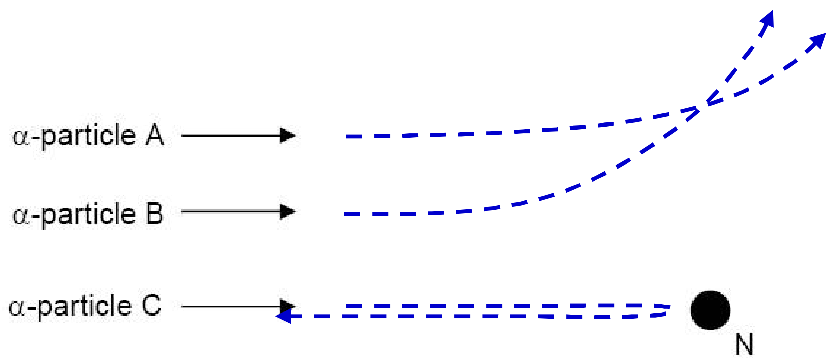
Chapter 20

NUCLEAR PHYSICS

Self-Attempt Questions

1 **(a)** The figure below shows three alpha particles approaching a heavy stationary nucleus N.



(i) Complete the diagram to show the paths of alpha particles as they pass by, and move away from N.

(Particle C shows most deflection, follow by B, then A)

(ii) The nucleus N could be one of several different isotopes of gold.

1 State what is meant by the term isotope.

Isotopes are nuclei of atoms of a particular element containing the same number of protons but different number of neutrons.

2 Suggest, with an explanation, whether different isotopes of gold would give rise to different deviations of a particular α-particle.

All isotopes of gold have the same number of protons, hence the same charge. There will be no difference in the deviations of the α-particle.

(b) Classical experiments on alpha particle scattering were performed by Rutherford, Geiger and Marsden. State the experimental observation obtained from such experiments that proves that

(i) the nucleus is small,

Most of the alpha particles pass straight through the gold foil without being deflected. This shows that the nucleus is small and the atom is largely empty space.

(ii) the nucleus is massive and charged.

Some alpha particles were scattered significantly and a very small number (about 1 in 8000) were deflected by more than 90°. This shows that the nucleus is positively charged, and that the nucleus is massive with the mass of the atom concentrated at the nucleus.

2 State what is meant by

(a) relative atomic mass,

The relative atomic mass is the ratio of the mass of one atom of the substance to 1/12th the mass of a carbon-12 atom.

(b) nuclear binding energy.

The nuclear binding energy of a nucleus is defined as the energy needed to completely separate the nucleons in the nucleus.
(It is also defined as the energy released when a nucleus is formed from its constituent nucleons.)

3 The decay of $^{238}_{92}\text{U}$ to $^{239}_{93}\text{Np}$ by β-emission is not possible because

A $^{239}_{93}\text{Np}$ is not a stable isotope.

B mass number cannot increase in a β-decay process.
C atomic number cannot decrease in a β-decay process.
D mass number and atomic number must both decrease in a β-decay process.

Ans: B

4 Complete the following representations of nuclear transformation:

(a) $^{9}_{4}\text{Be} + \alpha \rightarrow {}^{12}_{6}\text{C} + {}^{1}_{0}\text{n}$

(b) $^{13}_{6}\text{C} + \text{p} \rightarrow {}^{14}_{7}\text{N} + \gamma$

(c) $^{15}_{7}\text{N} + \text{p} \rightarrow {}^{12}_{6}\text{C} + {}^{4}_{2}\text{He}$

5 The nuclide $^{238}_{92}U$ decays to a final stable product $^{210}_{84}Po$ through a series of radioactive nuclides. At each stage, an alpha or beta particle is emitted. Determine the number of alpha and beta particles emitted during the complete decay process.

$$^{238}_{92}U \xrightarrow{\text{after chain decay}} {}^{210}_{84}Po$$

Since the mass number decreases by 28, the number of alpha particles emitted = 28 / 4 = 7. Thus the atomic number should decrease by 2 x 7 = 14.

However the atomic number only decreases by 8. Therefore, 6 beta particles must be released.

$$^{238}_{92}U \rightarrow {}^{210}_{84}Po + 7\ {}^{4}_{2}He + 6\ {}^{0}_{-1}e$$

6 Antimony-124 undergoes radioactive decay, with a half-life of 60 days, to become tin-124. Tin-124 is stable. Initially, a sample of antimony-124 contains no tin-124. For this sample, after what period of time will the ratio $\dfrac{\text{number of tin-124 nuclei}}{\text{number of antimony-124 nuclei}}$ be equal to 6?

A between 60 days and 120 days

B 120 days

C between 120 days and 180 days

D 180 days

Ans: **C**

Let initial number of antimony-124 nuclei be X_o. Let number of antimony-124 nuclei and tin-124 nuclei at time *t* be X and T respectively.

$T + X = X_o$

$$\frac{T}{X} = \frac{X_o}{X} - 1$$

$$X = X_o e^{-(\ln 2)\left(\frac{t}{60}\right)}$$

$$\Rightarrow \frac{X_o}{X} = e^{(\ln 2)\left(\frac{t}{60}\right)}$$

Hence $\dfrac{T}{X} = e^{(\ln 2)\left(\frac{t}{60}\right)} - 1$

$$6 = e^{(\ln 2)\left(\frac{t}{60}\right)} - 1$$

$t = 168.4$ days

ATOMIC STRUCTURE AND NUCLEUS

1 **(a)** In an experiment, ${}^{13}_{6}C$ was bombarded by protons of kinetic energy 2.00 MeV to produce ${}^{13}_{7}N$. By using the masses of the isotopes given below, determine whether this process is possible.
[mass of ${}^{13}C$ = 13.003355 *u*; ${}^{1}H$ = 1.007825 *u*; ${}^{13}N$ = 13.005739 *u*; ${}^{14}N$ = 14.003074 *u*; ${}^{1}_{0}n$ = 1.008665 *u*; ${}^{17}O$ = 16.999134 *u*]

$${}^{13}_{6}C + {}^{1}_{1}H + 2.00\text{ MeV} \rightarrow {}^{13}_{7}N + {}^{1}_{0}n$$

Total mass of ${}^{13}_{6}C$ and ${}^{1}_{1}H$

$$= 13.003355\,u + 1.007825\,u + \frac{2.00\times10^{6}(1.6\times10^{-19})}{(3.0\times10^{8})^{2}(1.66\times10^{-27})}u = 14.013322\,u$$

Total mass of ${}^{13}_{7}N$ and ${}^{1}_{0}n$ = 13.005739 *u* + 1.008665 *u* = 14.014404 *u*

Total mass of reactant < Total mass of products
This process is not possible.

(b)

$${}^{14}_{7}N + {}^{4}_{2}He \rightarrow {}^{17}_{8}O + {}^{1}_{1}H$$

In the nuclear reaction represented by the above equation, nitrogen was bombarded with α-particles of kinetic energy 7.68 MeV. The oxygen nucleus and the proton produced have kinetic energy of 0.56 MeV and 5.93 MeV respectively. Calculate the mass of the α-particle.

By conservation of energy,

$$m_N c^2 + m_{He} c^2 + K_{He} = m_O c^2 + K_O + m_H c^2 + K_H$$

$$m_{He} = \frac{(K_O + K_H - K_{He})}{c^2} + (m_O + m_H - m_N)$$

$$= \frac{(0.56+5.93-7.68)\times10^{6}\times1.60\times10^{-19}}{(3.00\times10^{8})^{2}}$$

$$+ (16.999134 + 1.007825 - 14.003074)\times(1.66\times10^{-27})$$

$$= 4.002611\times1.66\times10^{-27}$$

$$= 6.64\times10^{-27}\text{ kg}$$

2 **(a)** Identify the numbers and symbols represented by the letters, q, r, s, t, u and v in the nuclear equation

$$^{3}_{1}q + ^{2}_{r}H \rightarrow ^{4}_{s}He + ^{t}_{v}u + 17.5\text{ MeV}$$

q = H (tritium), r = 1 (atomic no. of deuterium), s = 2 (atomic no. of helium)
From conservation of mass number, 3 + 2 = 4 + t → t = 1
From conservation of atomic number, 1 + 1 = 2 + v → v = 0
Therefore u = n (neutron)

(b) The product particles each have a greater mass than when at rest. Account for this and calculate the overall difference in mass in this nuclear reaction.

Assume that energy released in this nuclear reaction is in the form of kinetic energy of the product particles.
From Einstein's mass-energy relation, the mass of the particles increased when they have additional energy.
The kinetic energy possessed by the particles (due to release of energy) contributed to the larger mass.
17.5 MeV = $\Delta m\, c^2$

$$\Delta m = \frac{17.5\times10^{6}(1.60\times10^{-19})}{\left(3.00\times10^{8}\right)^{2}} = 3.11\times10^{-29}\text{ kg}$$

(c) If 200 kg of mixed material (denoted by 'q' and H) were used each year to fuel a fusion power station with an overall conversion efficiency of 10%, estimate the electrical power output and the waste heat produced. Assume 1 mole of 'q' & H weigh 5.0 g together, and there are equal numbers of 'q' and H in the mixed material.

200 kg will contain $\frac{200\times10^{3}}{5} = 4.00\times10^{4}$ moles of 'q' & H.
No. of pairs of 'q' & H in 200 kg = $4 \times 10^{4} (6.02 \times 10^{23}) = 2.41 \times 10^{28}$
Total energy produced = $2.41 \times 10^{28} (17.5 \times 10^{6} \times 1.60 \times 10^{-19}) = 6.74 \times 10^{16}$ J

Total power (each year) = $\frac{6.74\times10^{16}}{3600\times24\times365} = 2.14\times10^{9}$ W

Useful power output = $0.10 \times 2.14 \times 10^{9} = 2.14 \times 10^{8}$ W
Waste heat produced = $0.90 \times 2.14 \times 10^{9} = 1.92 \times 10^{9}$ W

3 The energy of the sun produced by the thermonuclear reaction represented by

$$4\,{}^{1}_{1}\text{H} \rightarrow {}^{4}_{2}\text{He} + 2\,{}^{0}_{1}\text{e} + \text{particle } x$$

The masses of ${}^{1}_{1}H$ and ${}^{4}_{2}He$ are 1.00813 u and 4.00386 u respectively.

(a) State the name of particle *x*.

neutrino

(b) If the mean earth-sun distance is 1.5×10^{11} m and the energy of the sun falling on a unit area of the earth per second is 1.35 kW m^{-2}, determine the rate that hydrogen is converted to helium on the sun. [The mass and energy of the positrons, ${}^{0}_{1}\text{e}$ and particle *x* is negligible.]

Energy released during one nuclear reaction

$$= (\Sigma M_{\text{Reactants}} - \Sigma M_{\text{Products}}) \times c^2$$
$$= (4m_H - m_{He}) \times c^2$$
$$= \left[4(1.00813) - 4.00386\right](u) \times c^2$$
$$= 0.02866 \times \left(1.66 \times 10^{-27}\right) \times \left(3.00 \times 10^8\right)^2$$
$$= 4.282 \times 10^{-12}\,\text{J}$$

$$P = IA = I(4\pi r^2) = (1.35 \times 10^3)\left[4\pi(1.5 \times 10^{11})^2\right] = 3.817 \times 10^{26}\ \text{W}$$

In every second, 3.817×10^{26} J of energy is released from the sun.
Rate at which He is produced = $3.817 \times 10^{26} / 4.282 \times 10^{-12} = 8.914 \times 10^{37}\ s^{-1}$
Rate at each hydrogen is changed to He = $4\ (8.914 \times 10^{37}) = 3.57 \times 10^{38}\ s^{-1}$

4 A stationary radon nucleus may decay spontaneously into a polonium nucleus and an α-particle as shown:

$$^{222}_{86}\text{Rn} \rightarrow {}^{218}_{84}\text{Po} + {}^{4}_{2}\text{He}$$

It may be assumed that no γ-ray is emitted.

[mass of $^{222}_{86}\text{Rn}$ = 222.0176 *u*, mass of $^{218}_{84}\text{Po}$ = 218.0090 *u*, mass of $^{4}_{2}\text{He}$ = 4.0026 *u*]

(a) Calculate the total kinetic energy of the decay products.

Decrease in mass of nuclei after reaction

$= (\Sigma M_{\text{Reactants}} - \Sigma M_{\text{Products}}) \times c^2$

= 222.0176 *u* – 218.0090 *u* – 4.0026 *u*
= 0.0060 *u*

Total K.E. of products = energy released in reaction
= energy equivalence of loss in mass
= (0.0060 *u*) c^2
= $(0.0060)(1.66 \times 10^{-27})(3.00 \times 10^8)^2$
= 9.0×10^{-13} J

(b) Explain how the principle of conservation of momentum applies to this decay and calculate the speed of the α-particle.

When the radon nucleus decays to polonium by emitting an alpha particle, total momentum has to be conserved.
Hence the emitted alpha particle and the polonium nucleus must move in opposite directions.

Let *M* = mass of Polonium, *V* = velocity of Polonium, *m* = mass of α-particle, *v* = velocity of α-particle.

$$0 = MV - mv$$
$$MV = mv$$
$$M^2V^2 = m^2v^2$$
$$½\, MV^2 gM = ½\, mv^2 gm$$
$$\frac{K_{Po}}{K_{He}} = \frac{m}{M}$$
$$\frac{K_T - K_{He}}{K_{He}} = \frac{m}{M}$$
$$K_T - K_{He} = \frac{m}{M} K_{He}$$

$$K_{He} = \frac{K_T}{\left(1 + \frac{m}{M}\right)}$$
$$= \frac{8.96 \times 10^{-13}}{\left(1 + \frac{4.0026\ u}{218.0090\ u}\right)}$$
$$= 8.8 \times 10^{-13}\ \text{J}$$
$$K_{He} = \frac{1}{2} m_{He} v_{He}{}^2$$
$$v_{He} = \sqrt{\frac{2K_{He}}{m}}$$
$$= \sqrt{\frac{2(8.798 \times 10^{-13})}{4.0026(1.66 \times 10^{-27})}}$$
$$= 1.6 \times 10^7\ \text{m s}^{-1}$$

5 **(a)** **(i)** Sketch a graph to show the variation of nuclear binding energy per nucleon with the mass number of natural nuclides.

Refer to average binding energy per nucleon vs nucleon number graph in lecture notes.

(ii) Comment on how the binding energy per nucleon varies with nucleon numbers for

1 light nuclides and

For light nuclides with nucleon number ranging from 1 to 56, the binding energy per nucleon increases significantly with an increasing in nucleon number

2 heavy nuclides.

For heavy nuclides with nucleon number ranging from 56 to 238, the binding energy per nucleon decreases gently with an increasing in nucleon number

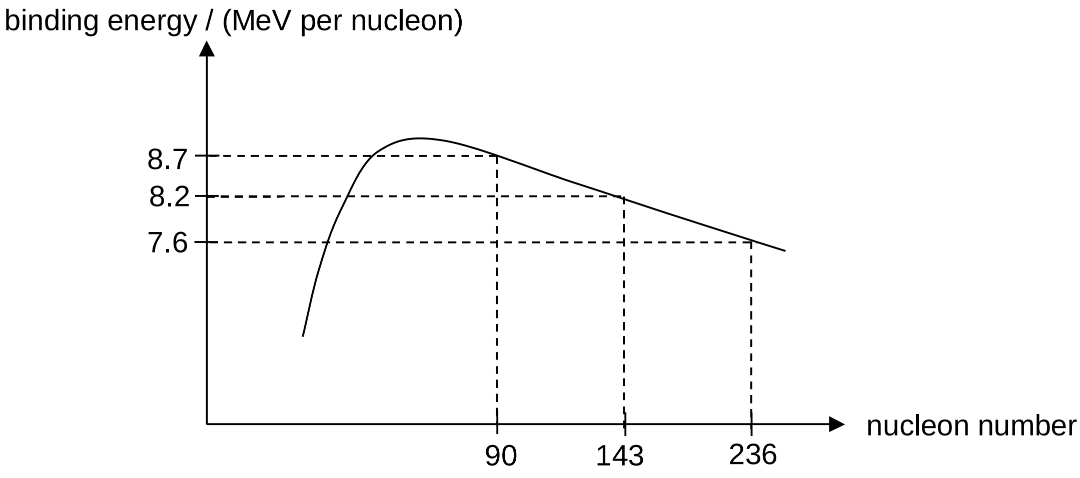
(b) The fusion of two deuterons produces an isotope of helium, ${}^{3}_{2}\text{He}$. As a result, energy is released. Write an equation to represent the reaction, and show how this reaction can be explained using your graph in **(a)** above.

$${}^{2}_{1}H + {}^{2}_{1}H \rightarrow {}^{3}_{2}He + {}^{1}_{0}n$$

Deuteron is a light nucleus with a low mass number of 2.
As such, it has a low binding energy per nucleon.

By fusing two deuterons together to form a product nucleus ${}^{3}_{2}He$, the binding energy of the product is more than the sum of the binding energies of the two lighter deuterons.
Since there is an increase in binding energy per nucleon in the product, there is a release of energy.

6 The simplified graph of binding energy per nucleon varies with nucleon number is shown in the figure below.



During one particular fission process, a Uranium-236 nucleus gives, among its fission products, a Strontium-90 nucleus and a Xenon-143 nucleus.

(a) Use the values on the figure to calculate the energy released during this fission process.

$^{236}U \rightarrow {}^{90}Sr + {}^{143}Xe + \text{other fission products}$

Energy released $= \Sigma$ B.E.$_{products}$ – Σ B.E.$_{reactants}$
= 90 (8.7) + 143 (8.2) – 236 (7.6)
= 162 MeV

(b) State any other fission particle(s) which are produced by this process. Explain why these particles are not taken into account in the calculation in **(a)**.

Other fission products may include neutrons, electrons etc. Since these particles are not nuclei, they possess no binding energy.

(c) Explain why a release of energy occur when there is an increase in the binding energy.

An increase in binding energy means that the product nuclei have higher binding energy per nucleon compared to the reactant nucleus. This means that the energy released when forming the products is more than the energy required when breaking the reactants into constituent particles. As a result, there is a net energy released.

7 A certain source has an activity of 5.00 × 10^6 Bq due to the decay of the radioactive isotope ^{14}C, which has a half-life of 1.81 × 10^{11} s.

(a) Determine the number of atoms of ^{14}C present in the source.

λ = ln2 / $t_{1/2}$ = ln 2 / 1.81 × 10^{11} = 3.830 × 10^{-12}
$N = A / \lambda$ = 5.00 × 10^6 / 3.830 × 10^{-12} = 1.31 × 10^{18}

(b) Calculate the mass of ^{14}C in the source.

6.02 × 10^{23} carbon-14 particles weigh 14 g
∴ 1.31 × 10^{18} carbon-14 particles weigh (1.31 × 10^8 / 6.02 × 10^{23})(14 × 10^{-3})
= 3.04 × 10^{-8} kg

(c) Calculate the activity per kilogram of ^{14}C.

A / mass = 5.00 × 10^6 / 3.04 × 10^{-8} = 1.65 × 10^{14} Bq kg^{-1}

8 Radioactive Potassium K-40 decays to form stable Argon Ar-40. Analysis of K-40 and Ar-40 atoms in a moon rock sample by a mass spectrometer shows that the ratio of the number of Ar-40 atoms present to the number of K-40 atoms is 10.3. The half-life for decay of the K-40 atoms is 1.25 × 10^9 years. Estimate the age of the rock, stating any assumptions made.

Let N_{Ko} be the number of Potassium atoms present at the time the rock is formed (i.e. t = 0).
The remaining number of Potassium atoms at time t is $N_K = N_{Ko}\, e^{-\lambda t}$, where t is the age of the rock.
Assumption: All the Argon atoms are formed from the decay of Potassium atoms.
Number of Argon atoms present, $N_{Ar} = N_{Ko} - N_K$
=>$N_{Ko} = N_{Ar} + N_K$
Therefore $N_K = (N_{Ar} + N_K)\, e^{-\lambda t}$

$$=> e^{-\lambda t} = \frac{N_K}{N_K + N_{Ar}}$$

$$-\lambda t = \ln N_K - \ln(N_K + N_{Ar})$$

$$\lambda t = \ln\left(\frac{N_K + N_{Ar}}{N_K}\right) = \ln\left(1 + \frac{N_{Ar}}{N_K}\right)$$

Since $\lambda = \dfrac{ln 2}{1.25 \times 10^9}$ yr^{-1}

$$=> \quad t = \frac{\ln(1+10.3)}{\frac{\ln 2}{1.25 \times 10^9}} = 4.37 \times 10^9 \text{ yrs}$$

The age of the rock is 4.37 × 10^9 years.

9 **(a)** State and explain the difference between the activity of a radioactive source and the count rate from a counter placed near that source.

Since the counter is able to detect only the radioactive radiation in its direction, its count rate is a fraction of the activity of the radioactive source.

(b) A count rate from a counter near a radioactive source is 7.6×10^{8} s^{-1}. The decay constant of the source is 4.6×10^{-3} s^{-1}.

Calculate

(i) the half-life of the source,

$$t_{\frac{1}{2}} = \frac{\ln 2}{\lambda} = \frac{\ln 2}{4.6 \times 10^{-3}} = 1.5 \times 10^{2} \text{ s}$$

(ii) the time taken for the count rate to fall to 8.3×10^{3} s^{-1}.

$$C = C_o e^{-\lambda t}$$
$$8.3 \times 10^{3} = 7.6 \times 10^{8} e^{-(4.6 \times 10^{-3})t}$$
$$t = 2.5 \times 10^{3} \text{ s}$$

(c) At a distance *x* from a radioactive source, a counter records an average rate of 234 counts per minute. Assuming that the source is radiating uniformly in all directions, deduce the average count rate when the counter is at a distance 3*x* from the source.

$$\frac{C_2}{C_1} = \frac{A_1}{A_2} = \frac{4\pi r_1^{2}}{4\pi r_2^{2}}$$
$$\frac{C_2}{C_1} = \left(\frac{r_1}{r_2}\right)^2 = \left(\frac{x}{3x}\right)^2 = \frac{1}{9}$$
$$C_2 = \frac{1}{9}(234) = 26 \text{ counts per minute}$$

10 The isotope Iron-59 is a β-emitter with a half-life of 45 days. In order to estimate engine wear, an engine component is manufactured from non-radioactive iron throughout which the isotope Iron-59 has been uniformly distributed. The mass of the component is 2.4 kg and its initial activity is 8.5×10^7 Bq.

The component is installed in the engine 60 days after manufacture of the component, and then the engine is tested for 30 days. During the testing period, any metal worn off the component is retained in the surrounding oil. Immediately after the test, the oil is found to have a total activity of 880 Bq. Calculate

(a) the decay constant for the isotope Iron-59,

$$\lambda = \frac{\ln 2}{45} = 0.0154 \text{ day}^{-1}$$

(b) the total activity of the component when it was installed,

$$A = 8.5 \times 10^7 e^{-\left(\frac{\ln 2}{45}\right)(60)} = 3.37 \times 10^7 \text{ Bq}$$

(c) the mass of iron worn off the component during the test.

Consider the activity per kg of iron after the test,

$$A = 8.5 \times 10^7 e^{-\left(\frac{\ln 2}{45}\right)(90)}$$
$$= 2.125 \times 10^7 \text{ Bq}$$
$$\frac{880}{A} = \frac{m}{2.4}$$
$$m = \frac{880}{A} \times 2.4$$
$$= \frac{880}{2.125 \times 10^7} \times 2.4$$
$$= 9.94 \times 10^{-5} \text{ kg}$$

11 **(a)** A nucleus of ${}^{214}_{82}\text{Pb}$ decays by β emission into ${}^{214}_{83}\text{Bi}$ with a half-life of 27 minutes. This Bismuth nuclide is itself radioactive with an unusual decay pattern. Sometimes it decays by α emission into Thallium (Tl) and sometimes by β emission into Polonium (Po).

Write down the nuclear equations for these two decays of ${}^{214}_{83}\text{Bi}$.

$${}^{214}_{83}\text{Bi} \rightarrow {}^{210}_{81}\text{Ti} + {}^{4}_{2}\text{He}$$

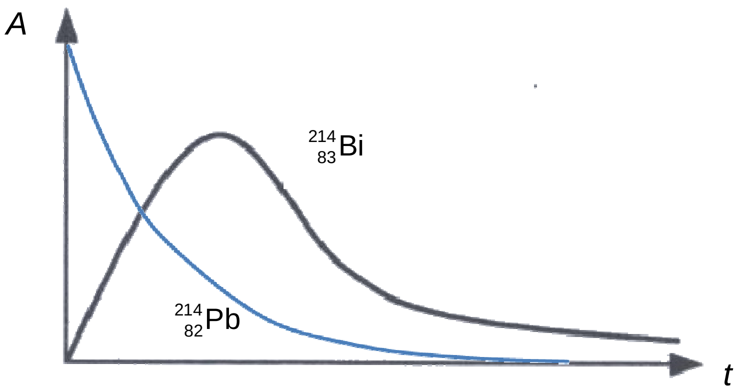
$${}^{214}_{83}\text{Bi} \rightarrow {}^{214}_{84}\text{Po} + {}^{0}_{-1}\text{e}$$

(b) The two decay patterns of $^{214}_{83}\text{Bi}$ each give rise to γ ray photons. Suggest why each of these photons has different energies.

γ ray photon is emitted when the excited nucleus of $^{214}_{83}\text{Bi}$ disintegrates into a nucleus with a lower energy state. The energy of the γ ray photon is equal to the difference in energy levels within the nucleus across which the transition takes place.
Since Ti and Po have different nuclear structures, their nuclear energy levels are different. Hence the energies of the γ ray photons given out by the respective decays are also different.

(c) **(i)** Without numerical values, sketch a labelled graph to show how the activity of $^{214}_{82}\text{Pb}$ changes with time.

(ii) On the same axes, sketch another graph to suggest how the activity of the $^{214}_{83}\text{Bi}$ nuclei formed from the $^{214}_{82}\text{Pb}$ nuclei changes with time.



(d) A sample of $^{214}_{82}\text{Pb}$ has mass 2.6 μg at time *t* = 0. Calculate

(i) the approximate number of atoms it contains,

$$N = \left(\frac{2.6\times10^{-6}}{214}\right)\left(6.02\times10^{23}\right) = 7.3\times10^{15}$$

(ii) its decay constant,

$$\lambda = \frac{\ln 2}{27\times60} = 4.3\times10^{-4}\ \text{s}^{-1}$$

(iii) its activity at time *t* = 0,

$$A = \lambda N = \left(4.3\times10^{-4}\right)\left(7.3\times10^{15}\right) = 3.1\times10^{12}\ \text{Bq}$$

(iv) the time at which its activity has fallen to 8.3 × 10^9 Bq.

$$8.3\times10^{9} = 3.1\times10^{12}e^{\left(4.3\times10^{-4}\right)t}$$

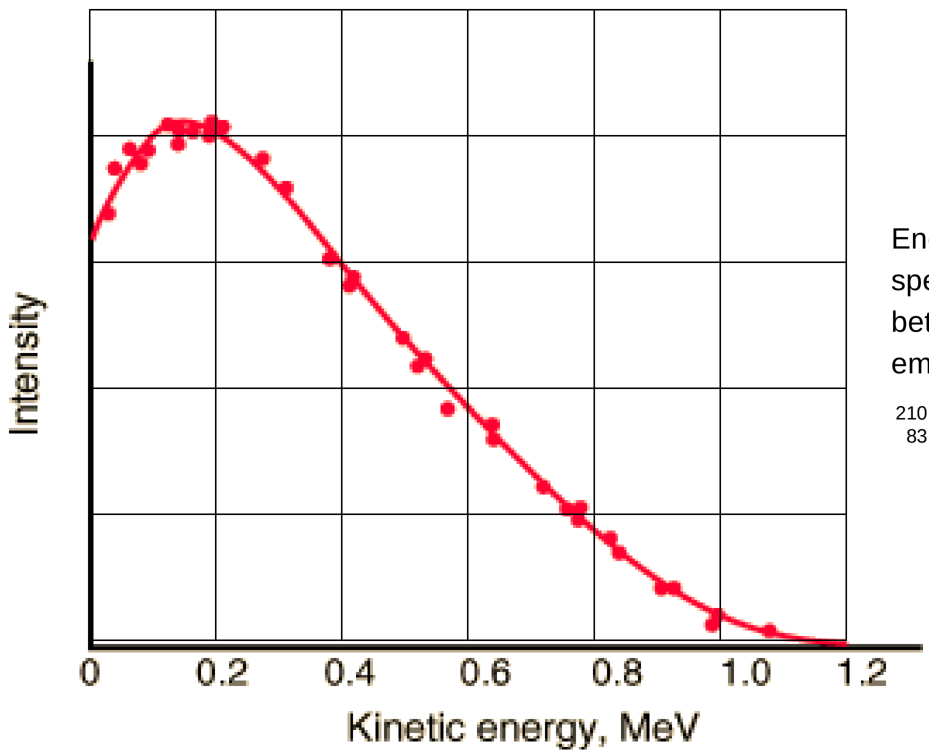
$t = 1.4\times10^5$ s

12 The nucleus of the isotope fluorine-18 decays into a nucleus oxygen-18 by the emission of a positron and neutrino. Outline how the nature of the β-decay energy spectrum of fluorine-18 suggests the existence of the neutrino involved in the decay.

The kinetic energy graph of beta decay shows a continuous energy spectrum up to a maximum value. By conservation of energy, the energy difference between the energy of any beta particle and the maximum value can only be accounted for by the energy of a third particle. As such, the spectrum provides evidence of the existence of a third particle, the neutrino.

13 James Chadwick, in some experiments conducted prior to World War I, used a Geiger Counter to study beta particles emitted from a source and deflected by a uniform magnetic field. He found that the beta particles had a wide range of radii of curvature in the field, indicating that the beta particles were emitted with a distribution of energies rather than with a distinct single value of energy.

The graph below shows the energy spectrum for beta particles emitted during the decay of $^{210}_{83}\text{Bi}$. The intensity (vertical axis) indicates the number of beta particles emitted with each particular kinetic energy (horizontal axis).



Energy spectrum of beta particles emitted from $^{210}_{83}\text{Bi}$

(a) Determine the maximum possible energy of the beta particle emitted from $^{210}_{83}\text{Bi}$.

Max possible energy $= (1.2\times10^{6})(1.60\times10^{-19}) = 1.92\times10^{-13}$ J

(b) State the most probable energy value with which the beta particle is detected.

0.16 MeV [Accept 0.14 to 0.17 MeV]

(c) It is noted that for stable isotopes of heavy elements, there is an optimal neutron-to-proton ratio. Unstable isotopes of a particular element will undergo radioactive decay in order to achieve this optimal ratio. Suggest, with a reason, whether $^{210}_{83}\text{Bi}$ has an excess of neutrons or protons, as compared to the optimal ratio.

Since $^{210}_{83}\text{Bi}$ undergoes radioactive decay to form $^{210}_{84}\text{Po}$ which has one more proton and one less neutron than $^{210}_{83}\text{Bi}$, it suggests that $^{210}_{83}\text{Bi}$ has an excess of neutrons compared to the optimal ratio.

(d) The continuous spectrum of kinetic energy values of the beta particle in the graph presented a problem to physicists in the early years.

Consider a stationary nucleus decaying into a beta particle and the daughter nucleus, assuming that the daughter nucleus is in a stable state that does not emit any gamma particles.

By considering both the conservation of linear momentum and energy, explain how the continuous spectrum of beta particle energies gave rise to this problem.

By conservation of linear momentum, since the parent nucleus is initially stationary, the daughter nucleus and beta particle must move in opposite directions, and the ratio of their kinetic energies is a constant.

Since the total energy released in the radioactive decay is a constant ($E = \text{B.E.}_{\text{daughter nucleus}} - \text{B.E.}_{\text{parent nucleus}}$), by conservation of energy, the KE of the beta particle must therefore also be a constant.

Therefore having beta particles with varying KEs presented a problem (since it then contradicts the conservation of energy, as well as linear momentum).

(e) Suggest what was hypothesized by physicists to resolve the problem in **(d)**.

The emission of another undetected particle with a certain kinetic energy during the radioactive decay will explain why the beta particles have a varying amount of kinetic energies, while ensuring that total energy and total momentum are conserved.

14 The GM tube was used to investigate the rate of decay of a radioactive isotope of protactinium. Counts were made over periods of ten seconds, with each count starting 30s after the previous count was completed. The results were recorded in Fig. 14.1 as follows:

Time interval /s	0-10	40-50	80-90	120-130	160-170	220-230
Total count	3440	2340	1650	1140	800	580
Total count minus background count	3410	2310	1620	1110	770	550
Count rate due to isotope / s^{-1}	341	231	162	111	77	55

Fig. 14.1

The average background count is 30 counts per 10 s.

(a) Complete Fig. 14.1.

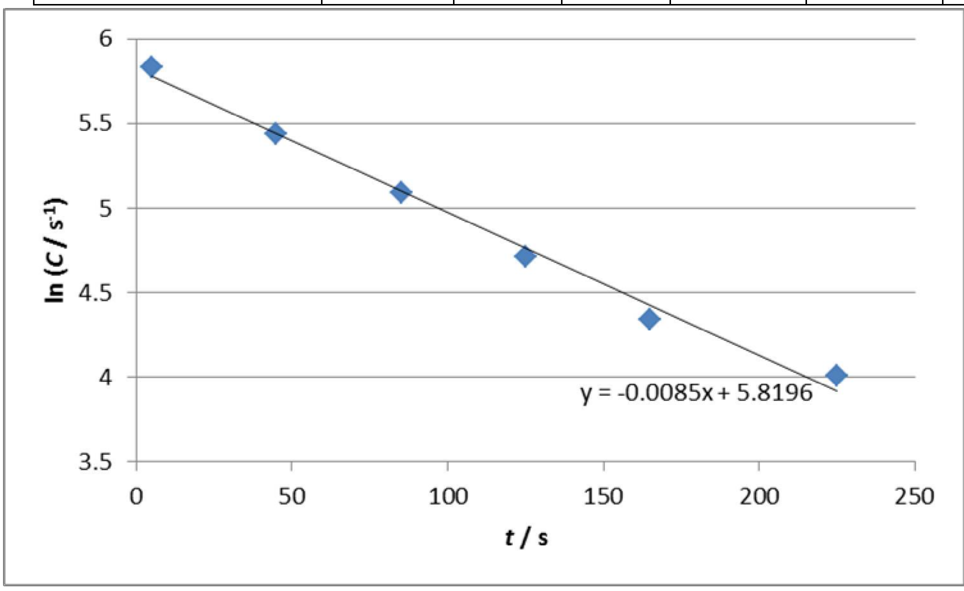
(b) By using the equation $C = C_o e^{-\lambda t}$ and plotting a suitable graph using the grid on the next page, determine the half-life of the isotope.

$C = C_o e^{-\lambda t}$ → $\ln C = -\lambda t + \ln C_o$

By plotting a graph of ln *C* against *t*, a straight line graph can be obtained, where gradient = $-\lambda$ = - ln 2 / $t_{1/2}$

vertical intercept = ln C_o

Time *t* / s	5	45	85	125	165	225
ln (C / s^{-1})	5.83	5.44	5.09	4.71	4.34	4.01



From the graph, λ = ln 2 / $t_{1/2}$ = 0.0085

=> $t_{1/2}$ = 81.5 s