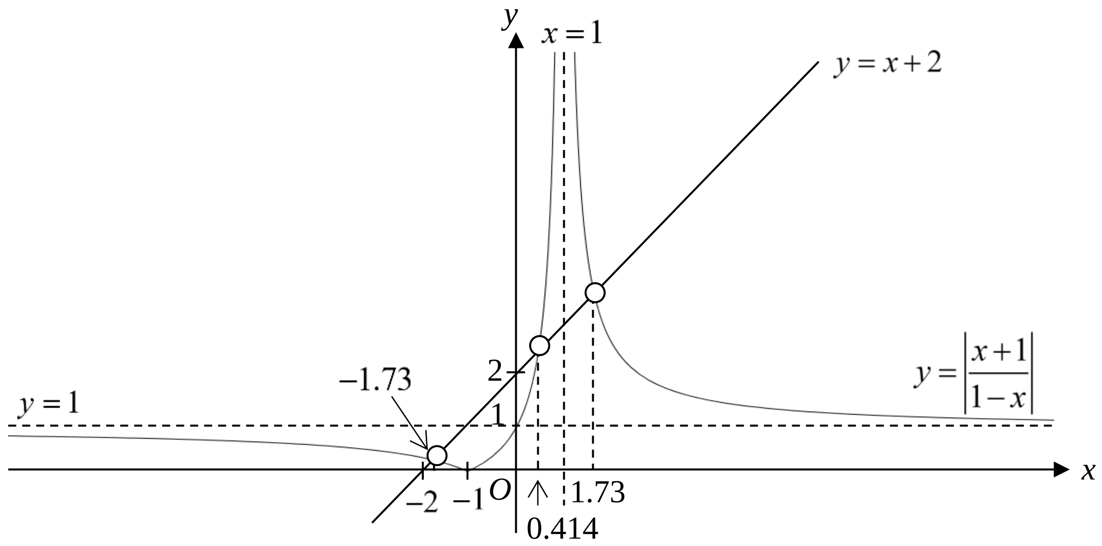
H2 Mathematics (9758)
Chapter 4 Equations and Inequalities
Extra Practice Solutions

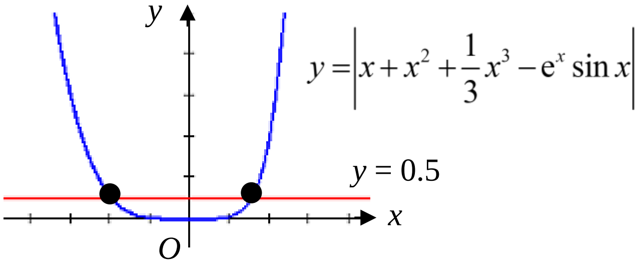
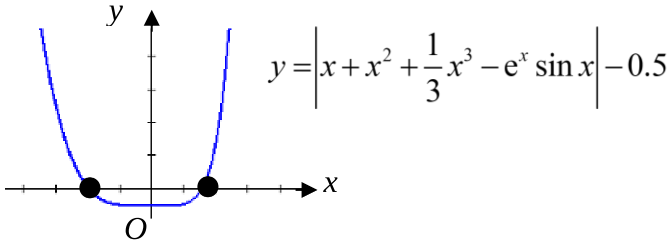
1	**2009/VJC Prelim/I/2**
	Let the price (per kg) for crab, lobster and bamboo clam for his first visit be c, *l*, b. 3.20c + 1.50*l* + 7b = 277.50 5.60c + 1.20 (1.1*l*) + 6.50b = 347 4.50c + 2 (1.1^2 *l*) + 6.50b = 395.18 From GC, c = 36.20, *l* = 79.9983, b = 5.95 Required price: $36.20, $79.9983 x 1.1^2 = $96.80 and $5.95 respectively.

2	**2010/SAJC Prelim/I/1**
	Let $u_n = an^3 + bn^2 + cn + d$ $u_1 = 63$: $a(1)^3 + b(1)^2 + c(1) + d = 63$ $a + b + c + d = 63$ $u_2 = 116$: $a(2)^3 + b(2)^2 + c(2) + d = 116$ $8a + 4b + 2c + d = 116$ $u_3 = 171$: $a(3)^3 + b(3)^2 + c(3) + d = 171$ $27a + 9b + 3c + d = 171$ $u_4 = 234$: $a(4)^3 + b(4)^2 + c(4) + d = 234$ $64a + 16b + 4c + d = 234$ Using GC, $a = 1,\ b = -5,\ c = 61,\ d = 6$. $\therefore u_n = n^3 - 5n^2 + 61n + 6$ Hence $u_{50} = (50)^3 - 5(50)^2 + 61(50) + 6 = 115556$

Qn 3	**2010/NJC Prelim/I/1**
	Let the unit digit be *z*. Let the tenth digit be *y*. Let the hundredth digit be *x*. $x + y + z = 15$ $(100x + 10y + z) - (100z + 10y + x) = 594 \Rightarrow 99x - 99z = 594$ $y + 4z = x + 5 \Rightarrow -x + y + 4z = 5$ Using GC , $x = 8, y = 5, z = 2$. Thus the number is 852

4	**2018/RI Promo/1**
	$\mathrm{f}(x)=x^3+ax^2+bx+c$ $\mathrm{f}(-1)=24 \Rightarrow a-b+c=25$ $\mathrm{f}(-2)=36 \Rightarrow 4a-2b+c=44$ $\mathrm{f}(1)=0 \Rightarrow a+b+c=-1$ By GC, $a=2, b=-13, c=10$ $\therefore \mathrm{f}(x)=x^3+2x^2-13x+10$

5	**2015/I/2**
(i)	$y=\left\lvert\frac{x+1}{1-x}\right\rvert=\left\lvert\frac{-(1-x)+2}{1-x}\right\rvert=\left\lvert-1+\frac{2}{1-x}\right\rvert$ 
(ii)	From the graph, Using GC, $-1.73 < x < 0.414$ or $x > 1.73$.

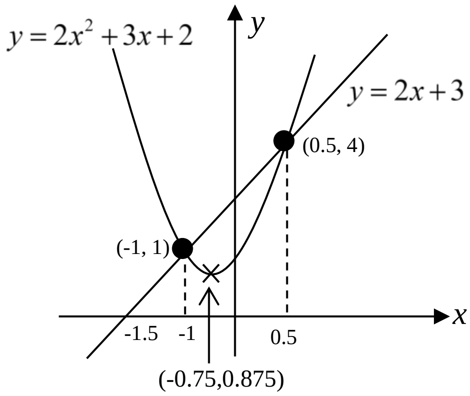
6	**2008/II/1 Modified**
	$\lvert \mathrm{g}(x)-\mathrm{f}(x) \rvert \leq 0.5$
	$\left\lvert x+x^2+\frac{1}{3}x^3-\mathrm{e}^x\sin x \right\rvert \leq 0.5$
	
	Alternative graph:
	
	From the graph, the solution set is $\{x\in\mathbb{R}: -1.96 \leq x \leq 1.56\}$.

7	2007/H1 A level/I/5(part)

Graphical Method 1:
Using intersection of 2 graphs

$2x^2+3x+2 \geq 2x+3$

$x \leq -1 \quad \text{or} \quad x \geq \frac{1}{2}$



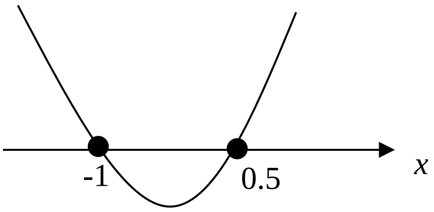
Graphical Method 2:
Draw 1 graph

$2x^2+3x+2 \geq 2x+3$

$2x^2+x-1 \geq 0$

$(x+1)(2x-1) \geq 0$

$x \leq -1 \quad \text{or} \quad x \geq \frac{1}{2}$



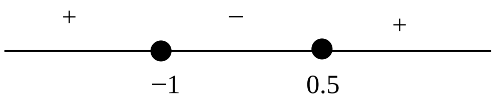
Algebraic Method:

$2x^2+3x+2 \geq 2x+3$

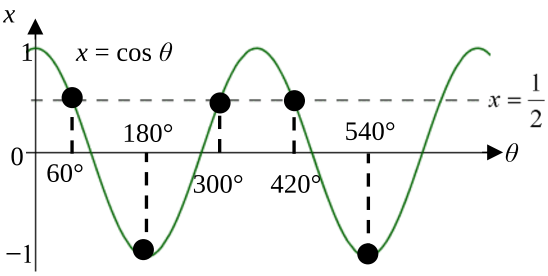
$2x^2+x-1 \geq 0$

$(x+1)(2x-1) \geq 0$

$\therefore x \leq -1 \quad \text{or} \quad x \geq \frac{1}{2}$



Let $x=\cos\theta$, $\Rightarrow \cos\theta \leq -1 \quad \text{or} \quad \cos\theta \geq \frac{1}{2}$

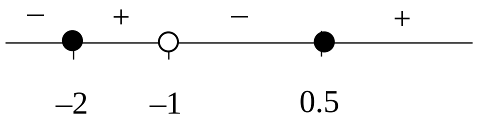


For $\cos\theta \leq -1$, $\theta = 180°, 540°$

For $\cos\theta \geq \frac{1}{2}$, $0° \leq \theta \leq 60° \quad \text{or} \quad 300° \leq \theta \leq 420°$

$\therefore \theta = 180° \;\text{ or }\; \theta = 540° \;\text{ or }\; 0° \leq \theta \leq 60° \quad \text{or} \quad 300° \leq \theta \leq 420°.$

8	2018/SAJC Promo/Q2
(i)	$x^2-2x+5=x^2-2x+1+4$ $=(x-1)^2+4$ Since $(x-1)^2\ge 0$ for all $x\in\mathbb{R}$, then $(x-1)^2+4>0$ for all $x\in\mathbb{R}$. Hence $x^2-2x+5>0$ for all $x\in\mathbb{R}$
(ii)	$\frac{4x^2-x+1}{x^2-2x+5}\le 1$ $\frac{4x^2-x+1}{x^2-2x+5}-1\le 0$ $\frac{4x^2-x+1}{x^2-2x+5}-\frac{x^2-2x+5}{x^2-2x+5}\le 0$ $\frac{3x^2+x-4}{x^2-2x+5}\le 0$ Since $x^2-2x+5>0$ for all $x\in\mathbb{R}$, $3x^2+x-4\le 0$ $(3x+4)(x-1)\le 0$ $-\frac{4}{3}\le x\le 1$ **<u>OR</u>** Since $x^2-2x+5>0$ for all $x\in\mathbb{R}$, $\frac{4x^2-x+1}{x^2-2x+5}\le 1\Rightarrow 4x^2-x+1\le x^2-2x+5$ $3x^2+x-4\le 0$ $(3x+4)(x-1)\le 0$ $-\frac{4}{3}\le x\le 1$
(iii)	As $x^2=\lvert x\rvert^2$, replacing x with $\lvert x\rvert$, $\frac{4x^2-\lvert x\rvert+1}{x^2-2\lvert x\rvert+5}\le 1$ has solution $-\frac{4}{3}\le\lvert x\rvert\le 1$ Since $\lvert x\rvert\ge 0$ for all $x\in\mathbb{R}$, then $0\le\lvert x\rvert\le 1$ $\lvert x\rvert\le 1$ $\therefore -1\le x\le 1$

9	**2018/YJC Promo/Q3**
	$\dfrac{2-x}{x+1} \geq 2x$ $\dfrac{2-x-2x(x+1)}{x+1} \geq 0$ $\dfrac{-2x^2-3x+2}{x+1} \geq 0$ $\dfrac{(2x-1)(x+2)}{x+1} \leq 0$  $x \leq -2$ or $-1 < x \leq \dfrac{1}{2}$
	$\dfrac{2+\ln x}{1-\ln x} \geq -2\ln x$ Replace x with $-\ln x$, $-\ln x \leq -2$ or $-1 < -\ln x \leq \dfrac{1}{2}$ $\ln x \geq 2$ or $-\dfrac{1}{2} \leq \ln x < 1$ (Note the change in equality when you /divide by -1) $x \geq e^2$ or $e^{-\frac{1}{2}} \leq x < e$

10	**2010/VJC Prelim/I/4**
(i)	Given $x \geq -\dfrac{1}{2}$ and $y = \sqrt{2x+1}$, $22-8\sqrt{2x+1}+2x = 22-8y+2\left(\dfrac{y^2-1}{2}\right)$ $= y^2-8y+21$ $=(y-4)^2+5>0$ Hence $22-8\sqrt{2x+1}+2x$ is always positive.
(ii)	Since $22-8\sqrt{2x+1}+2x$ is always positive and $(x-m)^2 \geq 0,\ x \neq m$, $\dfrac{22x-8x\sqrt{2x+1}+2x^2}{(x-k)(x-m)^2} \geq 0$ $\Rightarrow \dfrac{x\left(22-8\sqrt{2x+1}+2x\right)}{(x-k)(x-m)^2} \geq 0$ $\Rightarrow \dfrac{x}{x-k} \geq 0,\ x \neq k,\ x \neq m \quad \because (x-m)^2 > 0$ $\Rightarrow -\dfrac{1}{2} \leq x \leq 0$ or $x > k,\ x \neq m$