National Junior College
2016 – 2017 H2 Further Mathematics
Topic: Further Special Discrete Probability Distributions **Assignment**

Name:____ **Time Spent: __ h ___ min**

1 In a skiing resort, the probability that snow will fall on any day during the winter season is 0.2 on average. The first day of the winter season is 1 December.

(a) State two assumptions required for the number of days from 1 December for snow to first fall to be well-modelled by a geometric distribution. [2]

Assume for the remainder of the question that the assumptions you have stated in part **(a)** hold.

(b) Find, for the winter season,

(i) the probability that the first snow falls on 20 December, [1]

(ii) the probability that the first snow falls before 5 December, [1]

(iii) the probability that the first snow falls after 20 December, if it has not snowed yet and it is now 5 December. [2]

(iii) the earliest date in December such that the probability that the first snow falls on or before that date is at least 0.95. [3]

2 The number of radioactive particles emitted per 150-minute period by some material has a mean of 0.7.

(a) State two assumptions required for the number of radioactive particles emitted by the material in any hour to be well-modelled by a Poisson distribution. [2]

Assume for the remainder of the question that the assumptions you have stated in part **(a)** hold.

(b) Find

(i) the probability that more than 2 particles will be emitted during a randomly chosen 10 hour period. [2]

(ii) the probability that out of three randomly chosen 10 hour periods, more than 2 particles will be emitted in each of two of these 10 hour periods. [2]

(iii) the longest time period, in minutes, for which the probability that no particles are emitted is at least 0.99. [4]

(c) There is a probability of 0.324135 that 4 particles are emitted during the first n hours of a 20 hour period, given that 6 particles are emitted in total in this 20 hour period. Set up a polynomial equation in n, and show that there is exactly one integer solution for n. [6]

3 Show that if f is a geometric distribution function, then the function $g:\mathbb{Z}^+ \mapsto [0,1]$ defined by

$$g(x) = \left(\frac{2-f(1)}{1-f(1)}\right) f(2x)$$

is a probability distribution function. [5]

Find the mean and variance of a random variable with probability distribution function g in terms of p, where $p = f(1)$. [9]