CATHOLIC JUNIOR COLLEGE

General Certificate of Education Advanced Level

Higher 2

JC2 Preliminary Examination

CANDIDATE NAME			
CLASS		INDEX NUMBER	

MATHEMATICS

9758/02

Paper 2

13 Sep 2023

3 hours

Candidates answer on the Question Paper.

Additional Materials: List of Formulae (MF26)

READ THESE INSTRUCTIONS FIRST

Write your class, index number and name on the work you hand in.
Write in dark blue or black pen.
You may use a HB pencil for any diagrams or graphs.
Do not use staples, paper clips, glue or correction fluid.

Answer **all** the questions.
Write your answers in the spaces provided in the Question Paper.
Give non-exact numerical answers correct to 3 significant figures, or 1 decimal place in the case of angles in degrees, unless a different level of accuracy is specified in the question.
You are expected to use an approved graphing calculator.
Unsupported answers from a graphing calculator are allowed unless a question specifically states otherwise.
Where unsupported answers from a graphing calculator are not allowed in a question, you are required to present the mathematical steps using mathematical notations and not calculator commands.
You are reminded of the need for clear presentation in your answers.

The number of marks is given in brackets [ ] at the end of each question or part question.

Question	**1**	**2**	**3**	**4**	**5**	**6**	**7**	**8**	**9**	**10**	**Total**
Marks											
Total	7	8	12	13	8	8	10	10	12	12	**100**

This document consists of 3 printed pages, including this cover page.

[Turn Over

Section A: Pure Mathematics [40 marks]

1 Without using a calculator, solve the inequality $\dfrac{x^2+2x-5}{x^2-2x}<2$. [4]

Hence, solve the inequality $\dfrac{x^2+2|x|-5}{x^2-2|x|}>2$. [3]

2 It is given that $y=\mathrm{e}^{-x}\sin x+x-1$.

(a) Show that $\dfrac{\mathrm{d}^2y}{\mathrm{d}x^2}=k\mathrm{e}^{-x}\cos x$, where k is a constant to be determined. [2]

(b) By further differentiation of this result, find the Maclaurin series for y, up to and including the term in x^3. [3]

(c) By using the result in part **(b)** and standard series from the List of Formulae (MF26), find the expansion of $\dfrac{\mathrm{e}^{-x}\sin x+x-1}{\cos 2x}$ in ascending powers of x, up to and including the term in x^3, giving the coefficients in exact form. [3]

3 Following the popularity of the action role-playing game, Ginseng Impact, three years ago, developers have developed a strategy game, Ginseng Impactful. The number of people who download Ginseng Impactful, P (in thousands), in a particular city, at time t months, can be modelled by the differential equation

$$\frac{\mathrm{d}P}{\mathrm{d}t}=\frac{1}{26}P(13-2P).$$

There were 2000 people who download Ginseng Impactful when it is launched.

(a) Show that $P=\dfrac{26}{9\mathrm{e}^{-\frac{1}{2}t}+4}$. [6]

(b) Determine, the time taken, in months, for the number of people who download Ginseng Impactful to double since the launch. [2]

(c) Find the number of people that download Ginseng Impactful in the long run. [2]

(d) Hence sketch the graph showing the number of people that download Ginseng Impactful against time. [2]

4 The plane Π_1 and the line l have equations

$$\Pi_1: \mathbf{r} \cdot \begin{pmatrix} 2 \\ 1 \\ -1 \end{pmatrix} = 3 \quad \text{and} \quad l: \frac{x+2}{3} = \frac{4-y}{2} = z-3$$

respectively.

(a) Find the acute angle between Π_1 and l. [2]

(b) Find the coordinates of the point of intersection between Π_1 and l. [3]

(c) Find the perpendicular distance from $B(10,-4,7)$ to Π_1. [3]

The plane Π_2 contains l and is perpendicular to Π_1.

(d) Find a cartesian equation of Π_2. [3]

(e) Without using a calculator, find a vector equation of the line which lies in both Π_1 and Π_2. [2]

Section B: Probability and Statistics [60 marks]

5 During the Great Singapore Sale, a certain electronics store organises a lucky draw to attract more customers. The lucky draw is designed as follows:

A circular board is divided into four sectors labelled with numbers 1, 2, 3, 4 and has angles 144°, 108°, 72°, 36° respectively. The board has a spinner pivoted at the centre of the circular board. When a customer spins the spinner, the spinner comes to rest randomly in one of the four sectors.

Every customer who visits the store is allowed to play one round of the lucky draw. In each round of the lucky draw, the customer gets to spin the spinner twice.

The score, X, of the customer is

- the sum of the two numbers if the numbers from the two spins are different,
- three times the number if the numbers from the two spins are the same.

A customer wins a prize if the score, X, is more than 6.

(a) Show that $\mathrm{P}(X=6)=0.15$. [2]

(b) Find the probability distribution of X. [3]

(c) Find the probability that a customer scores less than 10, given that the customer wins a prize. [3]

6 The sales manager of a company that sells air conditioning systems presented the data of average daily temperature, t (°C) and sales, s (in hundred units) at a meeting.

Average daily temperature, t (°C)	18	20	23	26	30	32	33	34
Sales, s (in hundred units)	307	366	497	523	565	580	588	596

(a) Draw a scatter diagram of these data and calculate the value of the product moment correlation coefficient between s and t. Comment on whether a linear model would be appropriate, referring both to the scatter diagram and the value of the product moment correlation coefficient found. [3]

The marketing director proposes that the data should be modelled instead by the regression equation $s = a\ln t + b$, where a and b are constants.

(b) Find the values of a and b, giving your answers to 3 decimal places. [1]

(c) Calculate the product moment correlation coefficient between s and $\ln t$. [1]

(d) Using parts **(a)** and **(c)**, explain which is a better model. [1]

(e) Use the model proposed by the marketing director to estimate the number of units sold when the average daily temperature is 38°C and comment on its reliability. [2]

7 In a class of 18 students, there are 12 girls and 6 boys. A chairperson, a vice-chairperson and a secretary are chosen from the 18 students.

(a) Find the number of ways the chairperson, the vice-chairperson and the secretary can be chosen so that

(i) they are all girls, [1]

(ii) there are at least one girl and at least one boy. [3]

The 18 students sit at random in a circle for a lesson.

Find the probability that

(b) the chairperson, the vice-chairperson and the secretary are all separated from one another, [3]

(c) there are exactly 2 girls sitting between each boy. [3]

8 The store manager at CJStore keeps a bin of large number of oranges. It is known that, on average, p % of the oranges are rotten. The oranges are packed into packets of 10 oranges each. The number of rotten oranges in each packet is denoted by the random variable X.

(a) State, in context, two assumptions needed for X to be well modelled by a binomial distribution. [2]

Assume now that X follows a binomial distribution.

(b) It is given that $p = 20$.

(i) Find the probability that there are at least two rotten oranges in a randomly chosen packet. [2]

(ii) 100 packets of oranges are sold at a profit of \$2 per packet. The store offers a discount of \$$d$ for any packet of oranges that contains more than 1 rotten orange. By finding the expected number of packets of oranges that contains more than 1 rotten orange, find the range of values of d, correct to 2 decimal places, if the store manager is expecting a net profit. [3]

(c) The store manager wants to ensure that 95% of the packets of oranges contain at most one rotten orange. Write down an equation satisfied by p. Hence find the value of p. [3]

9 In this question, you should state the parameters of any normal distributions you use.

A supermarket sells apples and guavas. The masses, in grams, of the apples and guavas each follows a normal distribution. The means and standard deviations of the masses of the apples and guavas are shown in the following table:

	Mean (g)	**Standard deviation (g)**
Apples	152	28
Guavas	268	43

Assume that the masses of the apples and guavas are independent of one another.

(a) Three apples are randomly chosen. Find the probability that two of the apples each has mass less than 140g and one of the apples has mass more than 170g. [3]

(b) Find the probability that the total mass of five randomly chosen apples is less than the total mass of three randomly chosen guavas. [3]

The supermarket packs apples and guavas into "Family Packs" for sale. A Family Pack contains three apples and two guavas that are randomly selected.

(c) The probability that a randomly chosen Family Pack differs from the mean mass of Family Packs by less than m grams is 95%. Find the value of m. You may assume that the packing material has negligible mass. [3]

(d) Family Packs are sold at \$5 per kg. Find the probability that a randomly chosen Family Pack costs less than \$5.00. [3]

10 Vesla, an electric vehicle (EV) car manufacturer claims that their EV cars have an average travelling distance of 650 km on a single charge. A car reviewer on the MeTube online video sharing platform wants to test if the EV car manufacturer overstated its claim.

The MeTube car reviewer buys a Vesla EV car to conduct his own test drives. He records the travelling distance on a single charge, x km, on 50 different days over a year. The travelling distance on a single charge, x km, is summarised below:

$$\sum(x-650) = -34.39 \qquad \sum(x-650)^2 = 22769.98$$

(a) Calculate unbiased estimates of the population mean and variance of the travelling distance of an EV car on a single charge. [2]

(b) State hypotheses that the MeTube reviewer can use to test if Vesla has overstated its claim about travelling distance on a single charge, defining any symbols you use. Work out the test statistic in this case, and use it to carry out the test at the 5% level of significance, giving your conclusion in the context of the question. [5]

(c) Explain if the MeTube car reviewer needs to apply Central Limit Theorem for the test to be valid. [2]

Another car reviewer on the TokTik online video sharing platform wants to test if the EV car manufacturer's claim is true. He rents a Vesla EV car to conduct his test drives where he records the travelling distance on a single charge. He records a random sample of 6 test drives and calculates the mean travelling distance on a single charge.

(d) Explain whether the TokTik car reviewer should use a 1-tail or a 2-tail test. [1]

(e) State two assumptions that allow the TokTik car reviewer to carry out his test. [2]

– END –