Title	Junior College H2 Mathematics Materials Compilation – Statistics
Author	AprilDolphin
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Title	Junior College 'A' Levels H1/H2 – Binomial Distribution
Author	AprilDolphin
Date	25/3/2025

Conditions for a random variable to be modelled by a Binomial Distribution includes the following:

The experiment must consist of Bernoulli trials (Where there are two possible outcomes in the experiment, which we can call as "outcome" and "complement outcome".)

All trials in the experiment have to be independent (Where the probability of obtaining "outcome" of each trial isn't affected by a previous trial or will affect a future trial within the experiment).

All trials within the experiment have to be identically distributed. (Such that each Bernoulli trial has constant probability of obtaining "outcome" and "complement outcome".)

Example of common outcomes and complement outcomes as follows:

Outcome	**Complement Outcome**
Yes	No
No	Yes
Success	Failure
Failure	Success
Picking a red ball	Not picking a red ball

If the experiment in question satisfies the above requirements, it is said the follow a Binomial Distribution with parameters n and p,where,
n refers to the total number of trials.
p refers to the probability of obtaining the "outcome" of each trial.

When it is written in standard Binomial Notation, it looks like the following, where X refers to the random variable.

$$X \sim B(n, p)$$

The formula for Binomial Probability distribution of a specific number of trials to be calculated for is given below:

$$P(X = x) = \binom{n}{x} p^x (1-p)^{n-x}$$

The formula for Binomial Probability distribution from 0 up till x number of trials can be calculated as follows:

$$P(X \le x) = \binom{n}{0} p^0 (1-p)^{n-0} + \binom{n}{1} p^1 (1-p)^{n-1} + \binom{n}{2} p^2 (1-p)^{n-2} + \cdots + \binom{n}{x} p^x (1-p)^{n-x}$$

Since A Levels permit the use of Texas Instrument Graphing Calculators in exam condition, I would also have to demonstrate the two rather commonly used functionality in TI-84 Plus CE, namely BinomialPDF and BinomialCDF that is equivalent the above two respectively.

BinomialPDF can be used when you are tasked to find $P(X = x)$ given parameters n and p.

Example 1. Given the random variable $X \sim B\left(3, \frac{1}{6}\right)$, find $P(X = 2)$.

This case requires the use of BinomialPDF functionality, which can be accessed by pressing the following buttons on the TI-84 PLUS CE in the following order:

Press [2ND] then [VARS] in exact order as mentioned and press the down arrow key repeatedly until you see your calculator cursor reaching an option called "binompdf".

Press [ENTER] key on the calculator and you should see something similar to the below example on the screen

trials:
p:
x value:
Paste

Key in value of n into the number of trials, and press [ENTER]
Key in value of p into the field "p" and press [ENTER]
Key in number of trials being computed for into the field "x value" and press [ENTER]
Once the cursor is on the "Paste", you should have the following,

trials: 3
p:1/6
x value: 2
Paste

Press [ENTER] after checking if the values are correct and you should see the following on your graphing calculator screen

binompdf(3, 1/6, 2)

Press [ENTER] and the answer should appear as follows (If the calculator isn't in fraction mode):

binompdf(3, 1/6, 2)
.0694444444

Example 2. Given the random variable $X \sim B\left(3, \frac{1}{6}\right)$, find the probability that $P(X \leq 2)$. This case requires the use of BinomialCDF functionality, which can be accessed by pressing the following buttons on the TI-84 PLUS CE in the following order.

Press [2ND] key, then Press [VARS] in exact order as mentioned and press the down arrow key repeatedly until you see your calculator cursor reaching an option called "binomcdf".

Press [ENTER] key on the calculator and you should see something similar to the below example on the screen

trials:
p:
x value:
Paste

Key in value of n into the number of trials, and press [ENTER]
Key in value of p into the field "p" and press [ENTER]
Key in number of trials being computed for into the field "x value" and press [ENTER]
Once the cursor is on the "Paste", you should have the following,

trials: 3
p:1/6
x value: 2
Paste

Press [ENTER] after checking if the values are correct and you should see the following on your graphing calculator screen

```
binomcdf(3,1/6,2)
```

Press [ENTER] and the answer should appear as follows (If calculator isn't in fraction mode)

```
binomcdf(3,1/6,2)
                .9953703704
```

Using graphing calculator manipulation to solve problems involving binomial distribution:

(Questions mostly taken from Power Math H2 Second Edition Volume 2 by PK Lim and slightly changed)

Q1. In XYZ Junior College, 65% of the student population are male.
12 students are randomly selected from this school. Find the probability that

- (i) Exactly 3 of them are male.
- (ii) At most 3 of them are male.
- (iii) Not less than 3 of them are male.
- (iv) More than 5 of them are male.
- (v) Between 4 to 8 of them inclusively are male.

Written working	Graphing Calculator Actions Performed
(i) X: Number of male students selected, out of 12 students $X \sim B(12,0.65)$ $P(X=3)=0.00476$	Go to "binompdf" option and press enter Key in trials as 12, Key in p as 0.65 Key in x value as 3 Answer obtained = 0.00476

(ii) $P(X \leq 3) = 0.00561$	Go to "binomcdf" option and press enter. Key in trials as 12 Key in p as 0.65 Key in x value as 3 Answer obtained = 0.00561 Final Answer: 0.00561
(iii) $P(X \geq 3) = 1 - P(X \leq 2) = 0.999$	Go to "binomcdf" option and press enter. Key in trials as 12 Key in p as 0.65 Key in x value as 2 Answer obtained = 8.479084920E-4 Press 1 – 8.479084920E-4 to get 0.9991520915 Final Answer: 0.999
(iv) $P(X > 5) = 1 - P(X \leq 5) = 0.915$	Go to "binomcdf" option and press enter. Key in trials as 12 Key in p as 0.65 Key in x value as 5 Answer obtained = 0.0846320652 Press 1 – 0.0846320652 to get 0.9153679348 Final Answer: 0.915
(v) $P(4 \leq X \leq 8) = P(X \leq 8) - P(X \leq 3)$ $= 0.648$	Go to "binomcdf" option and press enter. Key in trials as 12 Key in p as 0.65 Key in x value as 8 Answer obtained = 0.6533473038 Go to "binomcdf" options and press enter.

	Key in trials as 12 Key in p as 0.65 Key in x value as 3 Answer obtained = 0.0056097523 Press 0.6533473038-0.0056097523=0.6477375515 Final answer: 0.648

Q2. A survey shows that only 60% of all drivers in a town uses their seatbelts.

(i) 5 drivers in the town are randomly selected. Let X be the number of drivers who use their seat belts, out of 5 drivers from this town. Find the exact standard deviation of X.

(ii) If a sample of 500 drivers are taken, what is the expected number of drivers who use their seat belts?

(i)

X: Number of drivers who uses their seat belts, out of 5 drivers selected from this town.

$X \sim B(5, 0.6)$

Variance of Binomial Distribution $\sigma^2 = np(1-p)$

Standard Deviation of Binomial Distribution $\sigma = \sqrt{np(1-p)} = \sqrt{5(0.6)(1-0.6)} = \sqrt{1.2}$

(ii)

Y: Number of drivers who uses their seat belts, out of 500 drivers selected from this town

$Y \sim B(500{,}0.6)$

Expected number of drivers who uses their seat belts $E(Y) = np = 500(0.6) = 300$

Q3. A random variable $X \sim B(n, p)$ has mean of 8 and variance of 6.

(i) Find the value of both n and p.

(ii) Find the probability that X lies within 1 standard deviation of the mean.

(i)

Equation 1: $np = 8$

Equation 2: $np(1-p) = 6$

Sub Equation 1 into Equation 2.
$8(1-p) = 6$
$8 - 8p = 6$
$-8p = 6 - 8 = -2$
Equation 3: $p = \frac{2}{8} = 0.25$

Substitute Equation 3 into Equation 1.
$n(0.25) = 8$
$$n = \frac{8}{0.25} = 32$$

(ii)
Let X represent the binomial random variable above.
$X \sim B(32, 0.25)$

$\sigma^2 = np(1-p) = 32(0.25)(1-0.25)$
$\sigma^2 = 6$
$\sigma = \sqrt{6}$

$\sqrt{6} = 2.449489743$

$\mu = np = 8$
Range of values as follows: $\mu - \sqrt{6} < X < \mu + \sqrt{6}$
$P(8 - \sqrt{6} < X < 8 + \sqrt{6})$
$P(5.5505 < X < 10.4495)$

Rewritten to fit a discrete probability distribution where X has to be **strictly** within 1 standard deviation from the mean, it looks like the following.
$P(6 \leq X \leq 10) = P(X \leq 10) - P(X \leq 5) = 0.8464053667 - 0.1530030994$
$= 0.693$

Q4. In a particular IT show, the probability that a customer bought the newest "Tiun" brand laptop is given by p. Sixty-five customers were randomly chosen.

Given that $p < 0.5$ and the variance of the number of "Tiun" laptop bought is 2.7, find the most probable of number of "Tiun" laptop bought by customers out of 65 randomly chosen customers.

Variance = $\sigma^2 = 2.7 = 65p(1-p) = 65p - 65p^2$
$-65p^2 + 65p = 2.7$
$p = 0.04342$ or $p = 0.95658$ (Rejected)

Since it is stated in question that $p < 0.5$ we have to reject one solution above as shown.

X: Number of customers who bought "Tiun" brand laptop out of 65 randomly chosen customers
$X \sim B(65, 0.04342)$

Graphing Calculator instructions for finding most probable number (AKA the mode) of the probability distribution above.

Press the [Y=] button
Press [2ND] and then press [VARS]
Scroll down repeatedly until you see "binompdf" and press [ENTER]

Key in "65" into the trials field. Key in 0.0434 as value of p. Key in letter X by pressing [Alpha] then [STO]. Press Enter twice and you should see the following on your graphing calculator

```
Plot1  Plot2  Plot3
\Y1= binompdf(65, 0.0434, X)
\Y2=
\Y3=
\Y4=
\Y5=
\Y6=
\Y7=
```

Once you reach the above stage, press [2ND] and then press [Graph] and the following should appear on the graphing calculator screen below.

X	Y1
0	.05583
1	.16473
2	.23927
3	.22807
4	.16064
5	.08886
6	.04033

In this case we already know that the most probable value of number of customers who bought the "Tiun" laptop is 2 as it has the highest probability (0.23927), but in other cases, you may need to scroll down all the way from 0 to n to locate the most probable value.

Q5. The random variable R denotes the number of red cars observed in a survey of n cars.

(i) Write down, in context, two assumptions needed to model R by a binomial distribution

It may be assumed that R has the distribution $B(20, p)$

(ii) Given that $p = 0.15$, find $P(4 \leq R < 8)$.

Written working and answers	Graphing calculator actions performed
(i) The event of observing red cars every time within the survey has to be independent and the probability of observing any one red car is constant for all observation within the survey.	
(ii) X: Number of red cars observed out of 20 cars $X \sim B(20, 0.15)$ $P(4 \leq R < 8) = P(X \leq 7) - P(X \leq 3)$ $= 0.351$	Go to binomcdf option and press enter Key in trials as 20 Key in p as 0.15 Key in X value as: 7 Press [ENTER] button twice Answer: 0.9940788545

<table>
<tr>
<td></td>
<td>Go to binomcdf option and press [ENTER]
Key in trials as 20
Key in p as 0.15
Key in X value as 3
Press [Enter] button twice
Answer: 0.6477251743
0.9940788545−0.6477251743=0.346

Final answer = 0.346</td>
</tr>
</table>

Q6. Given that $X \sim B(15{,}0.4)$,
Find the largest integer r, such that $P(X > r) > 0.1$

<table>
<tr>
<td>$P(X > r) > 0.1$
$P(X \leq r) < 0.9$
$r = 7$</td>
<td>Press [Y=] button
Press [2ND] button and then press [VARS] button
Scroll down until your cursor is on “binomcdf” and press [ENTER]

After pressing [ENTER],
Key in trials: 15
p: 0.4
x value: X

Press [ENTER] twice and you should see the following on your graphing calculator screen

<table>
<tr><td>Plot 1 Plot 2 Plot 3</td></tr>
<tr><td>\Y1= binomcdf(15, 0.4, X)
\Y2=
\Y3=
\Y4=
\Y5=
\Y6=
\Y7=</td></tr>
</table>
</td>
</tr>
</table>

<table>
<tr><td></td><td>Press [2ND] and then Press [Graph] and the following should appear on the screen.

<table>
<tr><td>X1</td><td>Y1</td></tr>
<tr><td>0</td><td>4.7E-4</td></tr>
<tr><td>1</td><td>.0517</td></tr>
<tr><td>2</td><td>.02711</td></tr>
<tr><td>3</td><td>.0905</td></tr>
<tr><td>4</td><td>.21728</td></tr>
<tr><td>5</td><td>.40322</td></tr>
<tr><td>6</td><td>.60981</td></tr>
</table>

Keep scrolling down until you find the largest integer that satisfy the condition stated and derived from the question

In this case, after scrolling, the largest integer $r = 7$ has the highest probability (0.7869 that satisfy the question of $P(X \leq r) < 0.9$

Final answer, largest integer $r = 7$</td></tr>
</table>

Q7. Given that $X \sim B(15, p)$ and that $p > 0.3$, find the value of p such that $P(X = 4) = 0.12$.

<table>
<tr><td>$X \sim B(15, p)$
$P(X = 4) = 0.12$
$p = 0.406$</td><td>Press [Y=] button
Press [2ND] followed by [VARS]
Scroll down to binompdf and press [ENTER]

Set the values as follow below
trials: 15
p: X
X value: 4
And press [ENTER] twice</td></tr>
</table>

	You should see the following on the screen by this time now. \Y1=binompdf(15, X, 4) \Y2= \Y3= \Y4= \Y5= \Y6= \Y7= Press [MATH] button and select "MATH" and press [ENTER] Scroll down to Numeric Solver and press [ENTER] You should see the following on the screen by this time now. E1: E2: In the E1 box field, press [ALPHA], followed by [TRACE] and a pop-up should appear, select Y1 and press [ENTER] In the E2 box field, key in the value of $P(X = 4)$ which is 0.12

	You should see the following on your screen: E1: Y1 E2: 0.12 OK Press [GRAPH] and you should see the following on your screen: Y1=0.12 X= bound = {-1E99, 1E99} Once at this stage, always set X = 0.5 and press [GRAPH] button. And the following should appear. Y1=0.12 X= 0.5 bound = {-1E99, 1E99} E1-E2 = 0 SOLVE

<table>
<tr><td></td><td>Set bounds as {0.3, 1}, since question mentioned $p > 0.3$ and probability cannot exceed 1 in value. And the following should appear.

Y1=0.12

X= 0.5
bound = {0.3, 1}
E1-E2 = 0

SOLVE

Press [ALPHA], followed by [ENTER} key

Y1=0.12

X= 0.40645273391598
bound = {0.3, 1}
E1-E2 = 0

SOLVE

Answer: $p = 0.406$</td></tr>
</table>

Q8. Given that the random variable $X \sim B(40, p)$ and find the value of p such that $P(X \leq 2) = 0.3$

<table>
<tr><td>$X \sim B(40, p)$
$P(X \leq 2) = 0.3$
$p = 0.0885746347538$</td><td>Press [Y=] button
Press [2ND] followed by [VARS]
Scroll down to binomcdf and press [ENTER]
Set the values as follows</td></tr>
</table>

	The following should appear on your screen \Y1=binomcdf(40, X, 2) \Y2= \Y3= \Y4= \Y5= \Y6= \Y7= Press [MATH] button and select "MATH" and press [ENTER] Scroll down to Numeric Solver and press [ENTER] You should have the following on your graphing calculator screen now E1: E2: In the E1 box field, press [ALPHA], followed by [TRACE] and a pop-up should appear, select Y1 and press [ENTER] In the E2 box field, key in the probability value as "0.3" and press [ENTER] and set the bound to "{0,1}" and you should have the following on the display screen.

	Y1=0.3 X=0 Bound {0, 1} Always set X=0.5 and press [ALPHA] key followed by [ENTER] key and you should see the following on the screen now. Y1=0.3 X=0.0885746347538 Bound {0,1} Answer: $p = 0.0886$

Title	Junior College 'A' Levels H1/H2 Mathematics – Normal Distribution
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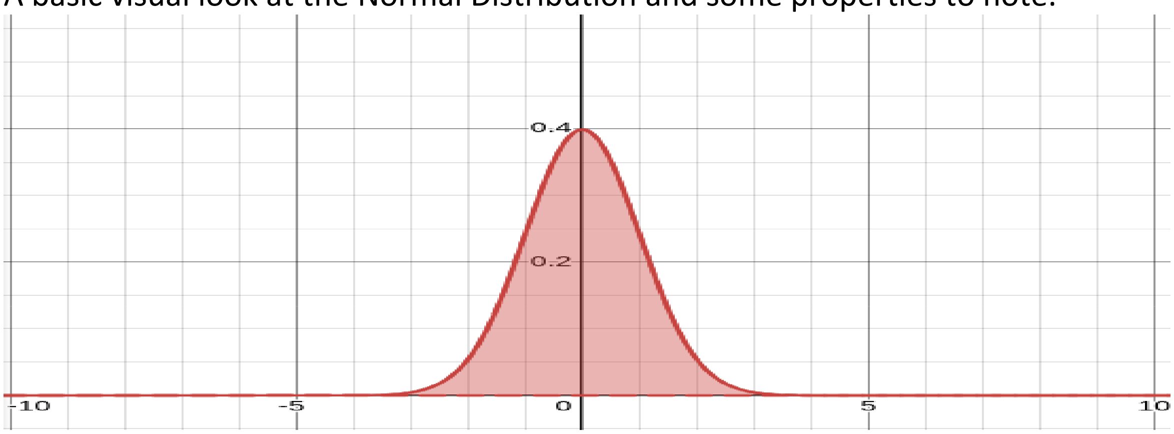
In many situations out there, many types of continuous random variables, such as test scores, height and weight of students of a certain age would show the following characteristics once data got collected and plotted as it would be on a probability distribution.

- You would realize that majority of the data are centred in the middle, with extreme small and big values tailing off symmetrically on the left and right of the average measurement respectively.

When this happens, the random variable X is said to follow a Normal Distribution with parameters μ and σ^2, where $X \sim N(\mu, \sigma^2)$, with μ referring to the mean and σ^2 referring to the variance of the Normal Distribution.

(*Be careful when dealing with notations, certain books, software and calculators deal with Normal Distribution using the Standard Deviation σ parameter rather than variance which is σ^2. In such cases, the obvious first step you should take is to square-root the variance to get the value of standard deviation σ.)

A basic visual look at the Normal Distribution and some properties to note.



- The probability value is the area between the curve and the x-axis. (Which is also the definite integral of the Normal Distribution in question.)
- The probability of getting a very specific value in a Normal Distribution is basically zero since area under curve cannot be created on a continuous random variable just with specific values. Instead of defining specific value on a Normal Distribution, we usually define a range of values to calculate probability in a Normal Distribution. [Therefore $P(X = x) = 0$]

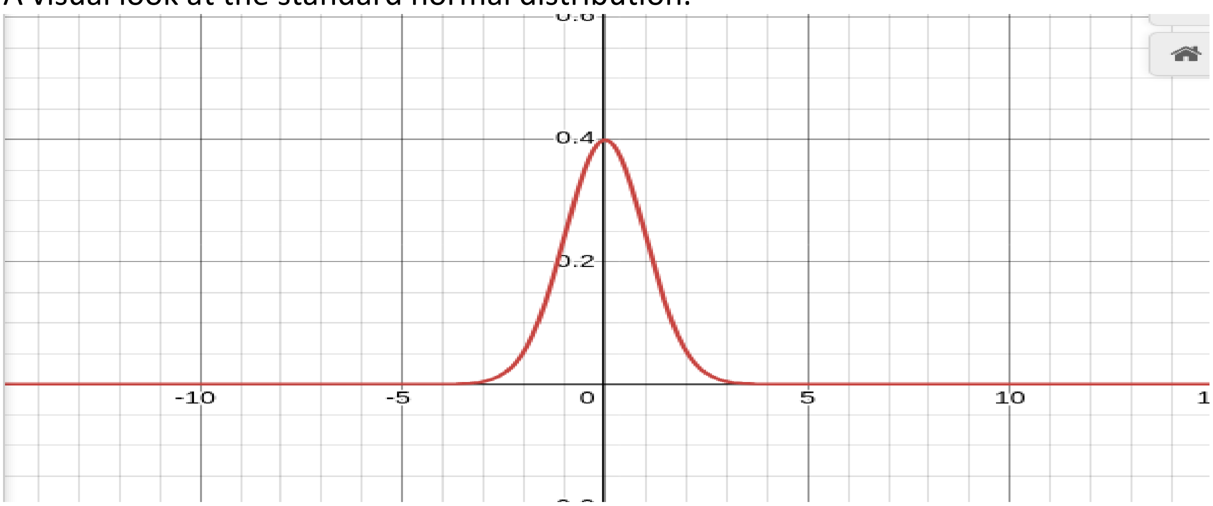
- Any Normal Distribution is symmetrical at the mean μ. (This property is important as certain questions you will encounter requires you to understand this symmetrical property of any Normal Distribution.)

Understanding the concept of a Standard Normal Distribution

- Any Normal Distribution can technically be transformed to a Standard Normal Distribution.
- A Standard Normal Distribution has the property of which the area under curve from negative infinity to positive infinity is exactly 1.
- A Standard Normal Distribution also has property of mean $\mu = 0$ and $\sigma^2 = 1$, for this reason, a Standard Normal Distribution will also have standard deviation $\sigma = 1$ as well.

(Note: In order to input "negative infinity" in Graphing Calculator, press -E99. In order to input a value of "positive infinity" in Graphing Calculator, press E99.)

A visual look at the standard normal distribution.



Understanding the concept of Z-score in Standard Normal Distribution

- Z-score refers to the number of standard deviations from the mean in a standardized normal distribution where the Z −score can take on any finite values. $(-\infty < Z < \infty)$
- Z-score of any normal distribution can be computed using the below formula.

$$Z = \frac{X - \mu}{\sigma}$$

Z refers to the Z-score
X refers to the position of the normal random variable on the X-axis as it is in original unstandardized form.
μ refers to the mean of the normal distribution as it is in original unstandardized form.
σ refers to the standard deviation as it is in the original unstandardized form.

Process of Finding Probability in a Standard Normal Distribution:
In order to find probability in a Standard Normal Distribution, we use the TI-84 graphing calculator in the following manner.
Example 1. Find the probability of the following

a) $P(Z < 1.96)$
b) $P(Z > -0.586)$
c) $P(0.43 < Z < 1.23)$

Procedures (Example 1a)	Output on Graphing Calculator Screen
Press [2ND] followed by [VARS]	DISTR DRAW 1:normalpdf (2:normalcdf (3:invNorm (4:invT (5:tpdf (6:tcdf (7:χ^2pdf (
Look for “normalcdf” option and press [ENTER]	normalcdf Lower: Upper: μ: 0 σ: 1 Paste
For probability values less than Z We key in -E99 in the field “lower” and we key in 1.96 in the upper field. Since Standard Normal Distribution has a value of $\mu = 0$ and $\sigma = 1$, we input $\mu = 0$ and $\sigma = 1$	normalcdf Lower: -E99 Upper: 1.96 μ: 0 σ: 1 Paste
Press the down arrow after checking the inputs and press down arrow until the cursor is on “Paste” and press [ENTER] twice. The probability is 0.975 (3sf)	normalcdf(-E99, 1.96, 0, 1) .9750021748

Procedures (Example 1b)	Graphing Calculator Output
Press [2ND] followed by [VARS]	DISTR DRAW 1:normalpdf (2:normalcdf (3:invNorm (4:invT (5:tpdf (6:tcdf (7:χ^2pdf (
Look for “normalcdf” option and press [ENTER]	normalcdf Lower: Upper: μ: 0 σ: 1 Paste
For probability values more than Z. We key in -0.586 in the “Lower” field and E99 into the “Upper” field. Since Standard Normal Distribution has a value of $\mu = 0$ and $\sigma = 1$, we input $\mu = 0$ and $\sigma = 1$.	normalcdf Lower: -0.586 Upper: E99 μ: 0 σ: 1 Paste
Press the down arrow after checking the inputs and press down arrow until the cursor is on “Paste” and press [ENTER] twice. The probability is 0.721 (3sf)	normalcdf(-0.586, E99, 0, 1) .7210622905

Procedures (Example 1C)	Graphing Calculator Output
Press [2ND] followed by [VARS]	DISTR DRAW 1:normalpdf (2:normalcdf (3:invNorm (4:invT (5:tpdf (6:tcdf (7:χ^2pdf (
Look for "normalcdf" option and press [ENTER]	normalcdf Lower: Upper: μ: 0 σ: 1 Paste
Since the question mentioned we have to find the probability for which the Z-score is in between 0.43 and 1.23. We key in 0.43 in the "Lower" field and 1.23 in the "Upper" field. We set $\mu = 0$ and $\sigma = 1$.	normalcdf Lower: 0.43 Upper: 1.23 μ: 0 σ: 1 Paste
After checking that the values are correct, we can press down arrow key on calculator until the cursor is on "Paste", press enter twice. The probability of obtaining Z-score between 0.43 and 1.23 is 0.224 (3sf)	normalcdf (0.43, 1.23, 0, 1) .2242492346

Example 2

Find the probability of the following

(a) $P(|Z| < 1.234)$

(b) $P(|Z| \geq 2.17)$

(c) $P(|Z-1| > 1.1389)$

2(a) $|Z| < 1.234$ is to be rewritten as $-1.234 < Z < 1.234$

Steps taken	Graphing Calculator Output
Enter “normalcdf” functionality of the graphing calculator	normalcdf lower: upper: μ: σ: Paste
Set “lower” as -1.234 Set “upper” as 1.234 Set μ as 0 Set σ as 1 Press down arrow key until the cursor is on “Paste”	normalcdf lower: -1.234 upper: 1.234 μ:0 σ:1 Paste
Press [ENTER] twice and the following should appear. The probability is 0.783 (3sf)	normalcdf(-1.234, 1.234, 0, 1) .7827969667

2(b) $|Z| \geq 2.17$ is to be written as $Z < -2.17$ OR $Z > 2.17$

Steps Taken	Graphing Calculator Output
Enter “normalcdf” functionality of calculator	normalcdf lower: upper: μ: σ: Paste

Input lower as - E99 Input upper as -2.17 Set $\mu = 0$ and $\sigma = 1$ Press arrow down key until the cursor is at paste.	normalcdf lower: -E99 upper: -2.17 μ:0 σ:1 Paste
Press [ENTER] twice.	normalcdf(-E99,2.17,0, 1) .150033693
Enter "normalcdf" functionality of calculator again. Input lower as 2.17 Input upper as E99 Set $\mu = 0$ and $\sigma = 1$ Press arrow down key until the cursor is at paste.	normalcdf lower: 2.17 upper: E99 μ:0 σ:1 Paste
Press [ENTER] twice.	normalcdf(2.17, E99, 0, 1) .150033693
$P(\|Z\| \geq 2.17)$ = 0.150033693+0.150033693 = 0.300 (3sf)	

2(c)

Rewrite $P(|Z-1| > 1.1389)$ as $Z - 1 < -1.1389$ as well as $Z - 1 > 1.1389$ which can be transformed as follows:

$Z < -1.1389 + 1$ OR $Z > 1.1389 + 1$

$Z < -0.1389$ OR $Z > 2.1389$

Steps Taken	Graphing Calculator Output
Enter “normalcdf” functionality of calculator	normalcdf lower: upper: μ: σ: Paste
Key in the values as follows Lower: -E99 Upper: -0.1389 μ:0 σ: 1	normalcdf lower:-E99 upper:-0.1389 μ:0 σ:1 Paste
Press down arrow until the cursor is at “paste” and press [ENTER] key twice	normalcdf(-E99, -0.1389, 0, 1) 0.447645561
Enter “normalcdf” functionality of calculator again	normalcdf lower: upper: μ: σ: Paste
Key in the values as follows Lower: 2.1389 Upper: E99 μ:0 σ: 1	normalcdf lower: 2.1389 upper: E99 μ:0 σ:1 Paste

Press down arrow until the cursor is on "Paste" and press [Enter] key twice	normalcdf(2.1389, E99, 0, 1) 0.0162218257

Therefore $P(|Z-1| > 1.1389) =$ 0.447645561+0.0162218257=0.461 (3sf)

Process of Finding Probability Values of Normal Distribution without consideration for Standardization.

Example 3.
Given that $X{\sim}N(23{,}9)$, find the following probabilities

(a) $P(20 < X < 25)$
(b) $P(X < 19)$
(c) $P(X \geq 14)$

(a)

Enter "normalcdf" functionality of calculator	normalcdf lower: upper: μ: σ: Paste
Key in the following values Lower: 20 Upper: 25 μ:23 σ: 3	normalcdf lower:20 upper: 25 μ:23 σ:3 Paste
Press down arrow until the cursor is on "Paste" and press [ENTER] twice to get probability value	normalcdf(20,25,23,3) .5888522734

$P(20 < X < 25) = 0.589$ (3sf)

(b)

Enter "normalcdf" functionality of calculator	normalcdf lower: upper: μ: σ: Paste
Key in the following values Lower: -E99 Upper: 19 μ:23 σ: 3	normalcdf lower: -E99 upper:19 μ:23 σ:3 Paste
Press down arrow until the cursor is on "Paste" and press [ENTER] twice to get probability value	normalcdf(-E99,19,23,3) .0912112819

$P(X < 19) = 0.0912$ (3sf)

(c)

Enter "normalcdf" functionality of calculator	normalcdf lower: upper: μ: σ: Paste
Key in the following values Lower: 14 Upper: E99 μ:23 σ: 3	

	normalcdf lower: 14 upper: E99 μ:23 σ:3 Paste
Press down arrow until the cursor is on "Paste" and press [ENTER] twice to get probability value	normalcdf(14,E99,23,3) .9986500328

$P(X \geq 14) = 0.999$(3sf)

Example 4.
Given that $X \sim N(15,3)$, find the following probabilities

(a) $P(|X - 15| < 4)$
(b) $P(|X - 15| > 2)$

4(a)
$P(|X - 15| < 4)$ can be rewritten as,
$P(-4 < X - 15 < 4)$
$P(11 < X < 19)$

Enter normalcdf functionality of calculator	normalcdf lower: upper: μ: σ: Paste

Key in the following values as follows Lower: 11 Upper: 19 μ:15 σ:$\sqrt{3}$	normalcdf lower:11 upper:19 μ:15 σ: $\sqrt{3}$ Paste
Press arrow down key until the cursor is on top of "paste" and press enter twice.	normalcdf(11, 19, 15, $\sqrt{3}$) .9790787186

$P(|X-15|<4)=0.979$ (3sf)

4(b) $P(|X-15|>2)$
Rewritten, it will look like the following
$P(X-15<-2)$ OR $P(X-15>2)$
$P(X<13)$ OR $P(X>17)$

Enter normalcdf functionality of graphing calculator	normalcdf lower: upper: μ: σ: Paste
Key in the following into the fields Lower: -E99 Upper: 13 μ:15 σ:$\sqrt{3}$	normalcdf lower:-E99 upper:13 μ:15 σ: $\sqrt{3}$ Paste
Press arrow down key until the cursor is on top of "paste" and press enter twice.	normalcdf(-E99, 13, 15, $\sqrt{3}$) .1241065934

Enter normalcdf functionality of graphing calculator again	normalcdf lower: upper: μ: σ: Paste
Key in the following into the fields as follows: Lower: 17 Upper: E99 μ:15 σ:$\sqrt{3}$	normalcdf lower:17 upper:E99 μ:15 σ: $\sqrt{3}$ Paste
Press arrow key until the cursor is on top of "Paste" and press [Enter] twice	normalcdf(17, E99, 15, $\sqrt{3}$) .1241065934

Adding both probability values, we get the following
0.1241065934+0.1241065934=0.248 (3sf)

Using inverse Normal Distribution functionality to find Z-Score or number of standard deviations away from mean by input of p-values from negative infinity of a normal distribution.

After studying how to find probability upon knowing the values of Z-Score or number of standard deviations away from mean, along with parameters μ and σ. It will be a logical next step to wonder if the reverse is also possible. The TI-84 family of calculator has a functionality that allows students to deduce the Z-score after knowing the probability value from negative infinity of the normal distribution up to the $Z - score$, along with parameters μ and σ.

The instructions below explain how to get into the "InvNorm" functionality of Ti-84 graphing calculators.

Steps Taken	Calculator Output

Press [2ND] followed by [VARS]	DISTR DRAW 1:normalpdf (2:normalcdf (3:invNorm (4:invT (5:tpdf (6:tcdf (7:χ^2pdf (
Press arrow down key until cursor is on top of "3:InvNorm" and press [ENTER]	InvNorm Area: μ: σ: Tail: LEFT CENTER RIGHT Paste:

Example 5

Given that $W \sim N(6,6)$, find the value of a or the range of values of a for each of the following:

(a) $P(W < a) = 0.00144$
(b) $P(W \geq a) > 0.25$
(c) $P(6 < W < a) > 0.4999$
(d) $P(|W| < a) = 0.01$
(e) $P(|W| \geq a) = 0.975$

(a)

Go to InvNorm Functionality of Graphing Calculator	Area: μ: σ: Tail: LEFT CENTER RIGHT Paste
Key in the fields as follows Area:0.00144 μ:6 σ: $\sqrt{6}$ Tail: Select "Left"	Area: 0.00144 μ:6 σ: $\sqrt{6}$ Tail: LEFT CENTER RIGHT Paste:
Press down arrow until the cursor is at "Paste" and press [ENTER] twice and the following should appear on the screen	invNorm (0.00144, 6, $\sqrt{6}$, LEFT) -1.300126258

$a = -1.30$ (3sf)

(b)

Go to InvNorm Functionality of Graphing Calculator	Area: μ: σ: Tail: LEFT CENTER RIGHT Paste
Key in the fields as follows Area: 0.25 μ: 6 σ: $\sqrt{6}$ Tail: RIGHT	Area: 0.25 μ:6 σ:$\sqrt{6}$ Tail: LEFT CENTER RIGHT Paste
Press down arrow until the cursor is at "Paste" and press [ENTER] twice and the following should appear on the screen	invNorm(0.25, 6, $\sqrt{6}$, RIGHT) 7.652155723

$a < 7.65$ (3sf)

(c)

Since $\mu = 6$ as well, we can agree that $P(W \leq 6) = 0.5$ which implies

$P(W < a) > 0.5 + 0.4999$

$P(W < a) > 0.9999$

Go to "invNorm" functionality of graphing calculator	InvNorm Area: μ: σ: Tail: LEFT CENTER RIGHT Paste:
Key in the fields as follows Area: 0.9999 μ: 6 σ: $\sqrt{6}$ Tail: Left	InvNorm Area: 0.9999 μ: 6 σ: $\sqrt{6}$ Tail: LEFT CENTER RIGHT Paste:
Press arrow down key until the cursor is on top of "Paste" and press [ENTER] key twice.	invNorm(0.9999,6,$\sqrt{6}$, LEFT) 15.10969287

$a > 15.1$(3sf)

(d)
$P(|W| < a) = 0.01$
Rewritten we get the following:
$P(-a < W < a) = 0.01$

Press [Y=] button and press [2ND] followed by [VARS] button. Select “normalcdf” option and fill in as follows: Lower: -X Upper: X μ:6 σ: $\sqrt{6}$	normalcdf lower: -X upper: X μ: 6 σ: $\sqrt{6}$ Paste
Scroll down until cursor is on top of “Paste” and press [Enter]	Y1= normalcdf(-X, X, 6, $\sqrt{6}$) Y2= Y3= Y4= Y5= Y6= Y7= Y8=
Press [MATH] and scroll down to look for numeric solver and press [ENTER]	Equation Solver E1: $\boxed{}$ E2: $\boxed{}$
Press [ALPHA] followed by trace and a pop up should appear, select Y1 and press [ENTER]	Equation Solver E1: $\boxed{\text{Y1}}$ E2: $\boxed{}$

Scroll down to the box named "E2" and key in 0.01	Equation Solver E1: Y1 E2: 0.01 OK
Press [GRAPH] and the following should appear	Y1 = 0.01 X=0 Bound= {-1E99, 1E99} Solve
Set X = 0.5	Y1 = 0.01 X=0.5 Bound= {-1E99, 1E99} Solve
Press [GRAPH] and the following should appear on the calculator	Y1 = 0.01 X=0.5883012575898 Bound= {-1E99, 1E99} Solve

$a = 0.588$(3sf)

(e)
$P(|W| \geq a) = 0.975$ can be rewritten as the following
$P(|W| < a) = 1 - 0.975 = 0.025$ and hence,
$P(-a < W < a) = 0.025$

Press [Y=] button and press [2ND] followed by [VARS] button. Select "normalcdf" option and fill in as follows: Lower: -X Upper: X μ:6 σ: $\sqrt{6}$	normalcdf lower: -X upper: X μ: 6 σ: $\sqrt{6}$ Paste

Scroll down until cursor is on top of "Paste" and press [Enter]	Y1= normalcdf(-X, X, 6, $\sqrt{6}$) Y2= Y3= Y4= Y5= Y6= Y7= Y8=
Press [MATH] and scroll down to look for numeric solver and press [ENTER]	Equation Solver E1: E2:
Press [ALPHA] followed by trace and a pop up should appear, select Y1 and press [ENTER]	Equation Solver E1: Y1 E2:
Scroll down to the box named "E2" and key in 0.025	Equation Solver E1: Y1 E2: 0.025 OK
Press [Graph] and the following should appear	Y1 = 0.025 X=0 Bound= {-1E99, 1E99} Solve

Set X=0.5	Y1 = 0.025 X=0.5 Bound= {-1E99, 1E99} Solve
Press [Graph]	Y1 = 0.025 X=1.2611029611347 Bound= {-1E99, 1E99} Solve

$a = 1.26$

Title	Normal Distribution – Operations Involving Linear Combination of Normal Random Variables and Sum of Multiple Independent Identically Distributed Normal Random Variables
Author	AprilDolphin
Date	17/5/2025

Given a Normal Random Variable X with parameters of μ as its mean and σ^2 as its variance, the linear function of $aX \pm b$ will have the following parameters and properties

1. The linear function of $aX \pm b$ is also normally distributed.
2. The linear function of $aX \pm b$ will have parameters of $a\mu + b$ and $a^2\sigma^2$ which can be written in the following way $aX \pm b \sim N(a\mu \pm b, a^2\sigma^2)$.

Given 2 independent random normal variable X and Y with the following parameters

Random Variable	Mean (AKA Expected Value, denoted as $E(X)$ or $E(Y)$) respectively	Variance, denoted as $Var(X)$ or $Var(Y)$ respectively
X	μ_X	σ_X^2
Y	μ_Y	σ_y^2

1. The linear combination of X and Y can be expressed as the following: $aX \pm bY \sim N(a\mu_X \pm b\mu_Y, a^2\sigma_X^2 + b^2\sigma_y^2)$
2. The linear combination of aX and bY is also normally distributed.

Given n independent identically distributed normal random variables which we can call $X_1, X_2, X_3 \ldots X_n$ where each variable has the parameter of μ and σ^2.

1. The sum of all independent identically distributed normal random variables is also normally distributed.
2. The sum of all independent identically distributed normal random variables shall have the parameters of $n\mu$ and $n\sigma^2$ which can also be written as the following in notation form.

$$X_1 + X_2 + X_3 \ldots + X_n \sim N(n\mu, n\sigma^2)$$

Important Note here:

$X_1 + X_2 + X_3 \ldots + X_n \neq nX$

Questions Taken from Power Math H2 Second Edition Book Authored by PK Lim

Q1. The mass of a certain grade of oranges is normally distributed with mass of 50g and standard deviation 6g.

(a) If 5 oranges are chosen at random,
Find the probability that their total mass will exceed 260g.

(b) If 1 orange is chosen at random,
Find the probability that 5 times the mass of the orange will exceed 260g.

Written Workings	Graphing Calculator Operation
(a) X: Mass of a certain grade of orange $X_1 + X_2 + X_3 + X_4 + X_5$ $E(X_1 + X_2 + X_3 + X_4 + X_5) = 5(50) = 250$ $Var(X_1 + X_2 + X_3 + X_4 + X_5) = 5(6^2)$ $= 180$ $X_1 + X_2 + X_3 + X_4 + X_5 \sim N(250, 180)$ $P(X_1 + X_2 + X_3 + X_4 + X_5 > 260) = 0.228$	After obtaining the mean and variance, we can proceed on calculation of probability of the mass exceeding 260g. Press [2ND] key, followed by [VARS] key and select "normalcdf" which should direct you to the following familiar interface. normalcdf lower: upper: μ: σ: Paste Key in the lower limit as 260 Key in the upper limit as E99 Key in μ as 250 Key in standard deviation as $\sqrt{180}$ Which should produce the following screen output normalcdf lower: 260 upper: E99 μ: 250 σ: $\sqrt{180}$ Paste

<table>
<tr><td></td><td>Press the down arrow key until the cursor is on top of “Paste” and press [ENTER] twice to get the answer which is 0.2280281968</td></tr>
<tr><td>(b)
X: Mass of a certain grade of orange
$X \sim N(50, 6^2)$
$E(5X) = 50 \times 5 = 250$
$Var(5X) = 5^2 \times 6^2 = 900$
$5X \sim N(250, 900)$
$P(5X > 260) = 0.369$</td><td>After obtaining the mean and variance we can proceed with the calculation of probability that 5 times the orange mass exceeds 260g

Press [2ND] key, followed by [VARS] key and select “normalcdf” which will bring you to this interface right here.

normalcdf
lower:
upper:
μ:
σ:
Paste

Key in lower as: 260
Key in upper as: E99
Key in μ as 250
Key in σ as $\sqrt{900}$
Which will produce the following output

normalcdf
lower: 260
upper: E99
μ: 250
σ: $\sqrt{900}$
Paste

Once done, press down key until the cursor is at “Paste” and press [ENTER] Key twice to get the answer to the question which is 0.3694414037</td></tr>
</table>

Q2. X and Y are independent normal random variables.
The means of X and Y are 10 and 12 respectively,
and their variances are 4 and 9 respectively.

(a) Find $P(3Y < X)$

(b) Find $P(4X + 5Y > 90)$

X_1 and X_2 are two independent observations of X,
and $Y_1, Y_2, Y_3 \dots \dots Y_{10}$ are ten independent observations of Y.

(c) Find the value of a such that $P(X_1 + X_2 < a) = \frac{1}{4}$

(d) Find $P(2Y - 1 > X_1 + X_2)$

(e) Find $P(|Y_1 + Y_2 + Y_3 \dots \dots + Y_{10} - 10Y| < 2)$

<table>
<tr>
<td>
(a)
$X \sim N(10, 4)$
$Y \sim N(12, 9)$

$E(3Y) = 12 \times 3 = 36$
$Var(3Y) = 3^2 \times 9 = 81$

$E(X) = 10$
$Var(X) = 4$

$P(3Y < X) = P(3Y - X < 0)$

$E(3Y - X) = 36 - 10 = 26$
$Var(3Y - X) = 81 + 4 = 85$

$3Y - X \sim N(26,85)$
$P(3Y - X < 0) = 0.00240$
</td>
<td>
After finding the expected value and variance of the normal random variable $3Y - X$, we use the graphing calculator to calculate $P(3Y - X < 0)$

Press [2ND] followed by [VARS] and select “normalcdf” and press [ENTER] to get to this screen as demonstrated just below.

normalcdf
lower:
upper:
μ:
σ:
Paste

Key in the parameters as follows
Lower: -E99
Upper: 0
μ: 26
σ: $\sqrt{85}$

normalcdf
lower: -E99
upper: 0
μ: 26
σ: $\sqrt{85}$
Paste
</td>
</tr>
</table>

	Press the down arrow key until the cursor is on top of “Paste” and press [ENTER] twice to get the answer 0.0024005266
(b) $X \sim N(10, 4)$ $Y \sim N(12, 9)$ $E(4X) = 4 \times 10 = 40$ $Var(4X) = 64$ $E(5Y) = 5 \times 12 = 60$ $Var(5Y) = 5^2 \times 9 = 225$ $E(4X + 5Y) = 100$ $Var(4X + 5Y) = 289$ $4X + 5Y \sim N(100, 289)$ $P(4X + 5Y > 90) = 0.722$	After obtaining the expected value and variance, press [2ND] and press [VARS] and choose “normalcdf” to get the following screen. normalcdf lower: upper: μ: σ: Paste Set lower as 90 Set upper as E99 Set μ as 100 Set σ as $\sqrt{289}$ and the following should appear normalcdf lower: 90 upper: E99 μ: 100 σ: $\sqrt{289}$ Paste Press down key until the cursor is on top of “Paste” and press [ENTER] twice to get the answer which is 0.7218128636

(c) $X \sim N(10,4)$ $E(X_1 + X_2) = 2 \times 10 = 20$ $Var(X_1 + X_2) = 2 \times 4 = 8$ $X_1 + X_2 \sim N(20,8)$ $P(X_1 + X_2 < a) = \frac{1}{4}$ $a = 18.1$	After getting the probability value of $\frac{1}{4}$, and obtaining the mean and variance, the next step will be to use invNorm functionality to get the value of a. Press [2ND] followed by [VARS] and select "InvNorm" Set area as $\frac{1}{4}$ Set μ: 20 Set σ: $\sqrt{8}$ Set Tail as Left Once done you should get the following output on screen InvNorm Area: $\frac{1}{4}$ μ:20 σ: $\sqrt{8}$ Tail: Left Centre Right Paste Press the arrow down until your cursor is on top of paste and press [Enter] twice to get the value of 18.0922549
(d) $Y \sim N(12,9)$ $X \sim N(10,4)$ $E(2Y - X_1 - X_2) = 24 - 2(10) = 4$ $Var(2Y - X_1 - X_2) = 2^2(9) + 2(4) = 44$ $P(2Y - X_1 - X_2 > 1) = 0.674$	After obtaining the mean and variance of the random variable $2Y - X_1 - X_2$, which is 4 and 44 respectively, press [2ND] followed by [VARS] and the following should appear on the graphing calculator. normalcdf lower: upper: μ: σ: Paste

<table>
<tr>
<td></td>
<td>Key in lower as 1
Key in upper as E99
Key in mean as 4
Key in standard deviation as $\sqrt{44}$

normalcdf
lower: 1
upper: E99
μ: 4
σ: $\sqrt{44}$
Paste

After checking your input, press down arrow until the cursor is on top of "Paste" and press [ENTER] twice to get the answer which is 0.6744616637</td>
</tr>
<tr>
<td>(e)
$P(|Y_1 + Y_2 + Y_3 \dots \dots + Y_{10} - 10Y| < 2) =$
$P(-2 < Y_1 + Y_2 + Y_3 \dots \dots + Y_{10} - 10Y < 2)$

$E(Y_1 + Y_2 + Y_3 \dots \dots + Y_{10}) = 10 \times 12$
$= 120$
$Var(Y_1 + Y_2 + Y_3 \dots \dots + Y_{10}) = 10 \times 9$
$= 90$

$E(10Y) = 10 \times 12 = 120$
$Var(10Y) = 10^2 \times 9 = 900$

$E(Y_1 + Y_2 + Y_3 \dots \dots + Y_{10} - 10Y)$
$= 120 - 120 = 0$

$Var(Y_1 + Y_2 + Y_3 \dots \dots + Y_{10} - 10Y)$
$= 900 + 90 = 990$

$E(Y_1 + Y_2 + Y_3 \dots \dots + Y_{10} - 10Y \sim N(0, 990)$

$P(-2 < Y_1 + Y_2 + Y_3 \dots \dots + Y_{10} - 10Y < 2)$
$= 0.0507$</td>
<td>Once the expected value and variance is obtained for$Y_1 + Y_2 + Y_3 \dots \dots + Y_{10} - 10Y$, on your graphing calculator, press [2ND] followed by [VARS] and select normalcdf to get to the screen below.
normalcdf
lower:
upper:
μ:
σ:
Paste

Once within the normalcdf screen, press the following into the graphing calculator

Lower: -2
Upper: 2
μ: 0
σ: $\sqrt{990}$</td>
</tr>
</table>

	The following should appear after you keyed in the values mentioned above. normalcdf lower: -2 upper: 2 μ:0 σ: $\sqrt{990}$ Paste Press [ENTER] twice and you will get the answer to the question which is 0.0506828817