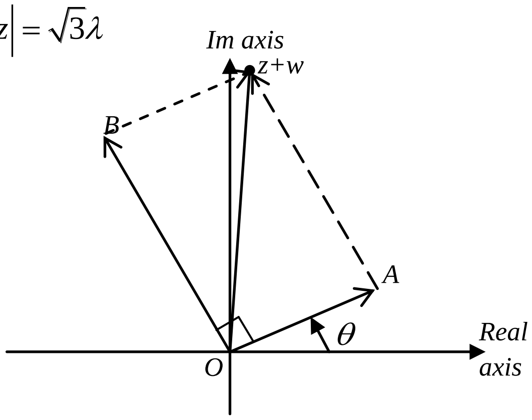
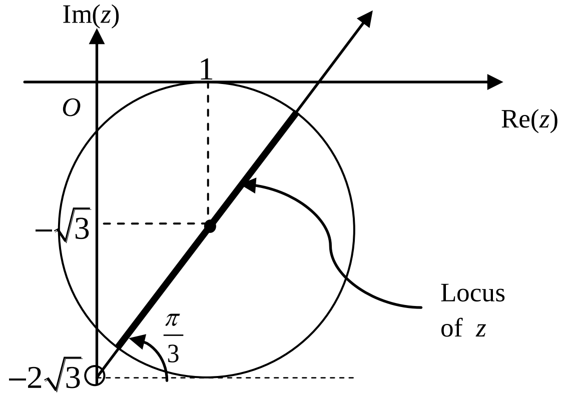
Qn	Suggested Solutions
1(a) (i)	$\lvert z+w\rvert = \lvert z+(\mathrm{i}\sqrt{3})z\rvert = \lvert(1+\mathrm{i}\sqrt{3})z\rvert = \lvert 1+\mathrm{i}\sqrt{3}\rvert\lvert z\rvert = 2\lambda$ $\arg(z+w) = \arg\left((1+\mathrm{i}\sqrt{3})z\right) = \arg(1+\mathrm{i}\sqrt{3}) + \arg(z) = \frac{\pi}{3}+\theta$ Alternatively, Given that $z=\lambda e^{\mathrm{i}\theta}$,where $\lambda>0$ and $0<\theta<\frac{\pi}{2}$, and $w=\mathrm{i}\sqrt{3}z$, find $\lvert z\rvert = \lvert\lambda e^{\mathrm{i}\theta}\rvert = \lambda$ $\lvert w\rvert = \lvert\mathrm{i}\sqrt{3}z\rvert = \sqrt{3}\lambda$  By vector addition, $z+w=\overrightarrow{OA}+\overrightarrow{OB}$ By Pythagoras Theorem, $\lvert z+w\rvert = \sqrt{\lambda^2+\left(\sqrt{3}\lambda\right)^2}$ $=\sqrt{4\lambda^2}$ $=2\lambda$
1(a) (ii)	$\arg(z+w) = \arg(z)+\tan^{-1}\left(\frac{\sqrt{3}\lambda}{\lambda}\right)$ $=\theta+\frac{\pi}{3}$

1(b) (i)



$\left|z-1+\mathrm{i}\sqrt{3}\right|=\sqrt{3}$ is a circle with centre $\left(1,\sqrt{3}\right)$and radius $\sqrt{3}$ units.

$\arg\left(2z+\mathrm{i}4\sqrt{3}\right)=\frac{\pi}{3}$

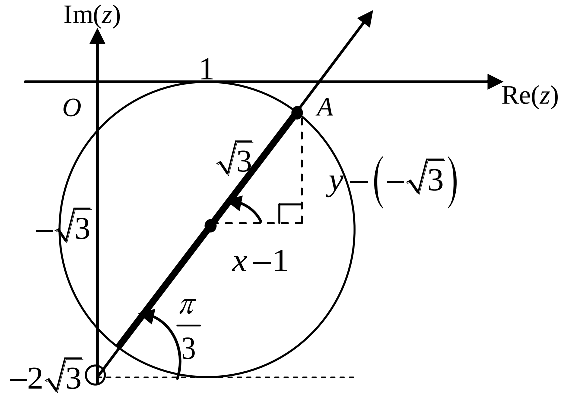
$\Rightarrow \arg\left(2\right)+\arg\left(z+\mathrm{i}2\sqrt{3}\right)=\frac{\pi}{3}$

$\Rightarrow \arg\left(z+\mathrm{i}2\sqrt{3}\right)=\frac{\pi}{3}$

$\arg\left(z+\mathrm{i}2\sqrt{3}\right)=\frac{\pi}{3}$ is a half-line with starting point at $\left(0,-2\sqrt{3}\right)$ that makes angle of $\frac{\pi}{3}$with the positive real axis.

Note that
1. the half line passes through the centre of circle.
2. The real axis is a tangent to the circle.

1(b) (ii)



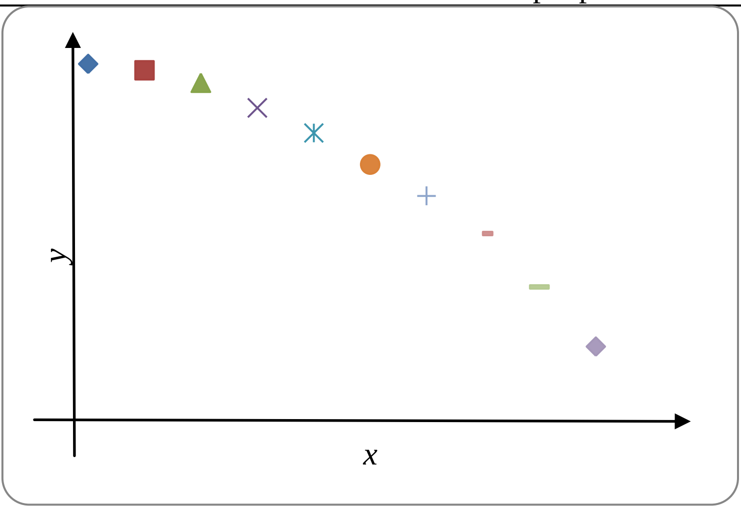
Let A denote the value of z that gives the greatest possible value of arg(z).
$A=x+\mathrm{i}y$

	$\sin\frac{\pi}{3}=\frac{y+\sqrt{3}}{\sqrt{3}} \Rightarrow \frac{\sqrt{3}}{2}=\frac{y+\sqrt{3}}{\sqrt{3}}$ $\therefore y=\frac{3}{2}-\sqrt{3}$ $\cos\frac{\pi}{3}=\frac{x-1}{\sqrt{3}} \Rightarrow \frac{1}{2}=\frac{x-1}{\sqrt{3}}$ $\therefore x=\frac{\sqrt{3}}{2}+1$ $\therefore z=\frac{\sqrt{3}}{2}+1+\mathrm{i}\left(\frac{3}{2}-\sqrt{3}\right)$
2(a)	x = distance of the car from the intersection y = distance between the car and truck By Cosine Rule, $y^2 = x^2 + 48^2 - 2x(48)\cos 120^\circ$ $y^2 = x^2 + 48x + 2304$ (shown)
	Differentiating implicitly with respect to time, $2y\frac{\mathrm{d}y}{\mathrm{d}t} = 2x\frac{\mathrm{d}x}{\mathrm{d}t} + 48\frac{\mathrm{d}x}{\mathrm{d}t}$ The rate of change of the car is $\frac{\mathrm{d}x}{\mathrm{d}t} = -60$ because it is traveling toward the intersection. When $x = 15$ $y^2 = x^2 + 2304 + 48x$ $y^2 = 15^2 + 2304 + 48(15) = 3249 \Rightarrow y = 57$ $2(57)\frac{\mathrm{d}y}{\mathrm{d}t} = (2(15) + 48)\frac{\mathrm{d}x}{\mathrm{d}t}$ $\frac{\mathrm{d}y}{\mathrm{d}t} = -41\frac{1}{19}$ $= -41.1$. (3sf) Hence, the distance between the car and the truck is decreasing at a rate of 41.1 km/h.
2(b) (1st part)	Cost for roof = \$$P$ per unit area; Cost for curved surface area and base = \$$3P$ per unit area. Total cost = \$$C$. Area $= 2\pi r^2+\pi r^2+2\pi rh$ (where h is the height of the cylinder) $\Rightarrow \quad C=7\pi r^2(P)+2\pi rh(3P)$ $\Rightarrow \quad \mathrm{C}=\pi rP(7r+6h)$

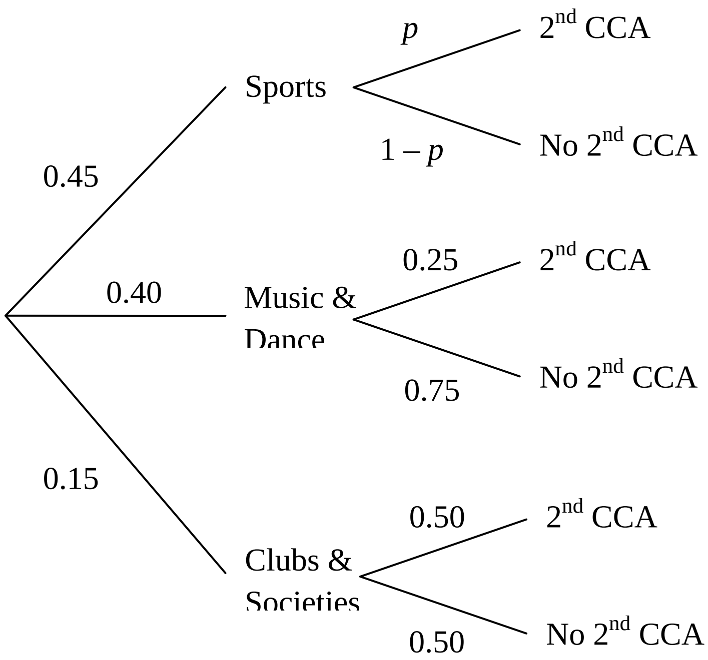
	$\Rightarrow \quad \dfrac{C}{6\pi rP} = 7r + 6h$ $\Rightarrow \quad h = \dfrac{C}{6\pi rP} - \dfrac{7r}{6}$
	$V = \dfrac{2}{3}\pi r^3 + \pi r^2 h$ $= \dfrac{2}{3}\pi r^3 + \pi r^2\left(\dfrac{C}{6\pi rP} - \dfrac{7r}{6}\right)$ $= \dfrac{2}{3}\pi r^3 + \dfrac{Cr}{6P} - \dfrac{7\pi r^3}{6}$ $= \dfrac{Cr}{6P} - \dfrac{\pi r^3}{2}$
2(b) (2nd part)	$\dfrac{\mathrm{d}V}{\mathrm{d}r} = \dfrac{C}{6P} - \dfrac{3}{2}\pi r^2 = 0$ $\Rightarrow r = \sqrt{\dfrac{C}{9P\pi}}$ $\dfrac{\mathrm{d}^2V}{\mathrm{d}r^2} = -3\pi r < 0$ when $r = \sqrt{\dfrac{C}{9P\pi}}$, the volume is maximum. Thus, cost of top $= \pi r^2 P = \pi\left(\dfrac{C}{9\pi P}\right)P$ $= \dfrac{C}{9}$
3(a)	$\displaystyle\int \frac{x+3}{x^2+4x+9}\,\mathrm{d}x$ $\displaystyle= \frac{1}{2}\int \frac{(2x+4)}{x^2+4x+9}\,\mathrm{d}x + \int \frac{1}{(x+2)^2+5}\,\mathrm{d}x$ $= \dfrac{1}{2}\ln\left\|x^2+4x+9\right\| + \dfrac{1}{\sqrt{5}}\tan^{-1}\dfrac{x+2}{\sqrt{5}} + c$ or $\dfrac{1}{2}\ln\left(x^2+4x+9\right) + \dfrac{1}{\sqrt{5}}\tan^{-1}\dfrac{x+2}{\sqrt{5}} + c$

3(b)	$\frac{\mathrm{d}^2y}{\mathrm{d}x^2}=\frac{2}{1+3x}$ $\frac{\mathrm{d}y}{\mathrm{d}x}=\int\frac{2}{1+3x}\ \mathrm{d}x=\frac{2}{3}\ln(1+3x)+c$ Since the curve has gradient value of 2 at the point(0,1), then we have $2=\frac{2}{3}\ln 1+c\Rightarrow c=2$ $\frac{\mathrm{d}y}{\mathrm{d}x}=\frac{2}{3}\ln(1+3x)+2$ $y=\int\frac{2}{3}\ln(1+3x)+2\ \mathrm{d}x$ $=\frac{2}{3}\left[x\ln(1+3x)-\int\frac{3x}{1+3x}\ \mathrm{d}x\right]+2x+D$ $=\frac{2}{3}\left[x\ln(1+3x)-\int 1-\frac{1}{1+3x}\ \mathrm{d}x\right]+2x+D$ $=\frac{2}{3}\left[x\ln(1+3x)-\left(x-\frac{1}{3}\ln(1+3x)\right)\right]+2x+D$ $=\frac{2}{3}\left[x\ln(1+3x)-x+\frac{1}{3}\ln(1+3x)\right]+2x+D$ Since curve passes through the point $(0,1)$, we have $1=\frac{2}{3}\left[0-0+\frac{1}{3}\ln(1)\right]+0+D\Rightarrow D=1$ $\therefore y=\frac{2}{3}\left[x\ln(1+3x)-x+\frac{1}{3}\ln(1+3x)\right]+2x+1$ $=\frac{2}{3}\left(x+\frac{1}{3}\right)\ln(1+3x)+\frac{4}{3}x+1$
4(i)	$y=\sqrt{4-x^2}$ is the equation of a semicircle centred at (0,0) with radius 2. Hence $\int_{-2}^{2}\sqrt{4-x^2}\mathrm{d}x=$ Area of semicircle $=\frac{1}{2}\pi(2)^2=2\pi$
4(ii)	Solving $x^2+(y-2)^2=4$ and $y=\left\|\frac{x}{2}\right\|+1$, we have Consider

	$x^2+\left(\left\lvert\frac{x}{2}\right\rvert-1\right)^2=4$ $\Rightarrow x^2+\frac{x^2}{4}-2\left\lvert\frac{x}{2}\right\rvert+1=4$ $\Rightarrow \frac{5}{4}x^2+\lvert x\rvert-3=0$ $\Rightarrow 5x^2+4\lvert x\rvert-12=0$ For $x<0$, $\Rightarrow 5x^2+4x-12=0$ $\Rightarrow x=-2$ or $\frac{6}{5}$ (rejected) Alternative by symmetrical properties, the other intersection is at $x=-2$. For $x>0$ $5x^2-4x-12=0$ $\Rightarrow x=2$ or $-\frac{6}{5}$(rejected)
4(ii) 1st part	$\text{Volume}=\pi\int_{-2}^{2}\left(\left\lvert\frac{x}{2}\right\rvert+1\right)^2-\left(2-\sqrt{4-x^2}\right)^2\ \mathrm{d}x$ $=\pi\int_{-2}^{2}\frac{x^2}{4}+2\left\lvert\frac{x}{2}\right\rvert+1-\left(4-4\sqrt{4-x^2}+4-x^2\right)\ \mathrm{d}x$ $=\pi\int_{-2}^{2}\frac{5}{4}x^2+\lvert x\rvert-7+4\sqrt{4-x^2}\ \mathrm{d}x$ (shown)
4(ii) 2nd part	$\text{Volume}=\pi\int_{-2}^{2}\frac{5}{4}x^2+\lvert x\rvert-7+4\sqrt{4-x^2}\ \mathrm{d}x$ $=\pi\left[\int_{-2}^{2}\frac{5}{4}x^2-7\ \mathrm{d}x+\int_{-2}^{2}\lvert x\rvert\ \mathrm{d}x+\int_{-2}^{2}4\sqrt{4-x^2}\ \mathrm{d}x\right]$ $=\pi\left[\frac{5}{12}x^3-7x\right]_{-2}^{2}-\pi\int_{-2}^{0}x\ \mathrm{d}x+\pi\int_{0}^{2}x\ \mathrm{d}x+4\pi(2\pi)$ $=-\frac{64}{3}\pi-\pi\left[\frac{x^2}{2}\right]_{-2}^{0}+\pi\left[\frac{x^2}{2}\right]_{0}^{2}+8\pi^2$ $=-\frac{64}{3}\pi+4\pi+8\pi^2$ $=8\pi^2-\frac{52}{3}\pi\ \text{unit}^3$

5(i)	Let n and N denote the sample size and population size respectively. $\frac{n}{N}\times 100\% = 2\%$ $\Rightarrow \frac{n}{N} = \frac{2}{100}$ $\Rightarrow k = \frac{N}{n} = \frac{100}{2} = 50$ Randomly select a starting point in the first 50 concert goers in the queue and then pick every 50th concert goer thereafter.
5(ii)	There may not be sufficient information to categorize the audience members in relevant strata/find out the numbers of people in each strata.
6(i)	 r-value $= -0.9728201266 = -0.973$ (3 sig. fig.)
6(ii)	(A) $y = ax + b$: r-value $= -0.973$ (B) $y = cx^2 + d$: r-value $= -0.999$ (C) $y = e\sqrt{x} + f$: r-value $= -0.930$ As x increases, y decreases at an increasing rate and (B) has r-value closest to -1, (B) is the best model.
6(iii)	Regression line of y on x^2 is $y = -0.358974359x^2 + 13.22512821$ $y = -0.359x^2 + 13.2$ When $x = 3.8$,

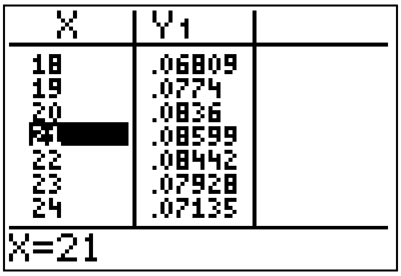
	$y = -0.358974359(3.8)^2 + 13.22512821$ $= 8.041538466$ $= 8.04$ (3 sig. fig.) Since $x = 3.8$ is within the data range given, and r-value indicates a strong negative linear correlation between y and x^2, the estimate is reliable.
7(a) (i)	Number of ways $= \dfrac{\overset{\text{8 letters}}{8!}}{\underset{\text{2 N's}}{2!}\ \underset{\text{2 A's}}{2!}} = 10080$
7(a) (ii)	Number of ways $=$ Number of ways without restrictions $-$number of ways that each pair is together $= \dfrac{8!}{2!2!} - 6!$ $= 9360$
7(a) (iii)	Number of ways $= \dfrac{4!\binom{5}{4}4!}{2!2!}$ $= 720$
7(b) (i)	Number of ways $= \binom{4}{3}\binom{3}{2}\binom{4}{2}\binom{4}{2}$ $= 432$
7(b) (ii)	Case 1: 3 trumpet players, 3 saxophone players Number of ways $= \binom{4}{3}\binom{4}{3}\binom{7}{3} = 560$ Case 2: 3 trumpet players, 4 saxophone players Number of ways $= \binom{4}{3}\binom{4}{4}\binom{7}{2} = 84$ Case 3: 4 trumpet players, 3 saxophone players Number of ways $= \binom{4}{4}\binom{4}{3}\binom{7}{2} = 84$

	Case 4: 4 trumpet players, 4 saxophone players Number of ways $= \binom{4}{4}\binom{4}{4}\binom{7}{1} = 7$ Total number of ways = 560 + 84 + 84 + 7 = 735
8	
8(i)	P(has second CCA | first CCA is Music and Dance) = 0.25
8(ii)	P(has second CCA) $= \left(\frac{45}{100}\right)p + \frac{40}{100}\left(\frac{1}{4}\right) + \frac{15}{100}\left(\frac{1}{2}\right)$ $= \frac{9p}{20} + \frac{1}{10} + \frac{3}{40} = \frac{9p}{20} + \frac{7}{40}$ $\frac{9p}{20} + \frac{7}{40} = \frac{13}{40}$ $p = \frac{1}{3}$
8(iii)	P(first CCA Clubs and Societies | no second CCA) $= \frac{\mathrm{P}\left(\text{first CCA Clubs and Societies} \cap \text{no second CCA}\right)}{\mathrm{P}\left(\text{no second CCA}\right)}$ $= \frac{\frac{15}{100}\left(\frac{1}{2}\right)}{1-\frac{13}{40}} = \frac{\frac{3}{40}}{\frac{27}{40}} = \frac{1}{9}$

8(iv)	$\mathrm{P}(A\cup B)=\mathrm{P}(A)+\mathrm{P}(B)-\mathrm{P}(A\cap B)$ $=2\left(\frac{40}{100}\right)\left(\frac{15}{100}\right)+\left(\frac{13}{40}\right)^2-2\left[\left(\frac{40}{100}\right)\left(\frac{1}{4}\right)\left(\frac{15}{100}\right)\left(\frac{1}{2}\right)\right]$ $=\frac{3}{25}+\frac{169}{1600}-\frac{3}{200}$ $=\frac{337}{1600}$
9 (i)	Let μ be the mean distance Kelly can throw her javelin. H_0: $\mu = 55$ H_1: $\mu > 55$ Level of significance: 5% (upper-tailed) Under H_0, $\bar{X} \sim N\left(55\ ,\ \frac{\sigma^2}{10}\right)$, where σ is the population standard deviation. Test statistic: $T=\frac{\bar{X}-55}{\left(\frac{s}{\sqrt{10}}\right)}\sim t(9)$ $\bar{x}=\frac{\sum x}{n}=\frac{569.5}{10}=56.95$ $s^2=\frac{1}{n-1}\left[\sum x^2-\frac{\left(\sum x\right)^2}{n}\right]$ $=\frac{1}{9}\left[32560.75-\frac{569.5^2}{10}\right]$ $=\frac{127.725}{9}=\frac{1703}{120}$ p-value = 0.0680407 > 0.05, hence we do not reject H_0. t-calculated = 1.636884, t_{critical} = 1.83311 There is insufficient evidence at 5% level of significance level to claim that the new training method has improved Kelly's performance.
	The probability of rejecting the claim that the new method has not improved Kelly's performance when it is actually true is 0.05. OR

	The probability of rejecting the claim that the Kelly's mean distance is equal to 55 metres wrongly is 0.05.
9(ii)	Given n = 60 and $\sigma^2 = 6^2$. Under H_0, $\bar{X} \sim \mathrm{N}\left(55\ ,\ \frac{36}{60}\right)$ Test statistic: $Z = \frac{\bar{X}-55}{\left(\frac{6}{\sqrt{60}}\right)} \sim \mathrm{N}(0,1)$ $z_{calculated} = \frac{\bar{x}-55}{\left(\frac{6}{\sqrt{60}}\right)}$ At 5% significance level, $z_{critical} = 1.644853626$ To reject H_0, we need $z_{cal} \geq z_{crit}$ $\frac{\bar{x}-55}{\left(\frac{6}{\sqrt{60}}\right)} \geq 1.644853626$ $\bar{x} \geq 56.27409814$ $\bar{x} \geq 56.3$ (3 sig. fig.)
10 (i)	Let X be the length of time a customer leaves a car in the vehicle park of the shopping centre on a weekday. $X \sim \mathrm{N}(95, 21^2)$ Required probability $= \mathrm{P}(X > 120)^2 \cdot \mathrm{P}(80 < X < 112) \cdot \frac{3!}{2!}$ $= (0.11692970104)^2 \cdot (0.5533679186) \cdot 3$ $= 0.02269786$ $= 0.0227$ (3 sig. fig.)
10(ii)	Let Y be the length of time a customer leaves a vehicle in the car park of the shopping centre on a weekend. $Y \sim \mathrm{N}(\mu, \sigma^2)$ Since $\mathrm{P}(Y \leq 120) = 0.5$, $\mu = 120$ minutes

	$\mathrm{P}(Y > 148) = 0.191$ $\mathrm{P}(Y < 148) = 0.809$ $\mathrm{P}\left(Z < \frac{148-120}{\sigma}\right) = 0.809$ $\frac{28}{\sigma} = 0.8742171608$ $\sigma = 32.02865518$ $\sigma = 32$ (to nearest minute)
10 (iii)	Let T be the amount of car park charges paid by Mr Teng in the particular week. $T = 0.04(X_1 + X_2) + 0.03Y + 2(2) + 5$ $= 0.04(X_1 + X_2) + 0.03Y + 9$ $T \sim \mathrm{N}(0.04(2)(95) + 0.03(120) + 9,\ 0.04^2(2)(21^2) + 0.03^2(32^2))$ $T \sim \mathrm{N}(20.2, 2.3328)$ $\mathrm{P}(T \le 18) = 0.0748767929$ $= 0.0749$ (3 sig. fig.)
10 (iv)	Let L be the amount of car park charges paid by Miss Low in the particular week. $L = 0.04(X_1 + X_2) + 0.03Y$ $L \sim \mathrm{N}(0.04(2)(95) + 0.03(120),\ 0.04^2(2)(21^2) + 0.03^2(32^2))$ $L \sim \mathrm{N}(11.2, 2.3328)$ $T - L \sim \mathrm{N}(9, 4.6656)$ $\mathrm{P}(T - L \ge 10) = 0.3216954711$ $= 0.322$ (3 sig. fig.)

11(a) (i)	Let X denote the random variable for the number of demands per hour for a court in this sports hall on a weekend. Then $X \sim \text{Po}(7.2)$ P(courts are fully booked on a particular time slot on a Saturday) $= \text{P}(X \geq 6)$ $= 1 - \text{P}(X \leq 5)$ $= 0.7241025$ $\approx 0.724\text{(shown)}$ Note students must indicate higher degree of accuracy before rounding off to 3 decimal places.
11 (a) (ii)	Let Y denote the random variable for the number of hours on an entire weekend for which the courts are fully booked. Then $Y \sim \text{B}(30,\ 0.7241025)$. Since $n = 30$ is large, $np = 21.723 > 5$, $n(1-p) = 8.2769 > 5$, $Y \sim \text{N}(21.723,\ 5.9933)$ approximately. P(the courts are fully booked for at least 20 randomly chosen hours on both Saturday and Sunday of a particular week) $= \text{P}(Y \geq 20)$ $= \text{P}(Y \geq 19.5)$ (by Continuity correction) $= 0.81807$ $= \underline{\underline{0.818}}$ (to 3 significant figures)
11(b)	By plotting $\text{Y}_1 = \text{poissonpdf}(21.6, x)$ in GC and using the "Table" function (as below)  X | Y1 18 | .06809 19 | .0774 20 | .0836 21 | .08599 22 | .08442 23 | .07928 24 | .07135 X=21 From the GC, most probable value is 21.

11(c)	Let $\overline{X}$ denote the average number of demands for a court between 0700 and 0800 per Sunday for 52 randomly chosen Sundays. Since n = 52 is large, by Central Limit Theorem, $$\overline{X} \sim N\left(7.2,\ \frac{7.2}{52}\right)\text{approx}.$$ $P\left(\overline{X} \le 7\right) = 0.2954667$ ≈ 0.295 **Alternative method:** $X_1 + X_2 + \ldots + X_{52} \sim Po(374.4)$ Since $\lambda = 374.4(>10)$, $X_1 + X_2 + \ldots + X_{52} \sim N(374.4,\ 374.4)\text{approx}$ $\mathrm{P}\left(\overline{X} \le 7\right) = \mathrm{P}\left(\frac{X_1 + X_2 + \ldots + X_{52}}{52} \le 7\right)$ $= \mathrm{P}(X_1 + X_2 + \ldots + X_{52} \le 364)$ $= \mathrm{P}(X_1 + X_2 + \ldots + X_{52} \le 364.5)$ $= 0.304450$ ≈ 0.304
11(d) (i)	The probability that one of the courts is booked will be affected by the event that another court is booked. Hence the trials do not occur independently (the probability of a court booked is not consistent) and so a binomial model would probably not be valid.
11(d) (ii)	As people have to work during weekdays, the average number of demands will be fewer on the weekdays. Hence the mean on the weekday is different from that on a weekend.