energy due to an object moving along a path.

5.2.3 Potential Energy

Types of Potential Energy

Potential energy (represented as E_P or U) is the energy due to the position or shape of an object. The calculation of potential energy requires a reference point at which the potential energy is usually set to zero. This might be the floor, the lowest point of a pendulum or an electric charge at an infinite distance from another charge.

Three types of potential energy are specified in the H1/ H2 A-Level syllabus:

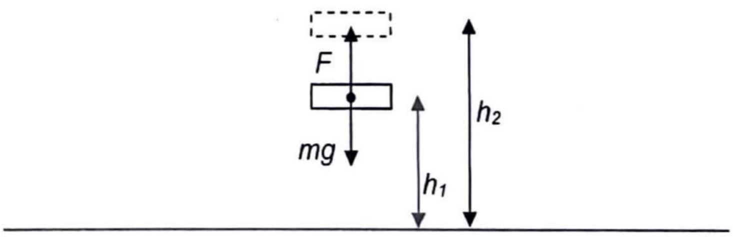
	Type of Potential Energy	Description
1	Gravitational potential energy (GPE)	The energy due to the position of a mass in a gravitational field of another mass. Note that GPE belongs to the system of the two masses, not to either mass alone.
2	Elastic potential energy	The energy stored in objects which have had their shapes changed elastically.
3	Electric or electrostatic potential energy	The energy due to the position of a charge in an electric field of another charge/ a configuration of charges. Similar to GPE, electric potential energy belongs to the system of charges, not to any charge alone.

5.2.3.1 Gravitational Potential Energy

Gravitational Potential Energy

The gravitational potential energy of an object of mass m at a height h above the surface of the Earth is $E_P = mgh$, where g is the acceleration of free fall near the Earth's surface (where the gravitational field is assumed to be uniform).

Derivation
Consider an object being raised upwards, at a **constant velocity**, from a height of h_1 to a height of h_2 near the Earth's surface. Since the velocity at which the object moves is constant, the force F required to lift the object must be equal to its weight mg.



Work done by F in displacing the center of mass of object vertically upward,

$$\begin{aligned} W &= Fs \\ &= mg(h_2 - h_1) \\ &= mgh \quad \text{where } h = h_2 - h_1 \end{aligned}$$

In raising the object, the force F does work on it which results in a gain in gravitational potential energy of the object given by the expression above.

The **absolute** value of gravitational potential energy holds little significance, since it can be changed by simply choosing a different point of reference for h. Hence, the **change** in the gravitational potential energy arising from a change in position of the body is the more important quantity.

The point where gravitational potential energy is taken to be zero is **arbitrary**. Therefore, if the Earth's surface is taken as the reference level with zero gravitational potential energy, an object raised to a height h above the Earth's surface acquires a gravitational potential energy E_P given by,

Formula

$$E_P = mgh$$

Note

- The zero potential energy level can be assigned to any point that is convenient.
- The acceleration of free fall g is assumed to be constant near the Earth's surface.

5.2.3.2 Elastic Potential Energy

Elastic Potential Energy

From **5.1.3**, we see that work done in compressing or extending the spring is:

$$W = \frac{1}{2}kx^2$$

The elastic potential energy stored in a spring which is extended or compressed by a distance x is

Formula

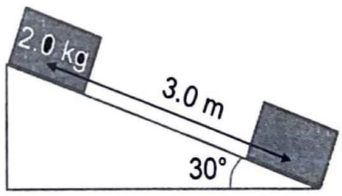
$$U_E = \frac{1}{2}kx^2$$

Note that the elastic potential energy of an unstretched or uncompressed spring is *zero*. This is unlike gravitational potential energy or electric potential energy, for which the point of zero energy can be arbitrarily chosen.

Example 4

A 2.0 kg box is released from the top of a frictionless slope. It slides a distance of 3.0 m down the slope, which is inclined at an angle of 30° to the horizontal. Find the work done by gravitational force on the box.

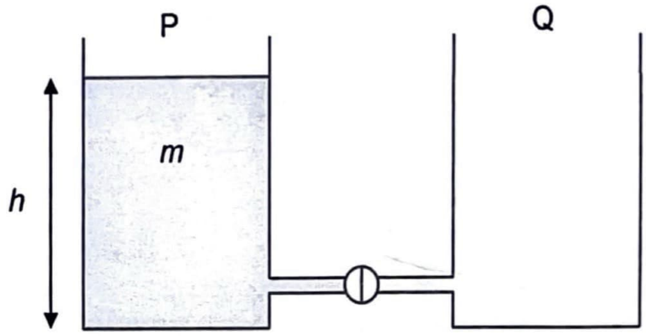
Work done by F_G
$= mg\,\Delta h$
$= (2.0)\,(9.81)\,(3.0 \sin 30°)$
$= 29.4$ J



Note: positive work done by gravitational force = loss in GPE = Gain in KE of box.

Example 5

The diagram shows two identical containers P and Q connected by a short pipe with a tap. Initially, P is filled with water of mass m to a depth h, and Q is empty.



Initial : $mg(h/2)$

Final: $(m/2)\,g\,(h/4) + (m/2)\,g\,(h/4)$
$= mg(h/4)$

PE lost $= mg(h/2) - mg(h/4)$
$= mg(h/4)$

When the tap is opened, water flows from P to Q until the water level in both containers is the same. How much potential energy is lost by the water?

A 0 **B** $\frac{mgh}{4}$ **C** $\frac{mgh}{2}$ **D** mgh

5.2.3.3 Relationship between Force and Potential Energy

Potential Energy in a Uniform Field

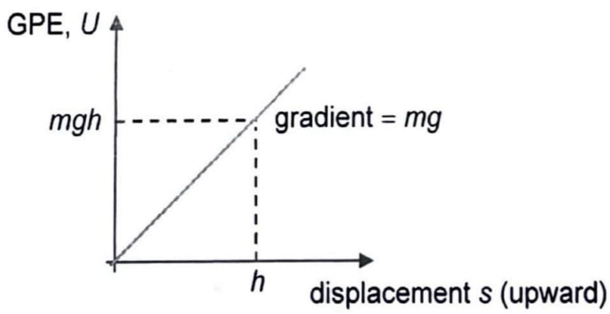
Formula

For a **field of force** (such as a gravitational field or an electric field), the relationship between the force F and the potential energy U for one-dimensional motion is given by

$$F = -\frac{dU}{dx}$$

From the relationship, we can infer:

1. The magnitude of the force at point x is equal to the gradient of the potential energy curve at x;
2. The direction of the force is the direction of decreasing potential energy.



1. Magnitude of gravitational force mg = gradient of potential energy vs displacement graph
2. The direction of the force is downwards, in the direction of decreasing potential energy.

Example 6

The graph shows the variation with distance x of the electric potential energy U of an electron in a uniform electric field. Find the electric force acting on the electron.

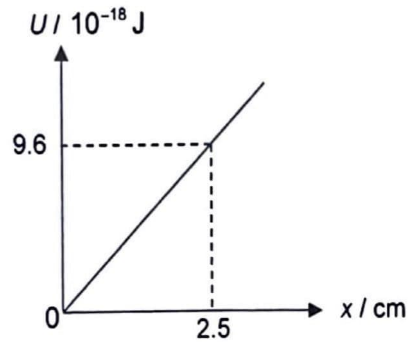
$$F = -\frac{dU}{dx}$$

Magnitude of electric force

$$F = \left| -\frac{\Delta U}{\Delta x} \right|$$

$$= \frac{9.6 \times 10^{-18}}{0.025}$$

$$= 3.84 \times 10^{-16} \text{ N}$$



5.3 Energy Conversion and Conservation

5.3.1 Principle of Conservation of Energy

Principle of Conservation of Energy

One may observe many cases of apparent "energy loss" in our daily life. But in fact, energy is never truly lost: it has merely been transformed into another form. For example, when a moving object experiences friction, it slows down while its temperature rises. Its kinetic energy has decreased, but the energy associated with the temperature of an object, namely the internal energy, has increased.

Statement

The principle of conservation of energy states that energy cannot be created or destroyed. It can only be converted from one form to another.

5.3.2 Total Mechanical Energy and Work Done by an External Force on a System

Total Mechanical Energy and Work Done by an External Force on a System

In a non-isolated system where there is work done W_F by an external force the energy equation is as follows:

$$(E_p + E_k)_{\text{initial}} + W_F = (E_p + E_k)_{\text{final}}$$

where E_p and E_k are the potential and kinetic energies of the system, respectively. Work is a means of transferring energy into or out of the system. Here W_F may be positive (an external force accelerating a cart, which increases total mechanical energy) or negative (friction slowing down the cart which decreases total mechanical energy).

If the resultant external force acting on the system is zero, i.e. $W_F = 0$, we call such a system an isolated system. The energy equation of the system becomes

Conservation of Mechanical Energy

$$(E_p + E_k)_{\text{initial}} = (E_p + E_k)_{\text{final}}$$

The sum of potential and kinetic energies is called mechanical energy (because they enable a system to do mechanical work). Thus the above equation states that, for an isolated system, the mechanical energy of a system is conserved.

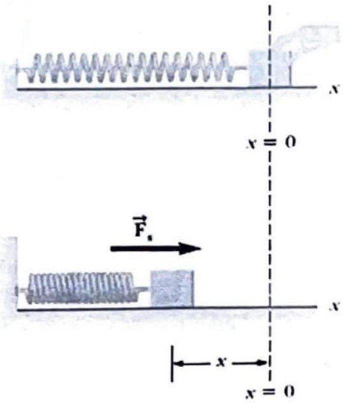
Example 7 A block of mass 0.50 kg is pushed against a spring of spring constant $k = 40\ \text{N m}^{-1}$. The spring is compressed by 10.0 cm and released. Calculate the maximum speed achieved by the block as it leaves the spring.

Initial : Elastic PE + KE = $\frac{1}{2}kx^2 + 0 = \frac{1}{2}(40)(0.100)^2$

Final : Elastic PE + KE = $0 + \frac{1}{2}(0.50)v^2$

$\frac{1}{2}(40)(0.100)^2 = \frac{1}{2}(0.50)v^2$

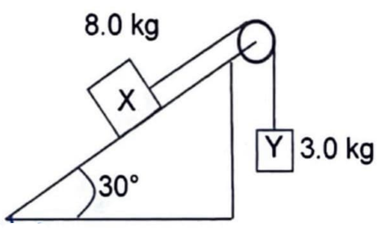
$v = 0.89\ \text{m s}^{-1}$



Example 8 A system of two bodies X and Y weighing 8.0 kg and 3.0 kg respectively are connected by a light cord that is passed over a light-free running pulley, as shown below. Starting from rest, X moves down a smooth plane inclined at 30° to the horizontal. When X has travelled 2.0 m along the plane, what is the total kinetic energy of the system?

(Gain in K.E of X and Y) + (increase in G.P.E of Y)
= (decrease in G.P.E of X)

Gain in K.E of X and Y
= decrease in G.P.E of X – increase in G.P.E of Y
$= m_x g h_x - m_y g h_y$
$= (8.0)(9.81)(2.0 \sin 30°) - (3.0)(9.81)(2.0)$
= **19.6 J**



5.3.3 Efficiency

Efficiency

When utilizing energy in our daily lives, not all energy input is converted into useful energy output. In the previous example, the work done by friction is converted to thermal and sound energy, which are useless for us. In practice, it is common that the useful energy output is less than the energy input due to the presence of dissipative forces such as friction or drag.

Efficiency gives a measure of how much of the total energy may be considered useful and is not "lost". It is given by.the expression:

Formula

$$\eta = \frac{\text{useful energy output}}{\text{total energy input}} \times 100\%$$

When solving efficiency problems, it is important to have a clear idea of what "useful energy output" is. For example, when accelerating a car, the useful energy output is the increase in kinetic energy of the car; when heating a pot of water, the useful energy output is the increase in the internal energy of the water in the pot, as measured by the temperature of the water.

Example 9

A car of mass 900 kg has an engine efficiency of 22%. Find the amount of petrol used to accelerate the car from rest to 20 m s^{-1}. Assume 1 kg of petrol provides 5.0×10^7 J of energy. Explain the energy conversion of the car.

$$\text{Increase in KE} = \frac{1}{2}mv^2 - \frac{1}{2}mu^2$$

$$= \frac{1}{2}(900)(20)^2 - 0$$

$$= 1.8 \times 10^5 \text{ J}$$

$$\eta = 22\% = \frac{\text{useful energy output}}{\text{energy input}} \times 100\%$$

$$22\% = \frac{1.8 \times 10^5}{\text{energy input}} \times 100\%$$

$$\text{energy input} = 8.18 \times 10^5 \text{ J}$$

Stored chemical potential energy in the petrol is converted to kinetic energy of the car, thermal and sound energy.

Amount of petrol required

$$= \frac{8.18 \times 10^5}{5.0 \times 10^7}$$

$$= 0.016 \text{ kg}$$

5.4 Power

5.4.1 Definition

Power

To illustrate the concept of power, consider two different motors which are used to lift the same object to the same height above the ground. The same amount of work can be done by a low-powered motor over a longer duration of time or a high-powered motor in a shorter time.

Definition

Power is defined as the rate of work done or energy conversion with respect to time.

Formula

$$P = \frac{dW}{dt}$$

If total work done is ΔW in a time interval Δt, the average power is $\langle P \rangle = \dfrac{\Delta W}{\Delta t}$.

The S.I. unit of power is the watt (W). Another common unit of power is horsepower. Note that kilowatt-hour (kWh) is a unit of energy, not of power.

Example 10

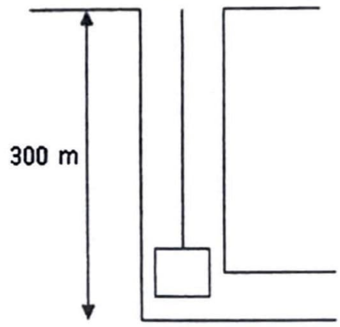
A machine is used to lift coal out of a coal mine.

(a) How much work is done by the machine in raising 500 kg of coal out of the mine?

(b) If the machine takes 20 minutes to accomplish the lift, what is its average power?

(c) Explain why the actual power needed will be greater than that in (b).

(d) If the power loss is 3.00 kW, what is the efficiency of this process of lifting coal?



A machine is used to lift coal out of a coal mine.

(a) How much work is done by the machine in raising 500 kg of coal out of the mine?

Work done = mgh = (500)(9.81)(300) = 1.47×10^6 J

(b) If the machine takes 20 minutes to accomplish the lift, what is its average power?

Power = Work / time = $(1.47 \times 10^6) / (20 \times 60)$ = 1.23 kW

(c) Explain why the actual power needed will be greater than that in (b).

There is energy loss due to friction between moving parts, joule heating in electrical cables (of machine), and energy used to lift the container and the cable.

(d) If the power loss is 3.00 kW, what is the efficiency of this process of lifting coal?

$$\eta = \frac{\text{useful power output}}{\text{total power input}} \times 100\% = \frac{1.23}{(1.23 + 3.00)} \times 100\% = 29\ \%$$

5.4.2 Relationship between Power, Force and Velocity

Power from a Constant Force

If a **constant force** F is applied and does work by moving its point of application a displacement s in time t, then the power supplied is given by the following derivation:

$$P = \frac{dW}{dt} = \frac{d(Fs)}{dt} = F\frac{ds}{dt},$$ (Note: F and s are parallel)

Formula

$$\boxed{P = Fv}$$

where $v = \frac{ds}{dt}$ is the velocity of the point of application.

Example 11
N14/1/9 modified

A locomotive develops a constant power. The locomotive pulls a train of total mass 3.0×10^5 kg against a constant frictional force of 5.0×10^4 N. The acceleration of the train is 0.50 m s^{-2} when travelling at 10 m s^{-1}. What is the maximum speed that the train can achieve on a level track?

If F_D is the driving force, and f_r = frictional force, then, the resultant force acting on the train is

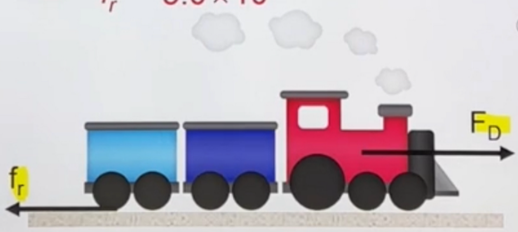
$$F_D - f_r = ma$$
$$F_D = f_r + ma$$

$$P = F_D v$$
$$= (f_r + ma)v$$
$$= (5.0 \times 10^4 + 3.0 \times 10^5 \times 0.50)(10)$$
$$= 2.0 \times 10^6 \text{ W}$$

At max speed, $a = 0$, $F_D = f_r$

$$P = F_D v_{max} = f_r v_{max}$$

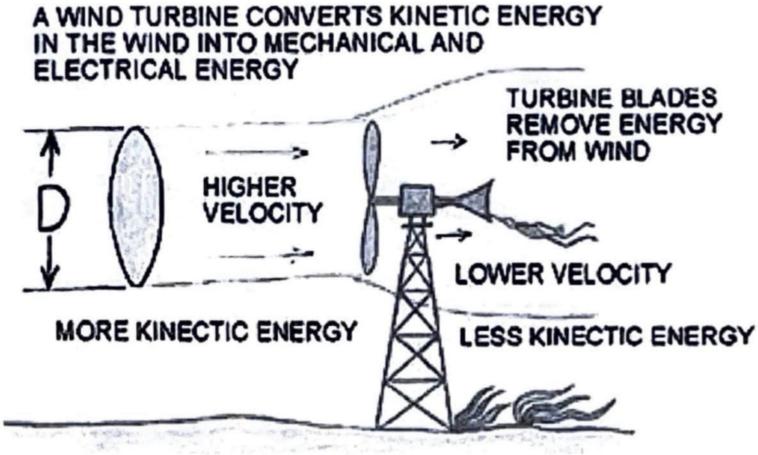
$$v_{max} = \frac{P}{f_r} = \frac{2.0 \times 10^6}{5.0 \times 10^4} = 40 \text{ m s}^{-1}$$



5.4.3 Wind Turbines

Operation of a Wind Turbine

The principle of operation of a wind turbine in which the kinetic energy of the wind is converted into electrical energy is shown below.



Example 12

One of the world's largest wind turbine have a diameter of 122 m. If at a given time the speed of the wind immediately before and after passing through the blades are 16 m s^{-1} and 14 m s^{-1} respectively, calculate

(a) the mass of air per second through the area swept out by the blades, given that the density of air before passing through the blades is 1.3 kg m^{-3},

(b) the energy removed from the wind is converted to electrical energy with an efficiency of 70%, calculate the power output.

$$\text{Mass per second} = \frac{dm}{dt}$$

$$= \frac{d(\rho A x)}{dt}$$

$$= \rho A \frac{dx}{dt}$$

$$= \rho A v$$

$$= 1.3 \times \pi \left(\frac{122}{2}\right)^2 \times 16$$

$$= 2.43 \times 10^5 \text{ kg s}^{-1}$$

Loss in KE of air in 1 s

$$= \frac{1}{2}(2.43\times10^5)(16^2) - \frac{1}{2}(2.43\times10^5)(14^2)$$

$$= 7.29 \times 10^6 \text{ J}$$

Therefore power input of turbine = 7.29×10^6 W

$$\eta = \frac{\text{power output}}{\text{power input}} \times 100\%$$

$$70\% = \frac{\text{power output}}{7.29} \times 100\%$$

power output = 5.10 MW

5.4.4 Graphs of variation with displacement of Energy and Power

Example 13 Consider the following scenarios and sketch the relevant graphs:

a) A car starts from rest and accelerates uniformly. Sketch a graph of the variation with displacement s of the car's

i. power output, P and

ii. kinetic energy E_K.

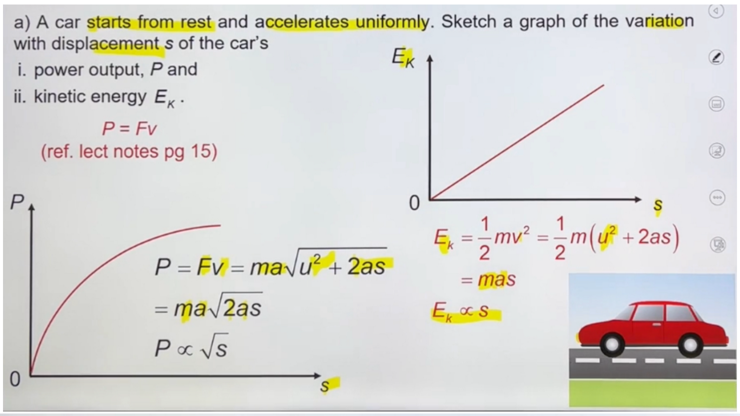
b) A spring is stretched by an external force that increases linearly with displacement. Sketch a graph of the variation with extension x of the elastic potential energy U_E.

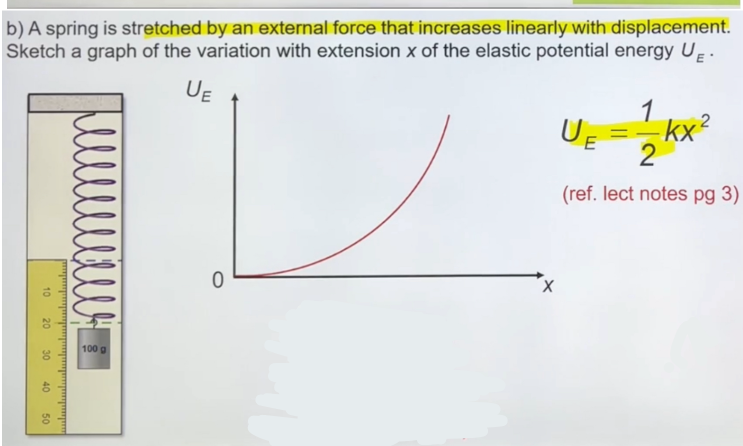
c) An object is dropped vertically from a height H. Sketch a graph of the variation with displacement s of its

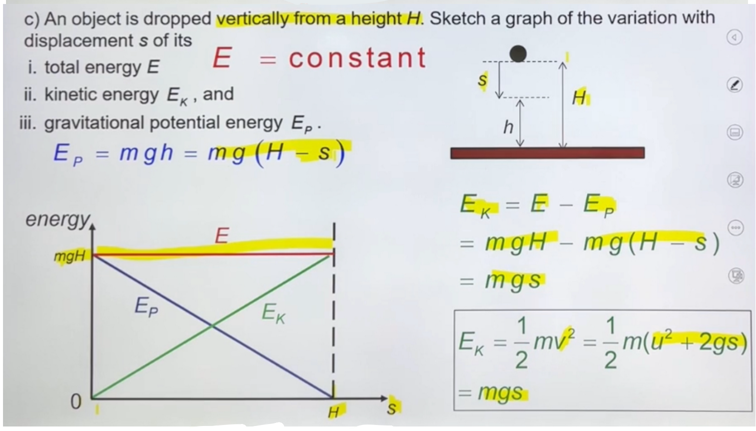
i. total energy E

ii. kinetic energy E_K, and

iii. gravitational potential energy E_P.







Appendix

Conservative Forces

The total mechanical energy of a system is constant only when conservative forces act within the system. Consider the work done by a conservative force that acts on an object as the object moves from an initial position to a final position along any arbitrarily chosen path. This work done is the same for all such paths for a conservative force, i.e. path-independent. If the work is not the same for all paths, the force is a nonconservative force, such as friction.

When work is done in a system by a conservative force, the configuration of its bodies changes, and so the potential energy of the system changes. The total work done by a conservative force in a round trip is equal to zero, because the bodies come back to the original location. We can associate a potential energy function with each conservative force.

Nonconservative forces

Forces that do not store energy are called nonconservative forces. Dissipative forces such as friction and air resistance are nonconservative forces. The work done by nonconservative forces varies for different paths. The energy that they remove from the system is no longer available to the system for kinetic energy.

The mechanical energy that is lost due to nonconservative forces, such as friction, is converted to internal energy (stored temporarily inside and within the surface of the bodies). At the subatomic scale, this internal energy is associated with the vibration of atoms about their equilibrium positions. Such internal atomic motion involves both kinetic and potential energy. If we include in our energy expression this increase in internal energy of objects that make up the system, we can easily see how the expression is consistent with the law of conservation of energy.

Tutorial 5 WORK, ENERGY AND POWER

Self-Check Questions

S1 Define work done by a force.

S2 Derive, from the equations for uniformly accelerated motion in a straight line, the formula $E_K = \frac{1}{2}mv^2$.

S3 Derive the formula $E_P = mgh$ for potential energy changes near the Earth's surface.

S4 State how the work done by a force can be deduced from the force-displacement graph.

S5 What does the area under the force-extension graph of an elastic material represent?

S6 State the relationship between force and potential energy in a field.

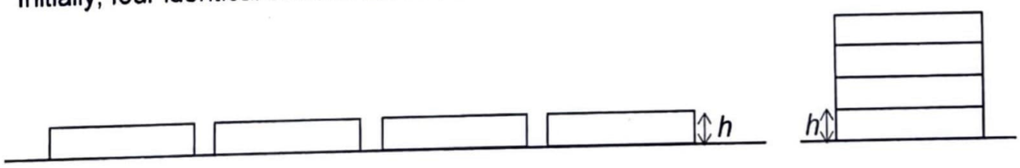
S7 State the principle of conservation of energy.

S8 Describe the energy transformations of a stone undergoing projectile motion in air, from the time when it is catapulted upwards at an angle from the ground to when it returns to the ground. Assume no air resistance throughout.

S9 Define power and derive power as the product of force and velocity in the direction of the force.

Self-Practice Questions

SP1 Initially, four identical uniform blocks, each of mass m and thickness h, are spread on a table



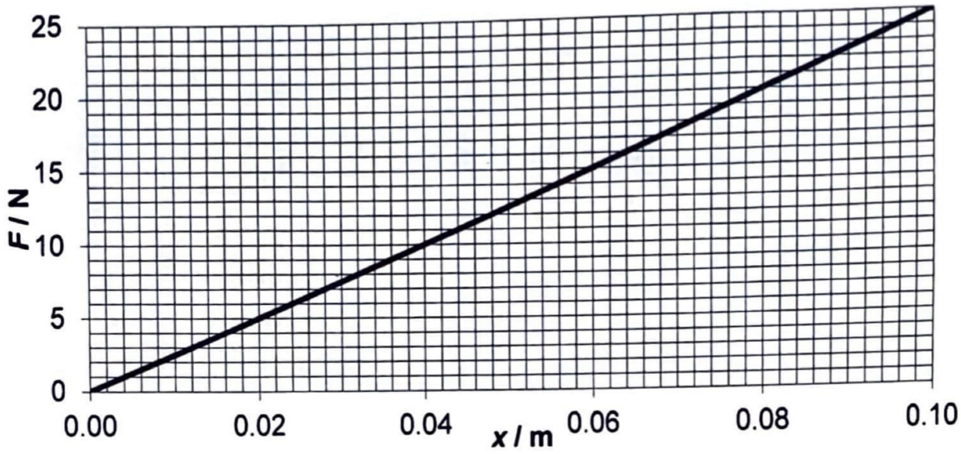
How much work is done on the blocks in stacking them on top of one another?

A 2 *mgh** **B** 3 *mgh* **C** 4 *mgh* **D** 6 *mgh*

(N04/I/8)

SP2 A man lowers a block of mass M through a vertical distance d at a constant downward acceleration of $\frac{g}{4}$. Find the work done by the man.

SP3 The graph shows the variation with extension x of the load F on a certain spring.



A load of 10 N is placed on the spring. How much additional elastic potential energy will be stored in the spring if it is then extended a further 0.040 m?

A 0.200 J **B** 0.450 J **C** 0.600 J **D** 0.800 J

J94/I/22 (modified)

SP4 A bullet of mass 2.0 g moving horizontally with velocity 200 m s^{-1} hits a wooden pole. It comes to rest after penetrating a depth of 100 mm. Calculate the average retarding force during the passage of the bullet within the pole. What happens to the work done by this force?

SP5 A bungee jumper has 24 kJ of gravitational potential energy at the top of his jump. He is attached to an elastic rope which starts to stretch after a short time of free fall. The values of gravitational potential energy, elastic potential energy and kinetic energy are given for the top and the bottom of the jump are given below.

	gravitational potential energy/ kJ	elastic potential energy/ kJ	kinetic energy/ kJ
top	24	0	0
bottom	0	24	0

Which row of the table below shows the possible values of these energies when the jumper is halfway down? Losses of energy through air resistance are negligible.

	gravitational potential energy/ kJ	elastic potential energy/ kJ	kinetic energy/ kJ
A	12	10	2
B	12	8	4
C	8	8	8
D	12	2	10

N07/I/11

SP6 An aircraft moving through air at velocity v experiences a resistive force F given by the expression

$$F = kv^2, \quad \text{where } k \text{ is a constant.}$$

What is the power required to keep the aircraft moving at this constant velocity?

A kv **B** kv^2 **C** kv^3 **D** kv^4

N03/I/7

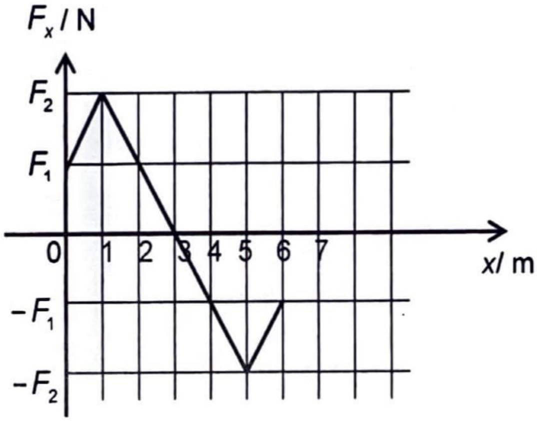
Discussion Questions

D1 The figure below shows the x-component F_x of a force F acting on a particle. If the particle begins from rest at $x = 0$, what is its displacement x when it has

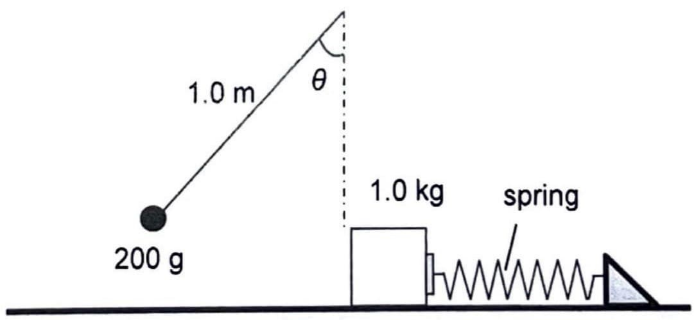
(a) **(i)** the greatest kinetic energy, and [1]

(ii) zero kinetic energy. [1]

(b) Sketch the corresponding graph of kinetic energy E_K against displacement x of the particle. [2]



D2 A 200 g rubber ball is tied to a 1.0 m long string and released from rest at an angle θ. At the lowest point, it has an *elastic* collision with a 1.0 kg block. The block is resting on a frictionless surface and is connected to a long spring of force constant 2.0 kN m^{-1}. The triangular support is fixed to the surface. After collision, the spring compresses a maximum distance of 2.0 cm.



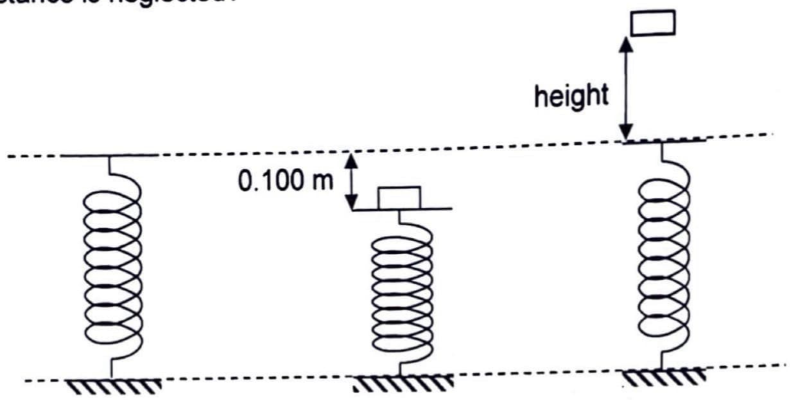
(a) Determine the potential energy stored in the spring at maximum compression. [2]

(b) Determine the speed of the block immediately after collision with the ball. [2]

(c) Determine the speed of the ball just before and after collision with the block [3]

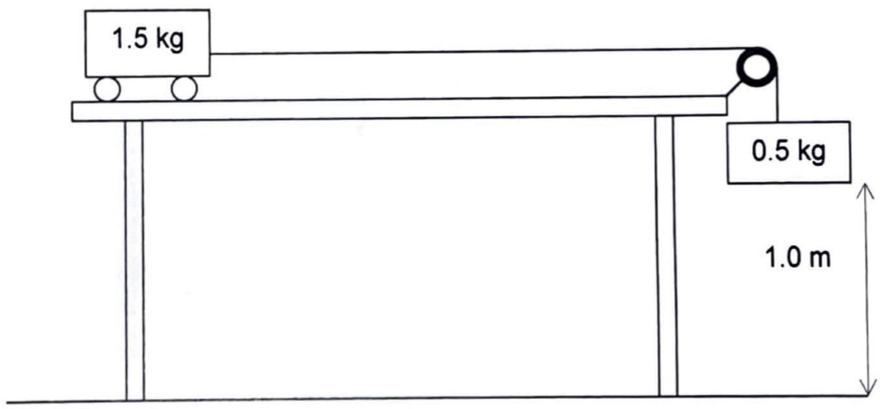
(d) Hence, determine the angle θ. [2]

D3 A block of mass 0.250 kg is placed on a light vertical spring of k = 500 N m^{-1} and pushed downwards, compressing the spring by 0.100 m. When the block is released, it leaves the spring and moves vertically upwards. What is the maximum height above the spring that the block reaches if air resistance is neglected?



[2]

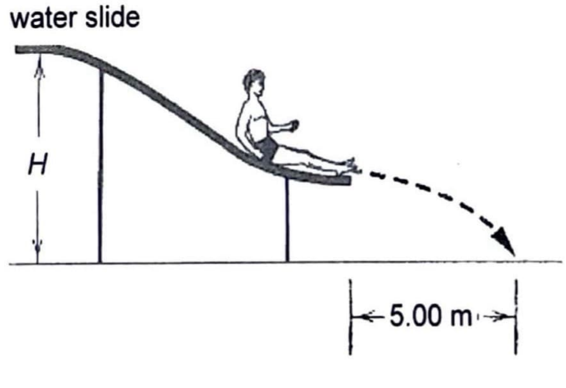
D4 The diagram shows a trolley being pulled from rest along a horizontal table by a falling mass. The mass of the trolley is 1.5 kg and the falling mass is 0.5 kg. The mass falls through 1.0 m. Assume the distance along the table is longer than 1.0 m.



What is the maximum kinetic energy of the trolley? [3]

N05/I/9

D5 A water slide is constructed so that swimmers, starting from rest at the top of the slide, leave the end of the slide traveling horizontally. The mass of the boy is 60 kg and the distance along the curved water slide is 10.0 m. As shown in the figure below, a boy is observed to hit the water 5.00 m from the end of the slide in 0.500 s after leaving the slide. Ignoring friction and air resistance, find the height H.

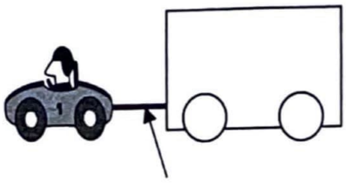


[3]

D6 A car of mass 900 kg tows a caravan of mass 750 kg. There is a force of 1700 N opposing the motion of the car and a force of 2000 N opposing the motion of the caravan. The car and the caravan move at a constant velocity of 20 m s^{-1}.

(a) State, with a reason, the value of the tension in the tow bar. [2]

(b) Calculate the power due to the engine of the car [2]

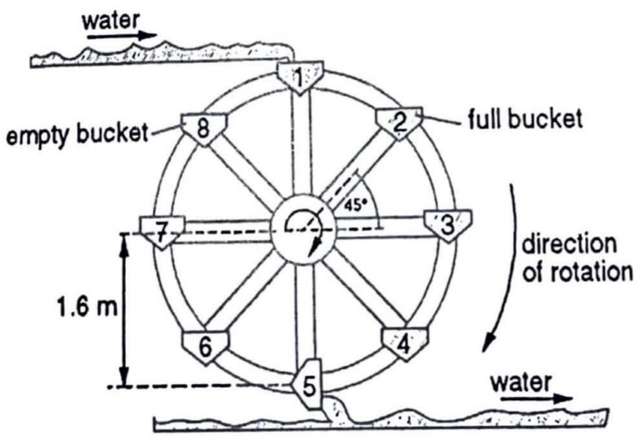


D7 A driving force of 200 N is needed for a car of mass 800 kg to travel along a level road at a speed of 20 m s^{-1}. What power is required to maintain the car at this speed up a gradient in which the car rises 1.0 m for each 8.0 m of travel along the same road? [3]

N09/I/12

D8 A cyclist is working at a rate of 0.20 kW when he is ascending a hill at a constant speed of 18 km h^{-1}. The mass of the cyclist and his bicycle is 80 kg and the resistive force acting on them is 10 N. Find the angle of inclination of the hill to the horizontal. [2]

D9 A water-wheel has eight buckets equally spaced around its circumference as shown in the figure below.



The distance between the centre of each bucket and the centre of the wheel is 1.6 m. When a bucket is at its highest point, the bucket is filled with a mass of 40 kg of water. The wheel rotates and the bucket is emptied at its lowest point. The wheel makes six revolutions per min.

(a) Calculate the total change in potential energy of the water in the buckets in one revolution of the wheel. [2]

(b) Calculate the average input power to the wheel. [2]

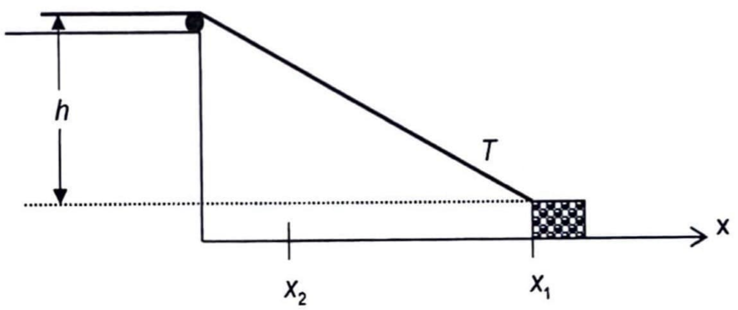
(c) Suggest why a larger number of small buckets is preferred to a smaller number of large buckets containing the same mass of water. [2]

Modified from N01/II/1b,c

D10 Wind is a useful source of energy for generating electrical power for people who live in heavily mountainous regions. Modern windmills have an overall efficiency of 40%; that is, 40% of the energy in the wind hitting the blades can be converted to electrical energy. What blade length would be needed to generate 5.00 kW in a 50.0 km h^{-1} wind? Density of air is 1.2 kg m^{-3}. [3]

Challenging Questions

C1 The figure below shows a cord attached to a cart that can slide along a frictionless horizontal rail aligned along the x-axis. The left end of the cord is pulled over a pulley of negligible mass and friction and at cord height $h = 1.20\text{ m}$, so that the cart slides from $x_1 = 3.00\text{ m}$ to $x_2 = 1.00\text{ m}$. During the motion, the tension T in the cord remains constant at 25.0 N. What is the change in kinetic energy of the cart during the motion?



C2 The engine of a car develops constant power when it is in motion and the car experiences a resistance which is proportional to the square of its velocity. The maximum speed of the car on a level road is v, while the maximum speed up a hill of constant slope is ½ v. Show that the maximum speed down the same hill is λv, where

$$\lambda^3 - \frac{7}{4}\lambda - 1 = 0$$

Answers

D1 (a) (i) 3 m **(ii)** 6 m
D3 0.919 m
D5 6.32 m
D7 23.6 kW
D9 (a) 1.00×10^4 J **(b)** 1.00 kW

D2 (a) 0.40 J **(b)** 0.894 m s^{-1} **(c)** 2.68 m s^{-1}, 1.79 m s^{-1} **(d)** 50.7°
D4 3.7 J
D6 (a) 2000 N **(b)** 74 kW
D8 2.2°
D10 1.57 m

C1 41.7 J

Tutorial 5 WEP Suggested Solutions

S1 Work done by a force on an object is defined as the product of the force and the displacement of the object in the direction of the force.

S2 Consider work done W, on a body by a constant net force F,

$W = Fs = (ma)s \quad \cdots (1)$

Body undergoes constant acceleration, rearranging $v^2 = u^2 + 2as$,

$s = \dfrac{v^2 - u^2}{2a} \quad \cdots (2)$

Substituting (2) into (1),

$$W = (ma)\left(\frac{v^2 - u^2}{2a}\right) = \frac{1}{2}mv^2 - \frac{1}{2}mu^2$$

The net work done on a body is associated with its change in energy due to motion.

Thus the quantity $\frac{1}{2}m(\text{velocity})^2$ is termed the kinetic energy of a body, i.e. $E_k = \frac{1}{2}mv^2$

S3 Work done by the force to raise an object from h_1 to h_2 at constant speed

$W = Fs$

$= mg(h_2 - h_1)$

$= mgh_2 - mgh_1$

= final gravitational potential energy – initial gravitational potential energy

S4 Area under the force-displacement graph gives the work done by the force.

S5 The area is the work done by the applied force to stretch the elastic material from the beginning (unstretched) till it is stretched by an extension e.

2 points to note:

i. If the elastic material can return to its original length, this work done is equal to the elastic potential energy stored.

ii. The force has to increase in order to increase the extension. This means the *work done per unit extension* increases with extension.

S6 $F = -\dfrac{dU}{dx}$ (negative sign means force is in the direction of decreasing potential energy)

S7 The law of conservation of energy states that energy cannot be created or destroyed. It can only be converted from one form to another.

S8 Elastic PE of the stretched catapult is transformed into the KE and GPE of the stone upon release of catapult's band. Taking zero GPE to be on the ground, the stone has an initial KE and zero GPE. As the stone moves up, KE is converted to GPE until it reaches maximum height where GPE is maximum and KE is minimum (non-zero) at this instant. When it moves down from maximum height, GPE is converted back to KE and KE reaches maximum just before hitting the ground where GPE is zero.

(Note: energy is scalar and KE has no direction, unlike velocity)

S9 Power is defined as the rate of work done or energy conversion with respect to time.

From $W = Fs$ and $P = \dfrac{dW}{dt}$,

$$P = \frac{d(Fs)}{dt} = F\frac{ds}{dt} = Fv$$

SP1 D.

Work done = $mgh + 2mgh + 3mgh = 6mgh$.

Alternatively, observe how much vertical distance the centre of mass of the system of 4 blocks has shifted (i.e. from $h/2$ to $2h$). $\Delta h = 1.5h \quad \therefore Mg\Delta h = 4mg(1.5h) = 6mgh$

SP2 $Mg - F_{man} = Mg/4$

$F_{man} = 3Mg/4$

Work done = $F_{man}\, d \cos 180°$

$= -0.75Mgd$

SP3 C.

With load, the spring would have extended by 0.040m.

Additional EPE stored in spring when it extends by a further 0.040m (0.040m to 0.080m)

$= \frac{1}{2}(10 + 20)(0.040) = 0.600$ J (= the additional area under the F-x graph)

SP4 $E_{k,initial} + W_{retarding\ force} = 0$ (final KE is 0)

$$\frac{1}{2}(0.0020)(200)^2 + F_r(0.100)(\cos 180^0) = 0$$

$$-0.100F_r = -40$$

$$F_r = 400 \text{ N}$$

The negative work done by retarding force causes a decrease in mechanical energy, which is converted into heat / gain in internal energy of the wooden pole and bullet.

SP5 D.

$GPE_{halfway}$ = ½ (GPE at top) = 12 kJ (reject C)

If rope starts to stretch the moment he jumps, (we know this does not happen, but we 'use' it just so we can set an upper limit for elastic PE)

Elastic $PE_{max} = \frac{1}{2}kx_0^2$ where x_0 is maximum extension of rope

Elastic $PE_{halfway} = \frac{1}{2}k\left(\frac{x_0}{2}\right)^2 = \frac{1}{4}$Elastic $PE_{max} = \frac{1}{4}(24) = 6\text{ kJ} < 8\text{ kJ}$ (reject A, B)

SP6 C.

Thrust produced by engine = Resistive force = kv^2

Power $= Fv = kv^2 \times v = kv^3$