1 The sum of the first 100 terms of an arithmetic progression is 10,000. The first, second and fifth terms of this progression are three consecutive terms of a geometric progression. Find the first term and the non-zero common difference of the arithmetic progression. [5]

2 The equation of a curve is $Ax^2 + By^2 + Cy = 8$, where A, B and C are constants. The curve has a tangent parallel to the y-axis at the point (2,1). Given that $\frac{\mathrm{d}y}{\mathrm{d}x} = \sqrt{\frac{3}{2}}$ and $\frac{\mathrm{d}^2 y}{\mathrm{d}x^2} = \frac{9}{4}$ at the point where the curve cuts the positive x-axis, find the values of A, B and C. [6]

3 Without using a calculator, solve the inequality $\frac{6x-4}{x-3} \le 1-2x$, $x \ne 3$. [4]

Hence find the exact range of values of θ for which $\frac{6-4cosec\,\theta}{1-3cosec\,\theta} \le 1-2sin\,\theta$ where $0 < \theta < 2\pi$. [3]

4 A curve with equation y = f(x) is transformed by a reflection in the y-axis, followed by a translation of 1 unit in the negative x-axis, followed by a scaling with factor 2 parallel to the y-axis.

The equation of the resulting curve is given by y = g(x), where

$$\mathrm{g}(x) = \frac{1}{\sqrt{4-x^2}}\ , \quad -2 < x < 0 .$$

(i) Find f(x). [3]

(ii) On the same diagram, sketch the graph of y = g(x) and $y = \mathrm{g}^{-1}(x)$. [3]

(iii) Without evaluating $\mathrm{g}^{-1}(x)$, find the exact area of the region bounded by the curve $y = \mathrm{g}^{-1}(x)$, the x-axis and the line x = 1. [4]

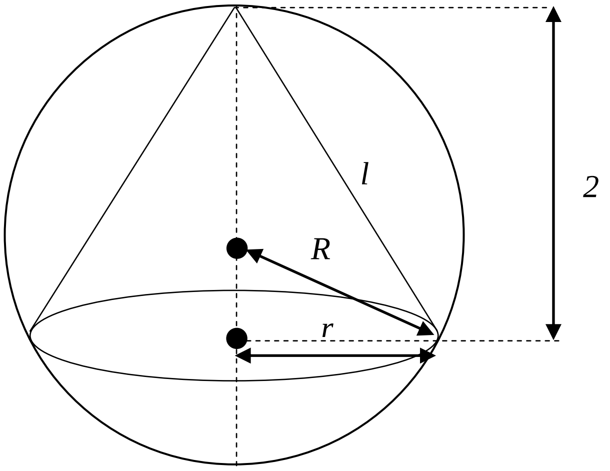
5 Consider $S_n = \frac{2}{1\times 2\times 3} + \frac{2}{2\times 3\times 4} + \frac{2}{3\times 4\times 5} + ... + \frac{2}{n(n+1)(n+2)}$

(i) Calculate the values of S_1, S_2 and S_3. [2]

(ii) Make a conjecture for S_n in the form $\frac{1}{2} - \frac{1}{(n+a)(n+b)}$, where a and b are integers. [1]

(iii) Prove your conjecture by the method of induction. [5]

6



The diagram shows a right circular cone of height of 2 units, radius r and slant height l inscribed in a sphere of radius R.

(i) Show that $A = 4\pi\sqrt{R^2 - R}$, where A is the curved surface area of the cone. [3]

(ii) If the volume of the sphere is increasing at a rate of 8 units^3/sec, find the exact rate of change of A at the instant when $R = 2$ units. [4]

[Curved surface area of a right circular cone with base radius r and slant height l is πrl. Volume of sphere = $\frac{4}{3}\pi R^3$ where R is the radius of the sphere.]

7 A sequence of numbers, x_n, satisfy the relation

$$x_{n+1} = \frac{1}{3}\left(2x_n + \frac{1}{x_n^2}\right), \quad n \in \mathbb{Z}^{+} \cup \{0\}, \quad 0 < x_0 < 1.$$

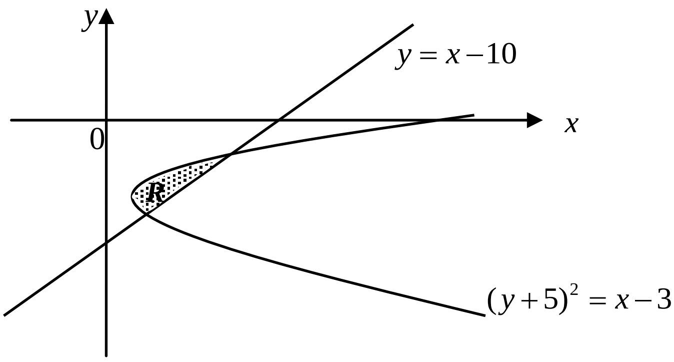
(i) If the sequence converges to a number L, find L. [2]

(ii) Given that $x_n > L$ for all integers $n \geq 1$, show that $x_{n+1} < x_n$. [3]

(iii) Describe, with reference to (i) and (ii), the behaviour of the sequence. [1]

The difference d_n between x_n and L is given by $d_n = x_n - L$. Show that $d_{n+1} \approx d_n^2$ and state the range of values of d_n for which this approximation is valid. [4]

8 (a) The diagram below shows a region R bounded by the curve $(y+5)^2 = x-3$ and the line $y = x-10$. Find the volume of solid formed when R is rotated through four right angles about the x-axis. [4]



(b) (i) Show that $\int e^{-2x} \cos x \ \mathrm{d}x = -\frac{2}{5} e^{-2x} \cos x + \frac{1}{5} e^{-2x} \sin x + C$,
where C is an arbitrary constant. [4]

(ii) A curve Γ is defined by the parametric equations

$$x = \sin t, \quad y = e^{-2t}, \text{ where } -\frac{\pi}{2} \le t \le \frac{\pi}{2}.$$

Find the exact area of the region bounded by the curve Γ, the tangent to the curve Γ at the y-axis and the line $x = 1$. [5]

9 The curve C has equation $y = \dfrac{2x^2 - a}{x+k}$ where $a, \ k > 0$.

(i) Given that $2k^2 \neq a$, find the range of values of k such that the curve C has at least one tangent parallel to the x-axis. [4]

(ii) What can be said about the graph of $y = \dfrac{2x^2 - a}{x+k}$ when $2k^2 = a$? [2]

For the range of values of k in (i),

(iii) sketch the curve of C, clearly indicating the equations of the asymptotes and any intercepts with the axis. [3]

(iv) The curve $y = \dfrac{2x^2 - a}{x+k}$ does not intersect the curve $(x+k)^2 - \left(\dfrac{y+4k}{b}\right)^2 = 1$ where $b > 0$. By drawing the 2 curves on the same diagram, find the range of values of b. [2]

10 The two planes p_1 and p_2, given by the equations $2x + y = 1$ and $8x + ay + z = 4$ respectively, meet at a line L which contains the point $A(0,1,0)$.

(i) Show that $a = 4$ and hence find the vector equation of the line L. [3]

(ii) Another point B lies in the plane p_1 such that AB is perpendicular to the line L.

Show that $\overrightarrow{AB}$ is parallel to $\begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix}$.

If the distance of B from p_2 is 5 units, find the possible position vectors of point B. [5]

(iii) Find the acute angle between line AB and p_2. [2]

(iv) A third distinct plane p_3 is given by the equation $2x + y + \beta z = 6$. Show that p_1, p_2 and p_3 do not meet at a common point for all values of β. Comment on the geometrical relationship of the three planes when $\beta = 0$. [3]

11 The point A represents the complex number a with $|a| = 1$ and $\theta = \arg(a)$, $0 < \theta < \frac{\pi}{4}$. The point B represents the complex number $\mathrm{i}a$.

The complex number z satisfies the relations

$$|z - \mathrm{i}a| \leq \sqrt{2} \quad \text{and} \quad |z - \mathrm{i}a| = |z - a|.$$

Sketch clearly, on a single Argand diagram, the locus of the point representing the complex number z, with reference to the points A and B. Find the angle that the locus of z makes with the positive real axis in terms of θ. [4]

By writing $2\mathrm{i}a - a$ as $\mathrm{i}a + (\mathrm{i}a - a)$, or otherwise, indicate the point C representing the complex number $2\mathrm{i}a - a$ on your diagram and state the geometrical relationship between the points A, B and C. [2]

Hence find

(i) the least value of $|z + a - 2\mathrm{i}a|$, and [1]

(ii) the largest value of $\arg(z + a - 2\mathrm{i}a)$. [3]

END OF PAPER