CANDIDATE NAME

ADDITIONAL MATHEMATICS

4049 / 01

Paper 1
4049-S4-PR-1-01

20 August 2024
Preliminary Examination

Public

2 hours 15 minutes

READ THESE INSTRUCTIONS FIRST

Write in dark blue or black pen.
You may use an HB pencil for any diagrams or graphs.
Do not use staples, paper clips, glue, or correction tape/fluid.

Answer **all** the questions.
Give non-exact numerical answers correct to 3 significant figures, or 1 decimal place in the case of angles in degrees, unless a different level of accuracy is specified in the question.
The use of an approved scientific calculator is expected, where appropriate.
You are reminded of the need for clear presentation in your answers.
DO NOT WRITE ON ANY MARGINS. THEY ARE SOLELY FOR THE MARKERS' USAGE.

The number of marks is given in brackets [ ] at the end of each question or part question.
The total number of marks for this paper is 90. **NO ADDITIONAL MATERIALS ARE NEEDED.**

Question	Mark attained	Maximum	Marker's Feedback
1		10	
2		5	
3		8	
4		9	
5		9	
6		6	
7		8	
8		7	
9		8	
10		3	
11		8	
12		6	
13		3	
TOTAL		90	
Marker's Signature			

1. ALGEBRA

Quadratic Equation

For the equation $ax^2 + bx + c = 0$,

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

Binomial expansion

$$(a + b)^n = a^n + \binom{n}{1} a^{n-1} b + \binom{n}{2} a^{n-2} b^2 + \cdots + \binom{n}{r} a^{n-r} b^r + \cdots + b^n$$

Where $(n > 0) \in \mathbb{Z}$ is a positive integer and

$$\binom{n}{r} = \frac{n!}{r!\,(n-r)!} = \frac{n(n-1)\ldots(n-r+1)}{r!}$$

2. TRIGONOMETRY

Identities

$$\sin^2(A) + \cos^2(A) = 1$$

$$\sec^2(A) = 1 + \tan^2(A)$$

$$\csc^2(A) = 1 + \cot^2(A)$$

$$\sin(A \pm B) = \sin(A)\cos(B) \pm \cos(A)\sin(B)$$

$$\cos(A \pm B) = \cos(A)\cos(B) \mp \sin(A)\sin(B)$$

$$\tan(A \pm B) = \frac{\tan(A) \pm \tan(B)}{1 \mp \tan(A)\tan(B)}$$

$$\sin(2A) = 2\sin(A)\cos(A)$$

$$\cos(2A) = \cos^2(A) - \sin^2(A) = 2\cos^2(A) - 1 = 1 - 2\sin^2(A)$$

$$\tan(2A) = \frac{2\tan(A)}{1 - \tan^2(A)}$$

Formulae for $\triangle ABC$

$$\frac{a}{\sin(A)} = \frac{b}{\sin(B)} = \frac{c}{\sin(C)}$$

$$a^2 = b^2 + c^2 - 2bc\cos(A)$$

$$\triangle = \frac{1}{2} bc \sin(A)$$

TURN OVER FOR QUESTION ONE

1 **(a)** Prove that

$$\tan(3\theta) = \frac{3\tan(\theta) - \tan^3(\theta)}{1 - 3\tan^2(\theta)}$$ [3]

(b) Differentiate $\ln(1 + \tan^2(k\theta))$ with respect to θ, where k is a constant. [2]

(c) **Hence**, evaluate

$$\int_0^{\frac{\pi}{9}} \frac{4\tan^3(\theta) - 12\tan(\theta)}{4 - 3\sec^2(\theta)}\, d\theta$$

Leave an **exact** answer. [5]

2 Let $y = \sqrt{3}\sin(x) + \cos(x) + 1$.

(a) Express y in the form $R\sin(x + \alpha) + 1$, where $R > 0$ and $0 < \alpha < 90°$. [3]

(b) Hence, sketch the graph of y for $0 \leq x \leq 360°$. [2]

(20 OCTOBER 2024) THIS QUESTION IS NOT REQUIRED

3 Solve the following logarithmic equations.

(a) $\log_5(4-\omega) - 2\log_5(\omega) = 1$ [2]

(b) The points P and Q lie on the curve

$$y = 6\log_2(x) - \log_2(7)\,,\ x > 0.$$

The x-coordinates of P and Q are 3 and 6 respectively.
Calculate the gradient k of the straight line passing through P and Q, and find the point on the curve where the gradient is equal to k. [6]

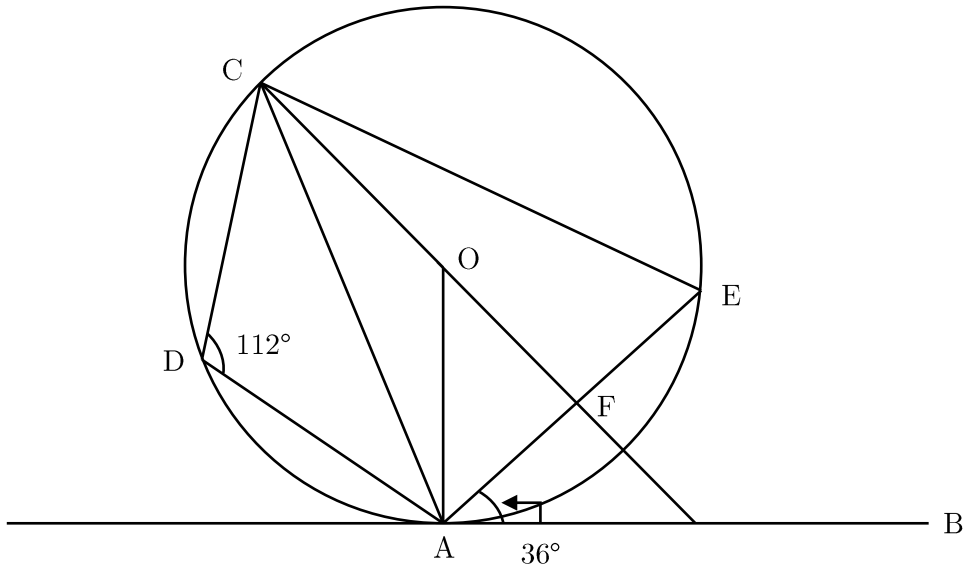
Continuation of working space for Question 3(b).

4 Suppose that a triangle ABC is inscribed in a circle. A, B lie on the line $y = \frac{1}{9}x + 1$; B, C lie on the line $7y = 35x + 18$; and A, C lie on the line $3y = -x - 5$.

Determine the equation of the circle. [9]

Continuation of working space for Question 4.

5



In the diagram above, $ADCE$ is a quadrilateral inscribed in a circle with centre O. F is the midpoint of the line AE which meets the tangent to the circle AB. Angle $CDA = 112°$ and angle $FAB = 36°$. O is due north of A, $OA = 5$ cm and O is located at the coordinates (2, 6).

Give a reason, if appropriate, for each statement you make.

(a) Find angle COA. [2]

(b) Find angle CEA. [1]

(c) Find angle ECF. [3]

(d) Find the coordinates of A. [1]

(e) Determine whether the circle passes through $G(8, 6)$.
(G is not on the diagram.) [2]

6 **(a)** Calculate the range of values of φ such that

$$\varphi^2x^2 + \varphi x + 4\varphi > 0$$

for all real x. [2]

(b) Solve the following simultaneous equations.

$$x + 4y = 9$$
$$y = \frac{x^2 - 3x + 2}{x + 3}$$

You must show appropriate working. Leave your answers in exact form. [4]

7 The polynomial $f\ x$ is a cubic polynomial such that $f\ -5\ +2=0$ and $x-5$ divides $f\ x$. Calculate u if

$$f\left(\frac{1}{u}\right)=0.$$

The graph $y=f\ x\ +9$ has a y-intercept of 4.

Leave your answer in **exact** form. [8]

Continuation of working space for Question 7.

8 **(a)** Consider the expansion until the first five terms, in ascending powers of k, of

$$\left(1+\frac{k}{20}\right)^{32}.$$

Hence, approximate 1.1^{32}, **leaving your answer in decimal form.** [4]

(b) Prove that in expansions of the form

$$\left(ax+\frac{b}{x}\right)^{n}$$

where n is a positive **even** integer, there is an independent term. Use the fact that for any even integer k there exists another integer j such that $k = 2j$, and fact that r is such that $0 \leq r \leq n$. [3]

9 **(a)** Express, in partial fractions

$$\frac{x^3 + 3x^2 - 6x + 9}{(x-1)^2(x^2+1)}$$

[5]

(b) Hence, find

$$\int \left(\frac{x^3 + 3x^2 - 6x + 9}{(x-1)^2(x^2+1)} - \frac{6x+7}{2(x^2+1)} \right) dx$$

[3]

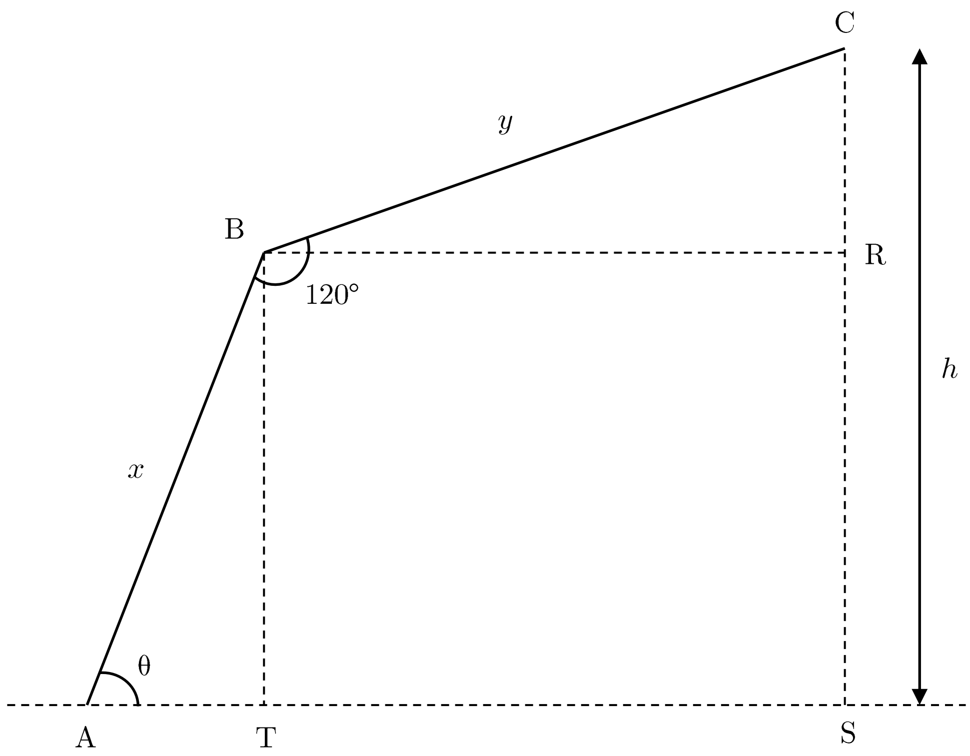
10 **Do not use a calculator in answering this question.**

Shao Kai runs a glass manufacturing business. He constructs a square glass pyramid of side length $3+\sqrt{5}$, and volume $17-\sqrt{30}$. If the height of the pyramid is

$$h = \frac{a - b\sqrt{30} - c\sqrt{5} + d\sqrt{150}}{4}$$

find $a+b+c+d$ if they are all positive integers. [3]

11



In the above diagram, $BTSR$ is a rectangle, and ABC is a bent rod which lies within a vertical plane. The vertical height CRS is h metres long and $\angle BAT = \theta$. $\angle ABC = 120°$.

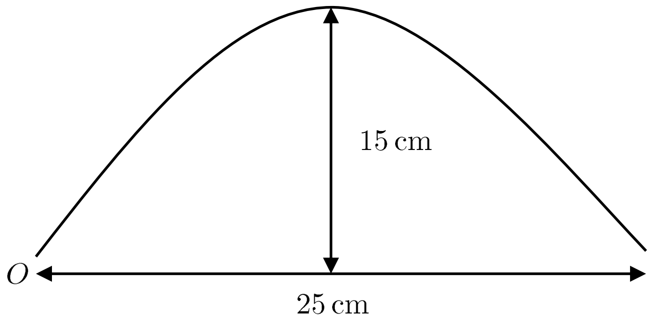
(a) Show that $h = x\sin(\theta) + q\sin(\theta - 60°)$, **stating appropriate reasons.** [4]

(b) Show that when AC is horizontal, the equation

$$\tan(\theta) = \frac{y\sqrt{3}}{2x + y}$$

holds. [4]

12 A parabolic road hump is constructed as shown below.



O is the start of the hump, which is 25 cm wide and 15 cm tall.

(a) Express the height of the arch, y cm, in terms of x cm, where $0 \leq x \leq 25$ is the distance from O. Leave your answer in the form $y = a(x - h)^2 + k$. [2]

(b) The road hump is 5 m long. Find the cost of constructing the hump with concrete, if $1\ \text{m}^3$ of concrete costs $160. [4]

13 The curve with equation $y = ax^2 + bx + a$ lies completely above the x-axis. Write down linear inequalities for a and b, and give possible values of a and b. Here, a, b are constants. [3]

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