Name:	Register No.:	Class:



CRESCENT GIRLS' SCHOOL SECONDARY FOUR PRELIMINARY EXAMINATION 2023

MATHEMATICS Paper 2

4052/02 22 Aug 2023 2 hours 15 minutes

Candidates answer on the Question Paper.

READ THESE INSTRUCTIONS FIRST

Write your name, register number and class on all the work you hand in.

Write in dark blue or black pen.

You may use an HB pencil for any diagrams or graphs.

Do not use staples, paper clips, glue or correction fluid.

Answer all the questions.

If working is needed for any question it must be shown with the answer.

Omission of essential working will result in loss of marks.

The use of an approved scientific calculator is expected, where appropriate.

If the degree of accuracy is not specified in the question, and if the answer is not exact, give the answer to three significant figures. Give answers in degrees to one decimal place.

For π , use either your calculator value or 3.142, unless the question requires the answer in terms of π .

The number of marks is given in brackets [] at the end of each question or part question. The total of the marks for this paper is 90.

For Examiner's Use

Question	1	2	3	4	5	6	7	8	9
Marks									

Table of Penalti	es	Question No.		
Presentation	-1			
Accuracy/ Units	-1		Parent's / Guardian's Signature	90

This document consists of **22** printed pages

Mathematical Formulae

Compound Interest

Total amount =
$$P\left(1 + \frac{r}{100}\right)^n$$

Mensuration

Curved surface area of a cone = πrl

Surface area of a sphere = $4\pi r^2$

Volume of a cone =
$$\frac{1}{3}\pi r^2 h$$

Volume of a sphere =
$$\frac{4}{3}\pi r^3$$

Area of triangle
$$ABC = \frac{1}{2}ab\sin C$$

Arc length = $r\theta$, where θ is in radians

Sector area =
$$\frac{1}{2}r^2\theta$$
, where θ is in radians

Trigonometry

$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$
$$a^2 = b^2 + c^2 - 2bc\cos A$$

Statistics

$$Mean = \frac{\sum fx}{\sum f}$$

Standard deviation =
$$\sqrt{\frac{\sum fx^2}{\sum f} - \left(\frac{\sum fx}{\sum f}\right)^2}$$

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1 (a) (i) Factorise $4x^2 - 20xy + 25y^2$.

Answer [1]

(ii) Hence, factorise $4x^2 - 20xy + 25y^2 - 12x + 30y$ completely.

(b)
$$\frac{x+ky}{2x-y} = -\frac{1}{4}$$

(i) Evaluate k when $x = -2$ and $y = 1$.

Answer k = [1]

(ii) Express y in terms of x and k.

Answer $y = \dots$ [3]

(c) Solve the equation
$$\frac{4}{x-3} - \frac{1}{6+x-x^2} = 0$$

- 2 A ship sails from Sunny Island to Windy Island and back to Sunny Island on a particular day. The distance between Sunny Island and Windy Island is 60 km and there is a constant current of speed *x* km/h flowing from Sunny Island to Windy Island. It is given that the speed of the ship in still water is 80 km/h and it takes the ship 30 minutes longer to travel from Windy Island back to Sunny Island.
 - (a) Show that $x^2 + 240x 6400 = 0$.

[3]

(b) Find the speed of the current, giving your answers correct to 3 decimal places.

Answer speed = $\dots km/h$ [3]

(c) Find the time taken by the boat to complete a journey of 300 km against the current. Leave your answer in hours and minutes, to the nearest minute.

Answer hours minutes [2]

(d) State one assumption that you made to solve this problem.

3 *TA* is a tangent to the circle *ABCD* at the point *A*. It is given that angle $BAD = 88^{\circ}$, angle $ABD = 46^{\circ}$ and BC = CD.



(a) Prove that AC is the diameter of the circle.

[3]

- (b) Find
 - (i) angle DAT,

Answer° [2]

(ii) angle DEC,

Answer° [2]

(iii) angle ATD.

Answer° [2]

(c) Name the quadrilateral *ABCD*.

4 The following is the table of values for $y = \frac{10}{x^2} + 2x - 5$.

x	1	1.5	2	2.5	3	4	5	6
у	7	2.4	1.5	1.6	2.1	3.6	р	7.3

(a) Calculate the value of *p*, correct to one decimal place.

Answer
$$p = \dots$$
 [1]

(b) On the grid, draw the graph of
$$y = \frac{10}{x^2} + 2x - 5$$
 for $1 \le x \le 6$.



(c) By drawing a tangent, find the gradient at the point where x = 1.5.

(d) Draw the graph of y = 7 - 0.5x on the grid in part (b) for $0 \le x \le 6$.

[1]

(e) (i) Write down the x coordinates of the points at which the two graphs intersect.

Answer x = or [1]

(ii) Given that a, b, c and d are integers, find the equation, in the form $ax^3 + bx^2 + cx + d = 0$, which is satisfied by the values of x found in (e)(i).

5 A container is formed by joining a cone of base radius 4 cm and a hemisphere of radius 4 cm as shown in Diagram 1. It is given that the volume of the cone is $\frac{3}{5}$ the volume of the hemisphere.



(a) Show that the height of the cone is 4.8 cm.

The container is partially filled with water as shown in Diagram 2. The surface of the water is 3 cm below the vertex of the cone.

(b) Calculate the volume of water in the container.

(c) This container is geometrically similar to a larger container. It is given that the height of the cone in the larger container is 6 cm. Without finding the radius of the cone or hemisphere of the larger container, find the surface area of the larger container.

[4]

6 *A*, *B* and *C* are three points on a horizontal ground. It is given that *AC* is 40 m and angle $ABC = 145^{\circ}$. *BD* is a vertical structure of height *h* m. The angle of elevation of *D* from *A* and *C* are 18° and 13° respectively.



(a) Express AB and BC in terms of h. Hence, show that h = 5.6527 m.

(b) E is a point on AC. Find the largest angle of elevation of D from E.

Answer° [4]

(c) Victoria claims that it is possible for *DE* to be 8 m. Do you agree? Justify your answer.

7 The heights of 200 students in Sunflower Secondary School were recorded and a cumulative frequency curve was drawn.



(a) Use your graph to estimate(i) the median height,

(ii) the interquartile range,

(iii) the number of students whose height is more than 158 cm.

Answer [1]

- (b) Two students are selected at random from the group of students. Find the probability that
 - (i) both students are shorter than 152 cm,

(ii) one student is shorter than 152 cm and one student is taller than 158 cm.

(c) The mass of the 200 students were also recorded. All students whose mass were above 65 kg underwent the school fitness programme for a period of 3 months. After the programme, their mass were taken again and the box and whisker plots of their mass before and after the fitness programme were drawn.



(i) Jessica claims that the fitness programme is effective in helping the students lose mass. Do you agree? Give two reasons to support your answer.

[2]

(ii) Jonathan claims that every student in the fitness programme lose mass. Do you agree? Justify your answer.



- (a) Express and simplify your answers in terms of **a** and **b**,
 - (i) \overrightarrow{BC} ,



(ii) \overrightarrow{AD} .

[Turn Over

(b) Given that
$$\frac{\text{area of } \Delta ODE}{\text{area of } \Delta ODA} = \frac{1}{4}$$
, express \overrightarrow{OE} in terms of **a** and **b**.

Answer $\overrightarrow{OE} = \dots$ [3]

(c) Calculate the value of $\frac{BE}{EC}$.

(d) Show that the area of $\triangle ODE$ and $\triangle OCE$ are equal.

9 Andy wants to sign up for a credit card from either Bank A, B or C. The following table shows the benefits of the credit cards from the different banks.

Credit card	Bank A	Bank B	Bank C
Cashback on all expenditures	0.5%	1.5%	0.1%
	+3.5% on F&B expenditures,		
Bonus cashback	+4.5% on online expenditures, if total expenditure exceeds \$400 for the month	+2% on all expenditures if total expenditure exceeds \$1000 for the month	+1.9% on all expenditures if total expenditure exceeds \$500 for the month

He estimates that he will spend \$400 on Food and Beverages (F&B), \$200 on online expenditures, \$100 on groceries and \$100 on public transport every month with his credit card.

(a) Calculate the total cashback he would get at the end of 1 year from each bank by charging all the above expenditures with the respective credit cards.

Answer	Bank $A = $ \$	Bank B = \$
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Bank C = \$

Banks A, B and C provide a base interest, and further bonus interests if certain criteria are met, for an initial amount deposited into their current accounts at the start of the year. Further deposits (such as salary credited) will not earn any interest for the year.

Current account	Bank A	Bank B	Bank C	
Base interest on initial deposit (per annum)	0.2%	0.2%	0.5%	
	+0.4% if salary of over \$2000 is credited monthly into account	+0.8% if salary of over \$3000 is credited monthly into account	+0.5% if salary of over \$3000 is credited monthly into account	
Bonus interest on initial deposit (per annum)	+0.4% if paying 3 or more bills monthly through bank account	+2% if paying 5 or more bills monthly through bank account	+1% if paying 3 or more bills monthly through bank account	
	+0.5% if account holder has a credit card from the bank with monthly expenditure over \$400	+1% if account holder has a credit card from the bank with monthly expenditure over \$500	+0.5% if account holder has a credit card from the bank with monthly expenditure over \$500	

Andy has \$20000 in savings at the start of the year. He earns a monthly salary of \$3500 which he plans to credit into the bank account, and he also uses his bank account to make 4 bill payments every month.

(b) What is the combination of bank account and credit card that he should use if he wants to maximise the total amount of cashback and interest earned at the end of 1 year? Justify your answer with clear mathematical calculations.

(c) What is the maximum amount of cashback and interest that Andy can earn at the end of 1 year?

Answer \$ [1]

END OF PAPER

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Qn	Answer
1(ai)	$(2x-5y)^2$
(ii)	(2x-5y)(2x-5y-6)
(bi)	$3\frac{1}{4}$
(ii)	$\frac{-6x}{4k-1}$ or $\frac{6x}{1-4k}$
(c)	$-2\frac{1}{4}$
2(b)	24.222
(c)	5 hours 23 minutes
(d)	The ship is travelling in the same direction as the current when it is travelling from Sunny Island to Windy Island. The ship is travelling in the opposite direction as the current when it is travelling from Windy Island to Sunny Island.
3(bi)	46°
(ii)	90°
(iii)	44°
(c)	Kite / Diamond
4(a)	5.4
(c)	-3.72 (accept -3.5 to -4.5)
(e)	1.05 or 4.6
(f)	$5x^3 - 24x^2 + 20 = 0$
5(b)	195 cm ³
(c)	280
6(b)	42.8°
(c)	Disgaree
7(ai)	161 cm
(11)	9.5 cm
(111)	132
(bi)	$\frac{13}{796}$
	858
(ii)	4075
(ci)	4775 A gree
(ii)	Do not agree
(11)	4
8(ai)	$\frac{1}{3}a-3b$
(11)	b-4a
(b)	$\mathbf{a} + \frac{3}{4}\mathbf{b}$
(c)	3
9(a)	Bank A = 324 , Bank B = 144 , Bank C = 192
(b)	Bank account: Bank A Credit Card: Bank C
(c)	\$724

