Answers to Examples, Quizzes & Mini-test [Measurement]

Quiz
Spot the odd one out:

kilogram	mass	length
time	temperature	

Answer: kilogram is a unit, while the others are physical quantities.

Example 6
Estimate the mass of a single sheet of A4 paper.
A 0.01 g **B** 0.3 g **C** 1 g **D** 30 g

Solution:

1. Estimate the mass of 1 ream of A4 size papers
   - **1 ream 'feels' heavier than a 1 litre bottle of water (1 kg) and lighter than a 5 kg bag of rice**
   - Estimated mass of 1 ream = 3 kg
2. 1 ream of A4 size paper has 500 sheets
3. Mass of 1 A4 size paper = $\frac{3\times1000}{500}$ g
   = 6 g **(10^0 - Order of magnitude)**

Answer: C

MINI-TEST 1

1. What are the SI base units of k so that the following equation is homogeneous?
   velocity = k x density

2. Convert 1.3 g cm^{-3} to SI base units.

3. Which is a reasonable estimate for the volume of a metre rule found in a school laboratory?
   A 1.5 cm^3 **B** 15 cm^3 **C** 150 cm^3 **D** 1500 cm^3

My solution:
1.
Apply rule: units of both sides of the equation must be the same

$$[\text{Velocity}] = [k][\text{density}]$$

$$[k] = \frac{[\text{Velocity}]}{[\text{density}]}$$

$$[k] = \frac{\text{m s}^{-1}}{\text{kg m}^{-3}} = \text{kg}^{-1}\,\text{m}^{4}\,\text{s}^{-1}$$

2.

ANS:

units	SI base units
g	kg
cm	m

$$1.3\,\text{g cm}^{-3} = 1.3 \times \frac{1\,\text{g}}{1\,\text{cm}^3}$$

Since $1\,\text{g} = 10^{-3}\,\text{kg}$

and $1\,\text{cm} = 10^{-2}\,\text{m} \rightarrow 1\,\text{cm}^3 = (10^{-2})^3\,\text{m}^3 = 10^{-6}\,\text{m}^3$

$$1.3\,\text{g cm}^{-3} = 1.3 \times \frac{10^{-3}\,\text{kg}}{10^{-6}\,\text{m}^3} = 1.3 \times 10^3\,\text{kg m}^{-3}$$

$$1.3\,\text{g cm}^{-3} = 1300\,\text{kg m}^{-3}$$

3.

Volume of ruler = length x width x thickness

Length of a metre ruler = 100 cm

Estimate width of a metre ruler = 2.5 cm

Estimate thickness of a metre rule = 0.5 cm

Estimated volume of a metre rule = 100 x 2.5 x 0.5 = 125 cm^3

ANS: C

MINI-TEST 2

1. The acceleration of free fall g was to be determined by measuring the period of oscillation T and the length *l* of a simple pendulum, and using the formula

$$g = \frac{4\pi^2 l}{T^2}$$

In the experiment, the uncertainties in measuring *l* and T were estimated to be 4% and 1% respectively. If the value of g is experimentally found as 9.697 m s^{-2}, determine the percentage uncertainty in g and express the value g with its uncertainty.

2. Errors in measurement may either be systematic or random.
Which of the following involves random error?
   - **A** Not allowing for zero error on a moving-coil voltmeter
   - **B** Using an incorrectly calibrated balance to weigh objects
   - **C** Stopping a stopwatch at the end of a race
   - **D** Using the value of g as 10 N kg^{-1} when calculating weight from mass

3. For the vectors shown in Fig. 3.1 and Fig. 3.2, draw the vector diagram for
   (a) $\vec{\mathbf{A}} + \vec{\mathbf{B}}$
   (b) $\vec{\mathbf{A}} - \vec{\mathbf{B}}$

Fig. 3.1

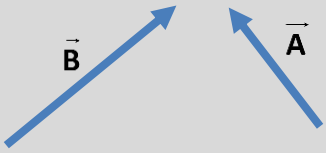


Fig. 3.2

4. Fig. 4.1 shows vectors $\vec{C}$ and $\vec{D}$.
   (a) Use geometry and trigonometry to determine the magnitude and direction of $\vec{E} = \vec{C} + \vec{D}$.
   (b) Use vector resolution to determine the magnitude and direction of $\vec{E} = \vec{C} + \vec{D}$.

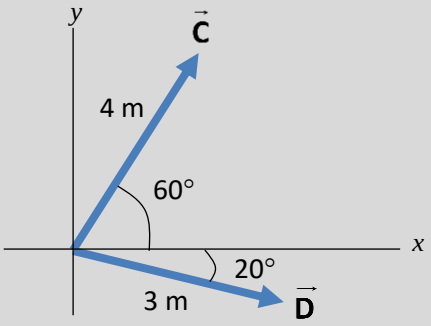


Fig. 4.1

My Solution:

1.

$$g = \frac{4\pi^2 l}{T^2} \quad \Rightarrow \quad \frac{\Delta g}{g} = \frac{\Delta l}{l} + 2\frac{\Delta T}{T}$$

$$\frac{\Delta g}{g} = \frac{\Delta l}{l} + 2\frac{\Delta T}{T} = 0.04 + 2(0.01) = 0.06$$

Percentage uncertainty of g = 0.06 x 100% = 6%

Since g = 9.697 m s^{-2},

Δg = 0.06 x g = 0.06 x 9.697 = 0.5818 = 0.6 ***(1 s.f.)***

g ± Δg = (9.7 ± 0.6) m s^{-2}

Same d.p.

2.

Random errors result in readings being scattered about a mean value.

Option A, B and D involve readings where there is an offset that is in a fixed direction and thus this will result in <u>systematic</u> error.

Due to errors in human judgment, the stopwatch could be stopped before or after the actual race. This will result in <u>random</u> errors

ANS: C

3.

(a)

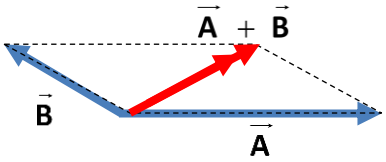


Fig. 3.1

(b)

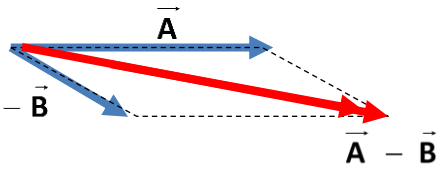


Fig. 3.1

(a)

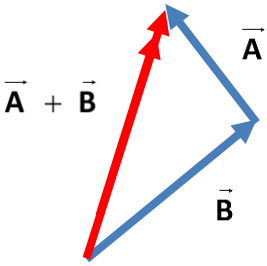


Fig. 3.2

(b)

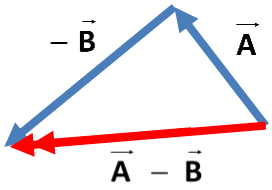
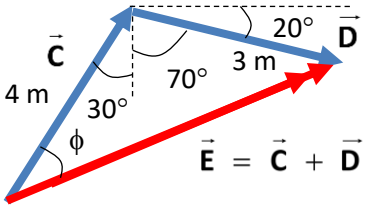


Fig. 3.2

4.

(a) Using geometry and trigonometry



Using cosine rule,

$$\left|\vec{\mathbf{E}}\right|^2 = \left|\vec{\mathbf{C}}\right|^2 + \left|\vec{\mathbf{D}}\right|^2 - 2\left|\vec{\mathbf{C}}\right|\left|\vec{\mathbf{D}}\right|\cos\theta$$

$$\left|\vec{\mathbf{E}}\right|^2 = 4^2 + 3^2 - 2(4)(3)\cos 100° = 29.168$$

$$\left|\vec{\mathbf{E}}\right| = \sqrt{29.168} = 5.40 \text{ m}$$

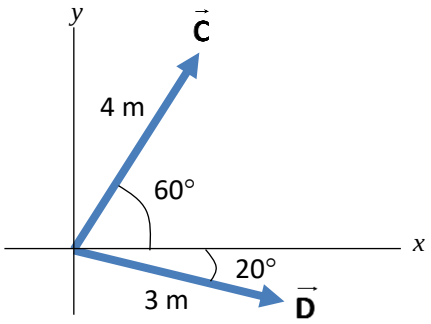
Using sine rule,

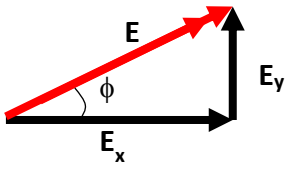
$$\frac{\sin\phi}{3} = \frac{\sin 100°}{5.40}$$

$$\phi = \sin^{-1}\left(\frac{\sin 100°}{5.40} \times 3\right) = \sin^{-1}(0.54712) = 33.2°$$

$\vec{\mathbf{E}}$ has a magnitude of 5.40 m and is in the direction of 26.8° (60° – 33.2°) anti-clockwise above the x-axis.

(b) Using vector resolution





Vector	x -component	y -component
$\vec{C}$	C_x = 4 cos 60° = 2 m	C_y = 4 sin 60° = 3.4641 m
$\vec{D}$	D_x = 3 cos 20° = 2.8191 m	D_y = – 3 sin 20° = – 1.0261 m
$\vec{E}$	E_x = 2 + 2.8191 = 4.8191 m	E_y = 3.4641 – 1.0261 = 2.438 m

$$|\vec{E}| = \sqrt{E_x^2 + E_y^2} = \sqrt{4.8191^2 + 2.438^2} = \sqrt{29.168} = 5.40 \text{ m}$$

$$\phi = \tan^{-1}\left(\frac{2.438}{4.8191}\right) = 26.8°$$

$\vec{E}$ has a magnitude of 5.40 m and is in the direction of 26.8° (60° – 33.2°) anti-clockwise above the x-axis.