**Candidate Name:** _____ **Index No:** ____ **Class:** _____

**ZHENGHUA SECONDARY SCHOOL**
**PRELIMINARY EXAMINATIONS 2023**
**SECONDARY FOUR EXPRESS/NORMAL (ACADEMIC) (O LEVEL)**
**MATHEMATICS** **4052/01**
**Paper 1**

22 AUGUST 2023

Candidates answer on the Question Paper. 2 hours 15 minutes



**READ THESE INSTRUCTIONS FIRST**

Write your index number and name on all the work you hand in.
Write in dark blue or black pen.
You may use an HB pencil for any diagrams or graphs.
Do not use staples, paper clips, glue or correction fluid.

Answer **all** questions.
The number of marks is given in brackets [ ] at the end of each question or part question.

If working is needed for any question it must be shown with the answer.
Omission of essential working will result in loss of marks.
The total of the marks for this paper is 90.

The use of an approved scientific calculator is expected, where appropriate.
If the degree of accuracy is not specified in the question and if the answer is not exact, give the answer to three significant figures. Give answers in degrees to one decimal place.
For $\pi$, use either your calculator value or 3.142.

Name of Setter:

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**This document consists of 1 printed pages.**

**[Turn over]**

## *Mathematical Formulae*

*Compound interest*

$$\text{Total amount} = P\left(1+\frac{r}{100}\right)^n$$

*Mensuration*

$$\text{Curved surface area of a cone} = \pi rl$$

$$\text{Surface area of a sphere} = 4\pi r^2$$

$$\text{Volume of a cone} = \frac{1}{3}\pi r^2 h$$

$$\text{Volume of a sphere} = \frac{4}{3}\pi r^3$$

$$\text{Area of triangle } ABC = \frac{1}{2}ab\sin C$$

$$\text{Arc length} = r\theta, \text{ where } \theta \text{ is in radians}$$

$$\text{Sector area} = \frac{1}{2}r^2\theta, \text{ where } \theta \text{ is in radians}$$

*Trigonometry*

$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$

$$a^2 = b^2 + c^2 - 2bc\cos A$$

*Statistics*

$$\text{Mean} = \frac{\sum fx}{\sum f}$$

$$\text{Standard deviation} = \sqrt{\frac{\sum fx^2}{\sum f} - \left(\frac{\sum fx}{\sum f}\right)^2}$$

Answer **all** the questions.

**1.** Calculate $\frac{\sqrt[3]{4\pi^2 - 15^2}}{0.63}$.

Leave your answer to 4 decimal places.

*Answer* ..... [1]

---

**2.** Factorise $9x^2 + 12x - 12$.

*Answer* ..... [2]

---

**3.** The sine of an angle is 0.427.

Give two possible values for the angle.

° °

*Answer* ..... or ..... [2]

---

**4.** 8 students took a test and their marks were recorded as follows.

8 10 9 13 10 9 $x$ $y$

The modal score for the 8 students is 10 and the median is 9.5.

If the range of the scores is 5, and $x \leq y$, determine the values of $x$ and $y$.

*Answer* $x$ = .....

$y$ = ..... [2]

---

**5.** A sum of money was shared among Allan, Ben and Chloe.

Allan receives $\frac{2}{5}$ of the money and the rest was shared among Ben and Chloe in the ratio 3:2.

Find the ratio of the money received in terms of Allan's : Ben's : Chloe's.

*Answer* ..... : ..... : ..... [2]

---

**6.** John is making two rectangular flower beds.

The dimensions of the larger rectangle will be four times the dimensions of the smaller rectangle.

There is going to be the same depth of soil in each flower bed.

John needs 120 kg of soil for the larger flower bed.

Work out how much soil John needs for the smaller flower bed.

*Answer* ..... kg [2]

---

**7.** $T = \sqrt{\frac{w}{d^3}}$ for a particular value of $w$ and $d$.

If $w$ is increased by 20%, and $d$ is decreased by 10%, determine the percentage change in $T$.

*Answer* ..... % [2]

---

**8.** Jamie is using the quadratic formula to solve a quadratic equation.

She substitutes values into the formula and correctly gets

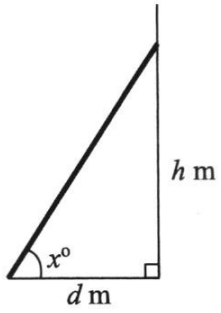
$$x = \frac{-9 \pm \sqrt{81 - 48}}{6}$$

Find the quadratic equation that Jamie is solving, giving your answer in the form $ax^2 + bx + c = 0$, where $a$, $b$ and $c$ are integers.

*Answer* ..... [2]

---

**9.**




A ladder is leaning against a vertical wall as shown in the diagram.

The bottom of the ladder is $d$ metres from the wall.

The top of the ladder is $h$ metres above the ground.

The angle between the ladder and the ground is $x°$.

**(a)** Some safety instructions say it is safe to climb the ladder when $h = 3.5d$.

Determine the value of $x$ when $h = 3.5d$.

*Answer* ..... [2]

**(b)** Some different safety instructions say the angle between the ladder and the ground should be $70°$. The ladder is then moved so that $x = 70°$.

Without further calculations, how does this change in angle affect the height, $h$ metres, of the top of the ladder above the ground?

.....

..... [1]

---

**10.** The table shows pairs of values of $x$ and $y$.

| $x$ | 4 | 5 |
|---|---|---|
| $y$ | 400 | 625 |

**(a)** Which of the following statement is correct?

Statement A: $y \propto x$

Statement B: $y \propto x^2$

Statement C: $y \propto x^3$

*Answer* Statement ..... [1]

**(b)** Write a formula for $y$ in terms of $x$.

*Answer* ..... [2]

---

**11.** Simplify.

**(a)** $3+p(3-p)$

*Answer* ..... [1]

**(b)** $9x^{-4} \div \frac{1}{2}x^{6}$

*Answer* ..... [2]

---

**12.** Solve the inequalities $x-3<\frac{3}{2}x-2\leq\frac{x+18}{6}$ and state the integer values of $x$ that satisfy them.

*Answer* ..... [3]

---

**13.** A bag contains 12 blue marbles and 8 red marbles.

**(a)** A marble is chosen at random and then replaced.

What is the probability that it is a red marble?

*Answer* ..... [1]

**(b)** How many red marbles must be placed in the bag so that the probability of choosing a red marble would be $\frac{3}{5}$?

*Answer* ..... [2]

---

**14.** Express as $\frac{3}{x-3} - \frac{4}{x^2-9}$ a single fraction in its simplest form.

*Answer* ..... [3]

---

**15.** A train 180 m long passes through a tunnel.

The average speed of the train is 42 km/h.

**(a)** Express 42 km/h in m/s.

*Answer* ..... m/s [1]

**(b)** The train passes through a 2.4 km tunnel.

Calculate the time for the train to completely pass through the tunnel.

Give your answer in minutes and seconds, to the nearest second.

*Answer* ..... min .....s [2]

---

**16.** $ab^2 + c = \dfrac{2c + b}{a}$

Rearrange the formula to make $c$ the subject.

*Answer* ..... [3]

---

**17.** **(a)** Factorise completely $3ax+12by-9ay-4bx$.

*Answer* ..... [2]

**(b)** Show that $(3n+5)^2-7$ is a multiple of 3 for all integer values of $n$.

*Answer*

[2]

---

**18.** The timings of 16 students running a 100 metre race is recorded below.

| Time, $x$ seconds | $12 \le x < 13$ | $13 \le x < 14$ | $14 \le x < 15$ | $15 \le x < 16$ |
|---|---|---|---|---|
| Frequency | 3 | 5 | 6 | 2 |

**(a)** Calculate an estimate for

**(i)** the mean running time of the students,

*Answer* ..... s [1]

**(ii)** the standard deviation of the running time.

*Answer* ..... s [2]

**(b)** It was found that there was a mechanical error in the stopwatch used to time the run.
An actual 10 seconds is measured as 9 seconds by the stopwatch.
Explain how the mean will change after the adjustment.

.....

.....

.....

..... [1]

---

**19.** Express as a product of prime factors, $168 = 2^3 \times 3 \times 7$ and $A = 2^p \times 3^3 \times 5$.
Using the above information, find

**(a)** the smallest integers $m$ and $n$ such that $\frac{168 \times m}{\sqrt{n}}$ is a perfect square and $m > n$,

*Answer* $m$ = .....

$n$ = ..... [2]

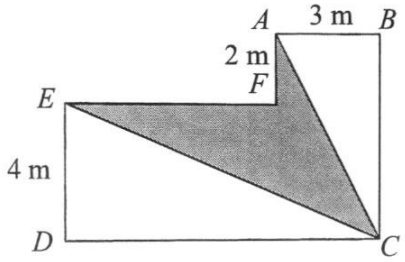
**(b)** the value of $p$ and of $A$ given that the highest common factor of 168 and $A$ is 12.

*Answer* $p$ = .....

$A$ = ..... [2]

---

**20.**




The diagram shows a shape *ABCDEF*.

All the corners of the shape are right angles.

$ED = 4$ m, $AF = 2$ m and $AB = 3$ m.

The perimeter of the shape is 38 m.

Find the area of the shaded part *ECAF* of the diagram.

*Answer* ..... $m^2$ [4]

**21.** The points $P$ and $Q$ have coordinates $(7, 2)$ and $(6, -3)$ respectively.

**(a)** Determine if the line $y + 5x = 4$ is parallel to $PQ$.

*Answer*

[2]

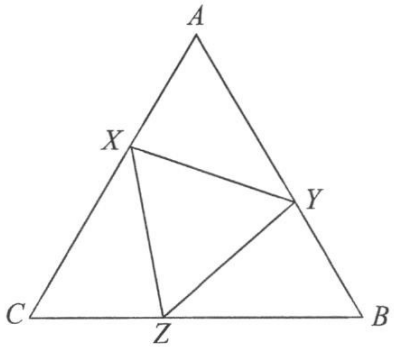
**(b)** **(i)** Find the column vector $\overrightarrow{PQ}$.

*Answer* ..... [1]

**(ii)** Find $\left|\overrightarrow{PQ}\right|$.

*Answer* ..... [2]

**22.**




In the diagram, triangle $ABC$ is an equilateral triangle.

$X$, $Y$ and $Z$ lie on lines $AC$, $AB$ and $BC$ respectively and $AX = CZ = BY$.

**(a)** Prove that triangle $AXY$ is congruent to triangle $CZX$.

*Answer*

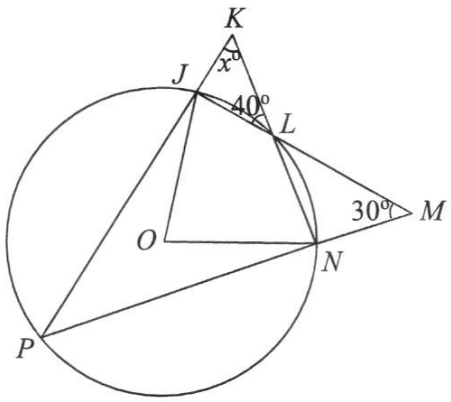
[3]

**(b)** Determine if triangle $XYZ$ is an equilateral triangle.

*Answer*

[2]

**23.**




In the diagram, $O$ is the centre of the circle. $P$, $N$, $L$ and $J$ lie on the circumference of the circle. The lines $PJ$ and $NL$ are extended to meet at $K$. The lines $JL$ and $PN$ are extended to meet at $M$. Angle $JLK$ is $40^\circ$, angle $LMN$ is $30^\circ$ and angle $JKL$ is $x^\circ$.

Write down an expression, in terms of $x$, for

**(a)** **(i)** angle $PJM$,

*Answer* ..... [1]

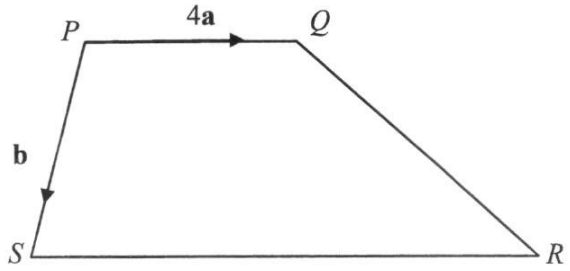
**(ii)** angle $PNL$.

*Answer* ..... [1]

**(b)** By considering triangle $JMP$, form an equation in $x$ and solve it.

*Answer* $x =$ ..... [2]

**24.**




In the diagram, $PQRS$ is a trapezium where $\overrightarrow{PQ} = 4\mathbf{a}$ and $\overrightarrow{PS} = \mathbf{b}$.

A point $T$ lies inside the trapezium where $QT : TS = 4 : 5$.

$SR$ is twice the length of $PQ$.

**(a)** Express, as simply as possible, in terms of **a** and/or **b**,

**(i)** $\overrightarrow{QS}$,

*Answer* ..... [1]

**(ii)** $\overrightarrow{QT}$,

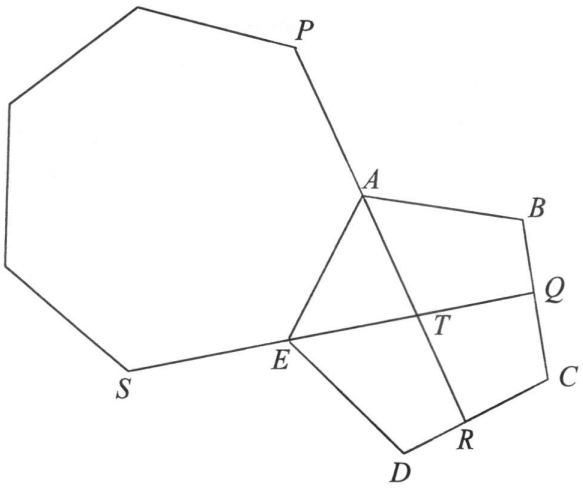
*Answer* ..... [1]

**(b)** Determine whether $P$, $T$ and $R$ lie on the same straight line.

*Answer*

[3]

**25.**




In the diagram, a regular heptagon and regular pentagon are fitted together at the edge *AE*.

*PAR* and *SEQ* are straight lines.

Find angle *TRD*.

°

*Answer* ..... [5]

**26.** The table below shows the distribution of three types of apartments in three blocks of flats.

| | 3-room | 4-room | 5-room |
|---|---|---|---|
| Block 1 | 10 | 20 | 50 |
| Block 2 | 25 | 45 | 10 |
| Block 3 | 30 | 50 | 20 |

The information can be represented by the matrix $\mathbf{T} = \begin{pmatrix} 10 & 20 & 50 \\ 25 & 45 & 10 \\ 30 & 50 & 20 \end{pmatrix}$.

The floor areas of the 3-room, 4-room and 5-room apartments are 60 m², 90 m² and 110 m² respectively and this information can be represented by the matrix $\mathbf{A} = \begin{pmatrix} 60 \\ 90 \\ 110 \end{pmatrix}$

**(a)** Evaluate **TA**.

*Answer* ..... [2]

**(b)** Explain what the elements in **TA** represent.

.....

..... [1]

**(c)** Maintenance fees are charged based on the floor area of the apartments in the three blocks of flats. The maintenance fee rates ($/m²) for Block 1, 2 and 3 are are $3, $4, and $6 respectively. Write down a 1×3 matrix **C** to show the maintenance fee rates for the three blocks of flats.

*Answer* ..... [1]

**(d)** Hence, using matrix multiplication, find the total estate management fee for these three blocks of flats.

*Answer* ..... [2]

---

**27. (a)** The first four terms of a sequence are 5, 9, 13 and 17.

The sum of the first $n$ terms of this sequence is given by $an^2 + bn$.

**(i)** When $n = 1$, $a + b = 5$.

Show that $4a + 2b = 14$.

*Answer*

[1]

**(ii)** Solve $a + b = 5$

$4a + 2b = 14$.

*Answer* $a$ = .....

$b$ = ..... [2]

**(b)** The $n$th term of another sequence is $n^2 + 4$.

Anthony says,

"The $n$th term of the sequence is always a prime number when $n$ is an odd number."

Anthony is wrong.

Give an example to show that Anthony is wrong.

*Answer* ..... [2]

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~ End of Paper ~

**Candidate Name:** _____ **Index No:** ____ **Class:** _____

**ZHENGHUA SECONDARY SCHOOL**
**PRELIMINARY EXAMINATIONS 2023**
**FOUR EXPRESS**
**FOUR NORMAL (ACADEMIC) (O LEVEL)**
**MATHEMATICS** **4052/02**
**Paper 2**

23 AUGUST 2023

Candidates answer on the Question Paper. 2 hours 15 min



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Name of Setter:

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**This document consists of 22 printed pages.**

**[Turn over]**

## *Mathematical Formulae*

*Compound interest*

$$\text{Total amount} = P\left(1+\frac{r}{100}\right)^n$$

*Mensuration*

Curved surface area of a cone = $\pi rl$

Surface area of a sphere = $4\pi r^2$

$$\text{Volume of a cone} = \frac{1}{3}\pi r^2 h$$

$$\text{Volume of a sphere} = \frac{4}{3}\pi r^3$$

$$\text{Area of triangle } ABC = \frac{1}{2}ab\sin C$$

Arc length = $r\theta$, where $\theta$ is in radians

$$\text{Sector area} = \frac{1}{2}r^2\theta, \text{ where } \theta \text{ is in radians}$$

*Trigonometry*

$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$

$$a^2 = b^2 + c^2 - 2bc\cos A$$

*Statistics*

$$\text{Mean} = \frac{\sum fx}{\sum f}$$

$$\text{Standard deviation} = \sqrt{\frac{\sum fx^2}{\sum f} - \left(\frac{\sum fx}{\sum f}\right)^2}$$

**1** **(a)** Solve $\frac{1}{x-1}+\frac{2}{x-2}=\frac{3}{x-3}$.

*Answer* $x=$ ..... [2]

**(b)** Simplify $\left(a^2b\right)^{\frac{1}{2}}\times\left(ab^{-3}\right)^{\frac{1}{3}}$.

*Answer* ..... [1]

**(c)** **(i)** Express $x^2+11x-15$ in the form $\left(x+a\right)^2+b$.

*Answer* ..... [1]

**(ii)** Hence solve the equation $x^2+11x-15=0$, giving your answers correct to two decimal places.

*Answer* $x=$ ..... or ..... [3]

**(d)** Simplify $\dfrac{3a^2+12a-36}{ab-2b+a^2-2a}$.

*Answer* ..... [3]

2 **(a)** Abigale has two possible options to invest a sum of money for 3 years.

Account A pays compound interest 6%, 8% and 10% for the 3 years respectively.
Account B pays compound interest 10%, 8% and 6% for the 3 years respectively.

Explain which, if any, is a better choice for Abigale's investment.

.....

.....

..... [1]

**(b)** An amount of \$3750 was placed in an account that pays simple interest.
After 3 years, the balance was \$4620.

**(i)** Find the rate of interest.

*Answer* .....% [2]

**(ii)** If the same interest is paid using compound interest, calculate the balance in the account after the same time period.

*Answer* \$..... [2]

**(c)** On 1 June 2022, Ben changed S$6200 into American dollars.
The exchange rate between American dollars (US$) and Singapore dollars (S$) was US$1 = S$1.37.

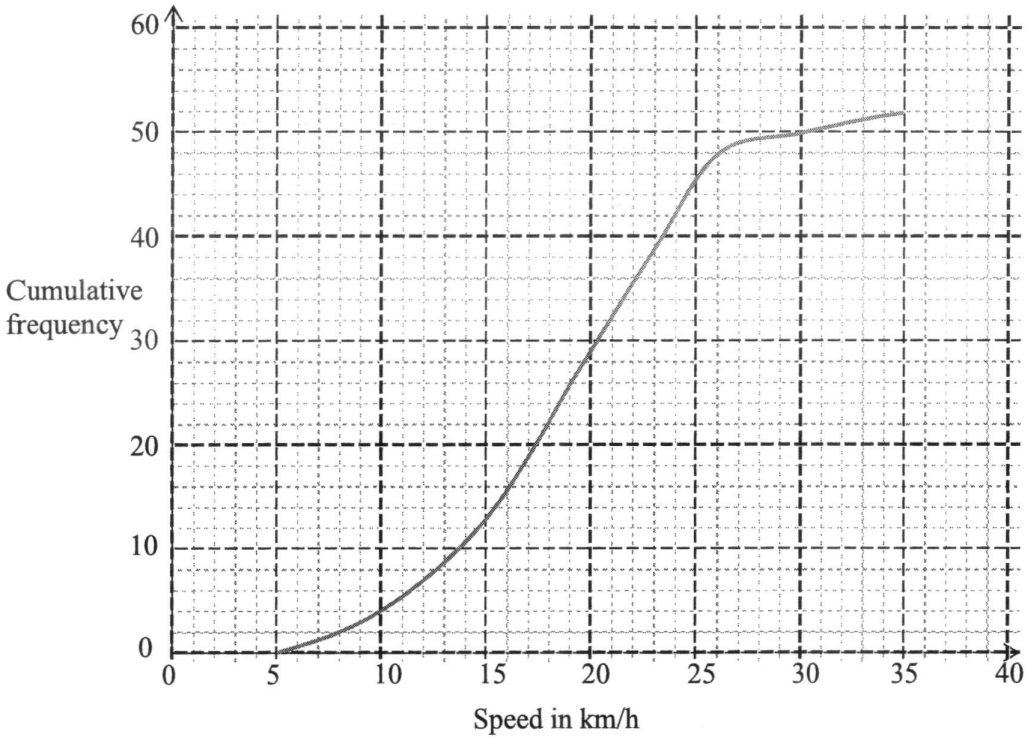
**(i)** Calculate the amount of American dollars he would receive.

*Answer* US$..... [2]

**(ii)** On a trip to the USA, Ben bought a tablet for US$1899 from a departmental store.
When Ben bought the tablet, the store made a profit of 8%.
Calculate how much the store paid for the tablet.

*Answer* US$..... [2]

3 The cumulative frequency graph below shows speeds in km/h of cyclists passing a certain point during a race.




**(a)** Use the curve to estimate

**(i)** the median speed of the cyclists,

*Answer* ..... km/h [1]

**(ii)** the percentage of cyclists having speed more than 21 km/h.

*Answer* ..... % [1]

**(iii)** the ninety-sixth percentile.

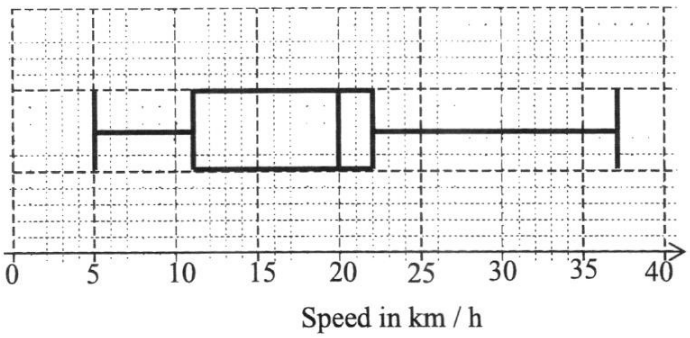
*Answer* ..... km/h [1]

**(iv)** Complete the grouped frequency table of the speeds of the cyclists passing a certain point during the race.

| Speed ($x$ km/h) | $5 \le x < 10$ | $10 \le x < 15$ | $15 \le x < 20$ | $20 \le x < 25$ | $25 \le x < 30$ | $30 \le x < 35$ |
|---|---|---|---|---|---|---|
| Frequency | 4 | | | | | |

[2]

**(b)** In the previous year, the same race had taken place.
The box-and-whisker plot shows the distribution of the speed of the cyclists.




Make two comparisons between the performances of the cyclists in the two years.
Use figures to support your answer.

.....

.....

.....

..... [3]

**4** The variables $x$ and $y$ are connected by the equation

$$y = \frac{x^2}{5} + \frac{6}{x}.$$

The table below shows some values of $x$ and the corresponding values of $y$ correct to 1 decimal place.

**(a)** Complete the table.

| $x$ | 1 | 1.5 | 2 | 3 | 4 | 5 | 6 |
|---|---|---|---|---|---|---|---|
| $y$ | 6.2 | 4.5 | 3.8 | 3.8 | 4.7 | 6.2 | |

[1]

**(b)** On the grid at the **next page**, draw the graph of $y = \frac{x^2}{5} + \frac{6}{x}$ for $1 \leq x \leq 6$.

[3]

**(c)** Use your graph to find the value(s) of $x$ in the range of $1 \leq x \leq 6$ for which $\frac{x^2}{5} + \frac{6}{x} - 4 = 0$.

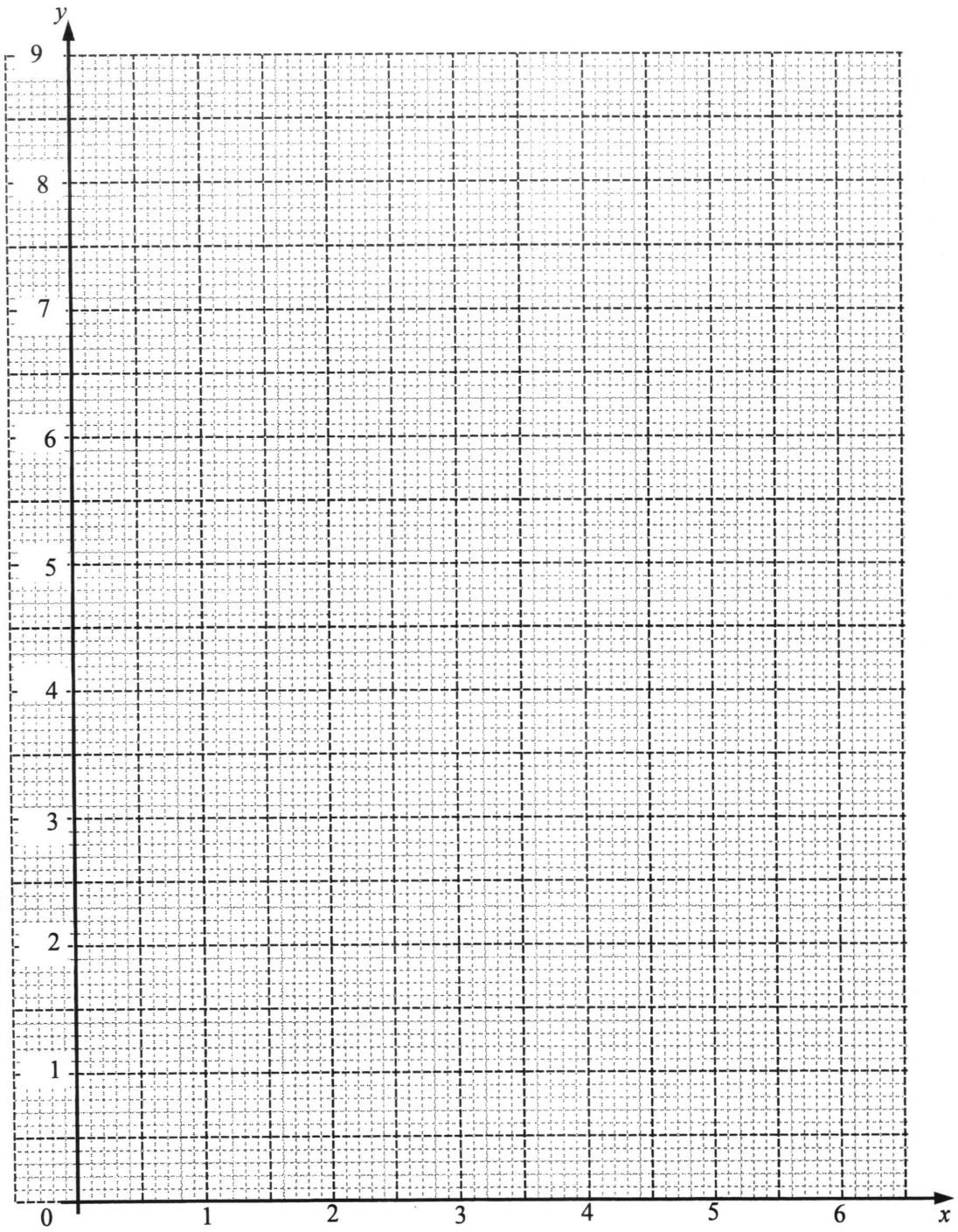
*Answer* $x =$ .....[2]

**(d)** By drawing a tangent, find the gradient of the curve at point $(4, 4.7)$.

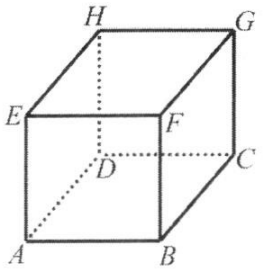
*Answer* ..... [2]

**(e)** By drawing the graph $y = \frac{1}{2}x + 4$ on the grid, find the equation in the form $2x^3 + ax^2 + bx + c = 0$, which is satisfied by the $x$-coordinates of the points at which the two graphs intersect.

*Answer* ..... [4]


y
9
8
7
6
5
4
3
2
1
0
1
2
3
4
5
6
x

**5** The diagram below shows a solid cube *ABCDEFGH*.



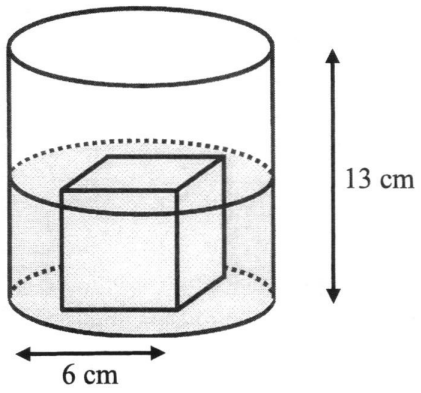

**(a)** The solid cube was produced after recasting a solid spherical toy with diameter 10 cm.
Show that the length of a side of the cube is approximately 8.06 cm.

*Answer* shown ..... cm [2]

**(b)** An ant is crawling from the point *A*.
Find the shortest distance that it can take from point *A* to point *G*.

*Answer* ..... cm [2]

**(c)** The solid cube is placed inside an open cylindrical container which has base radius 6 cm and height 13 cm.




$V$ cm$^3$ of water is poured into the container at a rate of 2.5 cm$^3$/s until it covers the cube as shown in the diagram above.

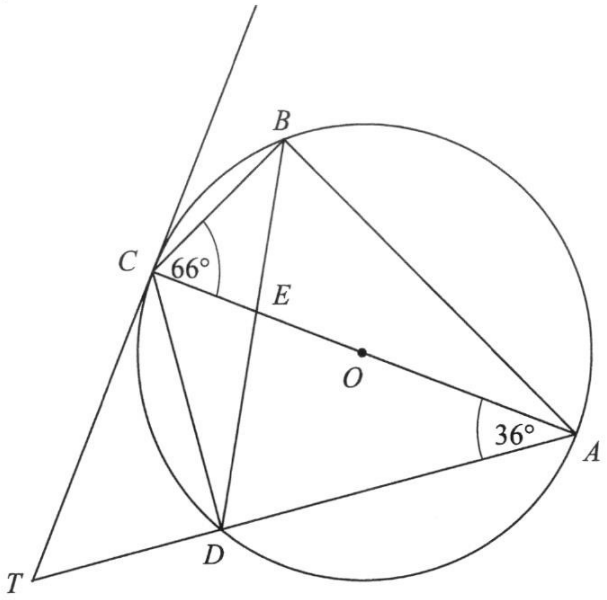
**(i)** Calculate the amount of time, in seconds, it takes for all the water to be poured in.

*Answer* ..... s [4]

**(ii)** When the cube is removed from the container, the water level drops by $x$ cm. Assuming that the water loss is negligible upon the removal of the cube, calculate the value of $x$.

*Answer* ..... cm [2]

**6** In the diagram, $A$, $B$, $C$ and $D$ are points on the circle with centre $O$.
The lines $AC$ and $BD$ intersect at $E$.
The tangent to the circle at $C$ meets $AD$ produced at $T$.
$\angle ACB = 66°$, $\angle CAD = 36°$ and $AC = 10\,\text{cm}$.




**(a)** Find the angle $DCT$.

*Answer* ..... [2]

**(b)** Show that the triangles $BCE$ and $ADE$ are similar.
Give a reason for each statement you make.

.....

.....

.....

.....

..... [2]

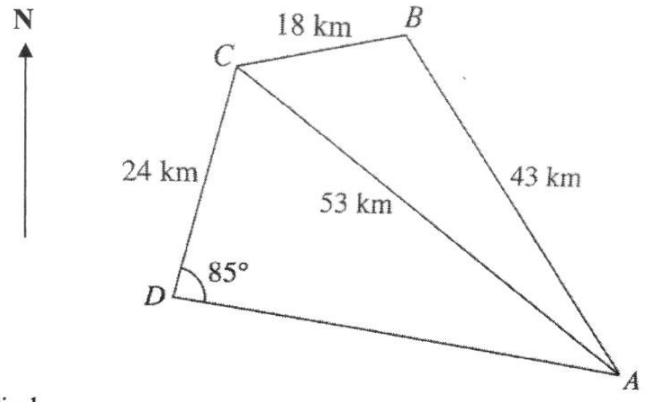
**(c)** Calculate $DE$.

*Answer* ..... cm [4]

**(d)** Calculate the length of the minor arc $BC$.

*Answer* ..... cm [3]

7 A city tour starting at Hotel $A$ is planned. The coach travels 43 km in the **north-west** direction to $B$, the zoo , where it stops for 3 hours. After the visit to the zoo, the coach continues its journey to a popular seafood restaurant at $C$, 18 km away, for lunch. After lunch, the coach brings the tourists to visit a museum at $D$, 24 km away and $\angle CDA = 85°$.




**(a)** Find

**(i)** $\angle BAC$,

*Answer* ..... [2]

**(ii)** $\angle CAD$.

*Answer* ..... [2]

**(b)** Find the bearing of $C$ from $A$.

*Answer* ..... [2]

**(c)** Calculate the shortest distance from $D$ to $AC$.

*Answer* ..... km [3]

**8** **(a)** $\xi = \{\text{integers } x\text{: } 0 < x < 12\}$
$P = \{\text{even numbers}\}$
$Q = \{\text{prime numbers}\}$

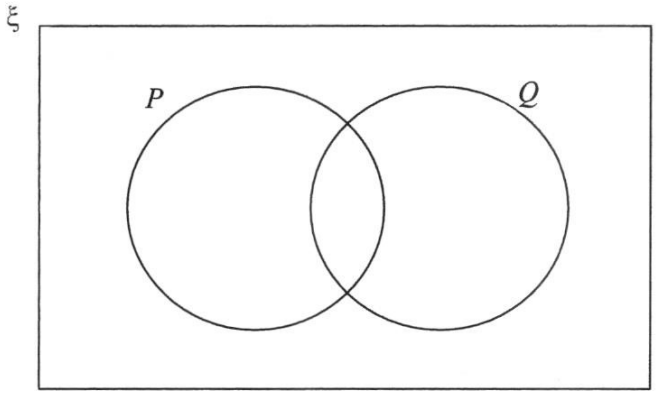
**(i)** List the elements in $P$.

*Answer* ..... [1]

**(ii)** List the elements in $(P \cup Q)'$.

*Answer* ..... [1]

**(iii)** Complete the Venn diagram showing all the elements of $\xi$.




[1]

**(iv)** Find the value of

**(a)** $\mathrm{n}[(P \cap Q) \cup Q']$

*Answer* ..... [1]

**(b)** $\mathrm{n}[(P \cup Q) \cap (P \cap Q)']$

*Answer* ..... [1]

**(b)** The table shows the number of cars owned by each of 25 families.

| | | | | |
|---|---|---|---|---|
| 1 | 3 | 0 | 2 | 1 |
| 3 | 1 | 6 | 0 | 1 |
| 4 | 2 | 1 | 3 | 2 |
| 0 | 1 | 3 | 2 | 4 |
| 2 | 0 | 2 | 1 | 3 |

**(i)** A family is chosen at random.
Find the probability that it owns 3 cars.

*Answer* ..... [1]

**(ii)** Two families are chosen at random.
Find the probability that one family owns 2 cars and the other own 3 cars.

*Answer* ..... [2]

**(iii)** A car is chosen at random.

Find the probability that it belongs to a family which owns 2 cars.

*Answer* ..... [2]

**9** Air-conditioners are designed to control both the air temperature and humidity. Choosing the right cooling capacity for your air-conditioner is important for proper comfort and energy optimisation.

An oversized air-conditioning system is costly and may cost more to operate. It might also not properly dehumidify the space. This is because the air-conditioner cools the space too quickly and then powers down before the moisture level is reduced. On the other hand, an undersized air-conditioning system will not provide adequate cooling and dehumidification.

To properly size your air conditioning system, a useful rule of thumb is as follows:

> **Cooling capacity required in kW* = Total aircon floor area ($m^2$) ÷ 5**
> *to convert to Btu/hr, multiply the cooling capacity figure in kW by 3,412*

Mr Wong has recently purchased an apartment and would like to instal air-conditioning units to the Main-bedroom, Bedroom, and the Living room. *(Refer to page 21 for the floor plan)*

**(a)** Find the total floor area of the rooms to be air-conditioned in $m^2$.

*Answer* ..... $m^2$ [2]

**(b)** Calculate the required Btu/hr for each of the rooms.

*Answer* Main Bedroom ..... Bedroom ..... Living room ..... [1]

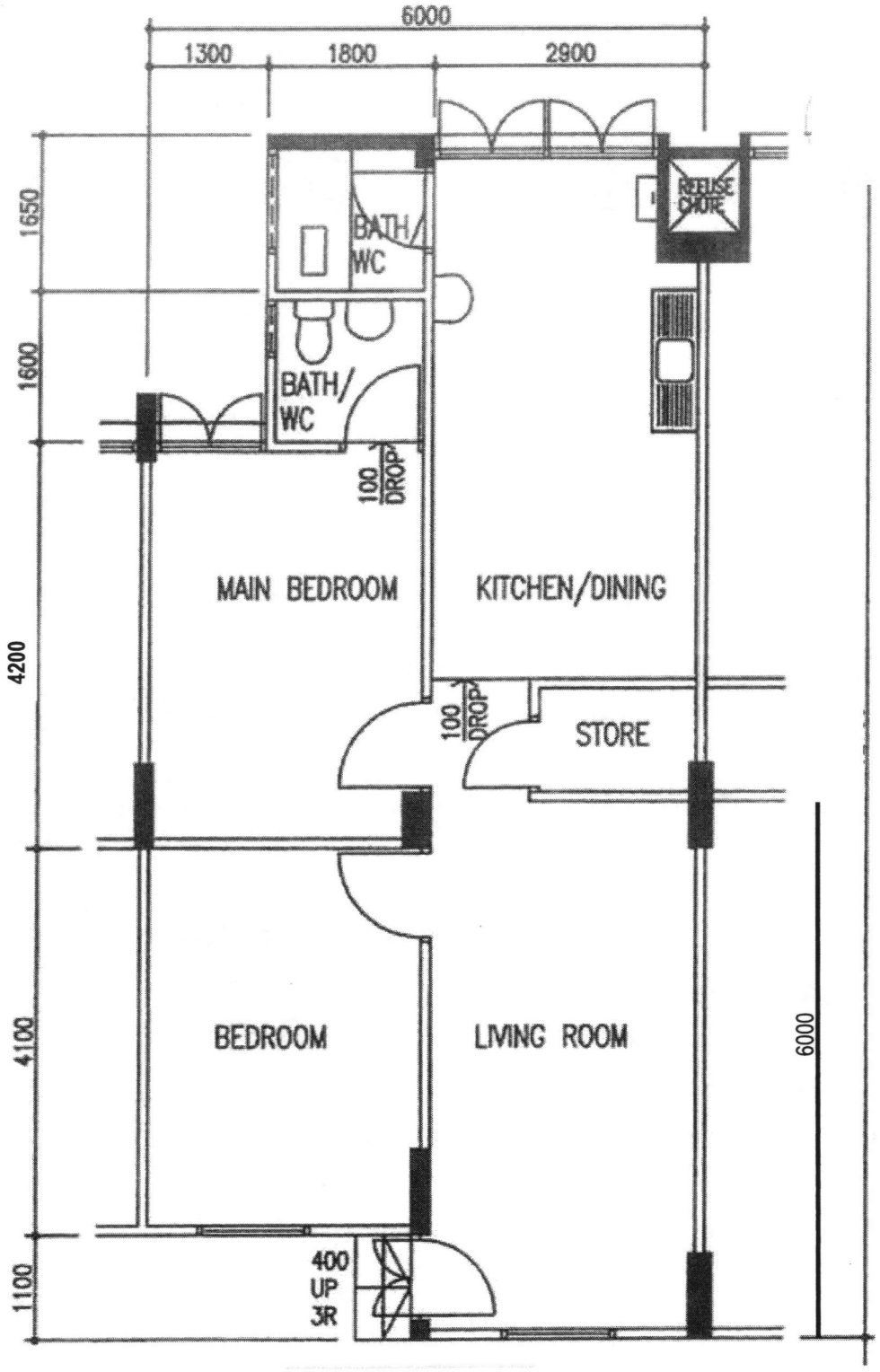
**(c)** Mr Wong's family use the air-conditioning in their bedrooms from 7pm to 6am and the living room from 5pm to 10pm daily. On Saturday and Sundays, Mr Wong will use the living room air-conditioning for an extra 3 hours before 5pm.

**(i)** Determine with explanation which of the packages is suitable for Mr Wong.
*(Refer to page 22)*

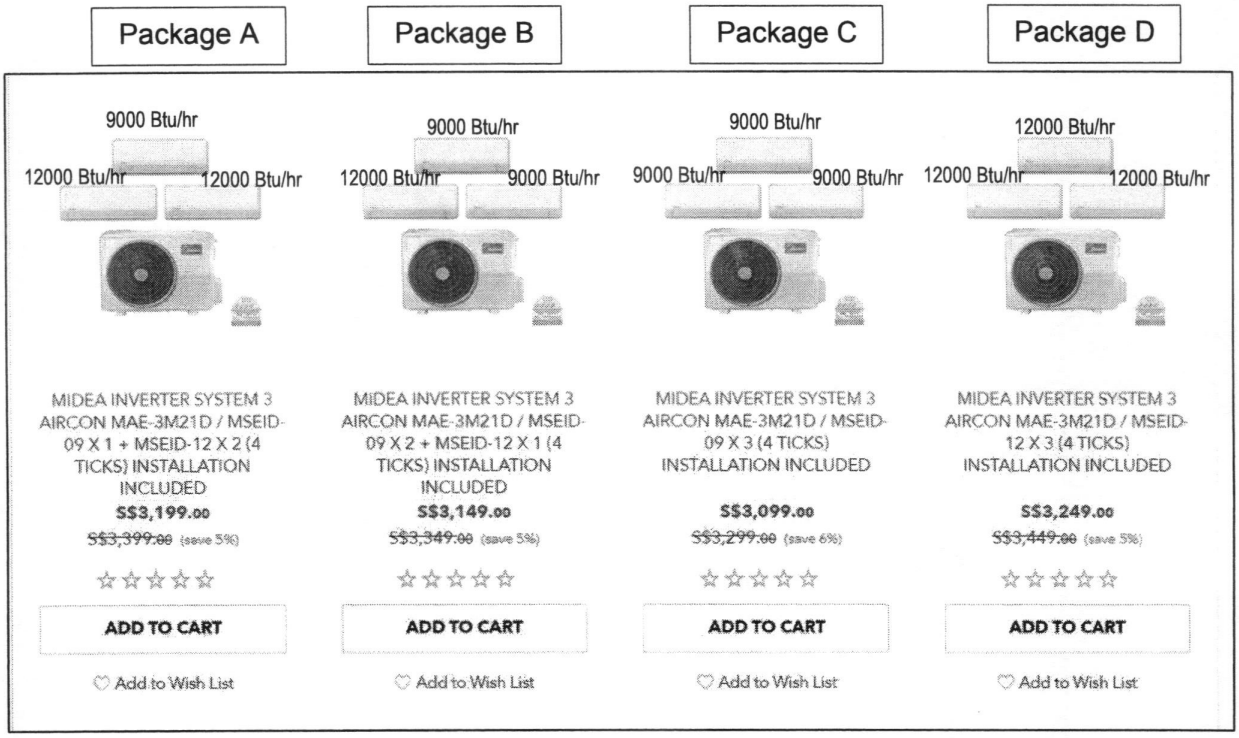
*Answer* Package ..... because ..... [1]

**(ii)** Calculate the cost of electricity usage (inclusive of GST 8%) by Mr Wong's family for air-conditioning for the month of July 2023.

*Answer* $..... [7]



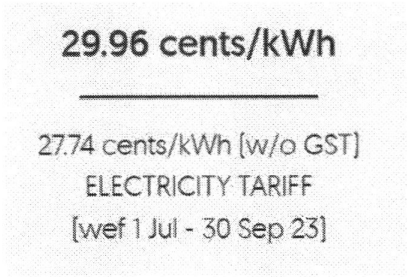

Floorplan of apartment. All measurements are measured in mm.




| Power consumption | 9000 btu/hr unit | 12000 btu/hr | Outdoor unit |
|---|---|---|---|
| | 20W | 24W | 1420W |

Electricity Consumption (kWh) = Power consumption(kW) × time (hours)

1kW = 1000W




**July 2023**

| Sunday | Monday | Tuesday | Wednesday | Thursday | Friday | Saturday |
|---|---|---|---|---|---|---|
| | | | | | | 1 |
| 2 | 3 | 4 | 5 | 6 | 7 | 8 |
| 9 | 10 | 11 | 12 | 13 | 14 | 15 |
| 16 | 17 | 18 | 19 | 20 | 21 | 22 |
| 23 | 24 | 25 | 26 | 27 | 28 | 29 |
| 30 | 31 | | | | | |

End of Paper

**Marking Scheme**

**1.** Calculate $\dfrac{\sqrt[3]{4\pi^2 - 15^2}}{0.63}$.

Leave your answer to 4 decimal places.

*Answer* −9.0530 [1]

---

**2.** Factorise $9x^2 + 12x - 12$.

*Answer* $3(3x-2)(x+2)$ [2]

---

**3.** The sine of an angle is 0.427.

Give two possible values for the angle.

$\sin^{-1} 0.427$

*Answer* 25.3° or 154.7° [2]

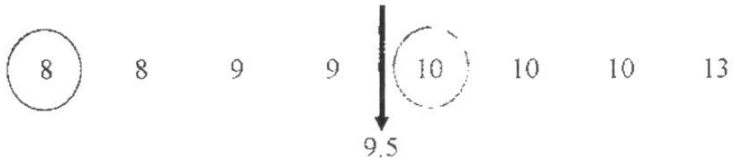
---

**4.** 8 students took a test and their marks were recorded as follows.

8 10 9 13 10 9 $x$ $y$

The modal score for the 8 students is 10 and the median is 9.5.

If the range of the scores is 5, and $x \leq y$, determine the values of $x$ and $y$.




*Answer* $x =$ 8

$y =$ 10 [2]

---

**5.** A sum of money was shared among Allan, Ben and Chloe.

Allan receives $\frac{2}{5}$ of the money and the rest was shared among Ben and Chloe in the ratio 3:2.

Find the ratio of the money received in terms of Allan's : Ben's : Chloe's.

Ben $= \frac{3}{5} \times \frac{3}{5} = \frac{9}{25}$

Charlie $= \frac{2}{5} \times \frac{3}{5} = \frac{6}{25}$

M1 for either

*Answer* 10 : 9 : 6 [2]

---

**6.** John is making two rectangular flower beds.

The dimensions of the larger rectangle will be four times the dimensions of the smaller rectangle.

There is going to be the same depth of soil in each flower bed.

John needs 120 kg of soil for the larger flower bed.

Work out how much soil John needs for the smaller flower bed.

Area ratio = 1 : 16 M1 for correct ratio identified

Soil needed = $\frac{1}{16} \times 120$

*Answer* 7.5 kg **[2]**

---

**7.** $T = \sqrt{\frac{w}{d^3}}$ for a particular value of $w$ and $d$.

If $w$ is increased by 20%, and $d$ is decreased by 10%, determine the percentage change in $T$.

New $w = 1.2w$

New $d = 0.9d$

New $T = \sqrt{\frac{1.2w}{(0.9d)^3}}$ M1

$= 1.28300\sqrt{\frac{w}{d^3}}$ (change from 1 to 1.28300 which is 28.3% change)

*Answer* 28.3 % **[2]**

---

**8.** Jamie is using the quadratic formula to solve a quadratic equation.

She substitutes values into the formula and correctly gets

$$x = \frac{-9 \pm \sqrt{81-48}}{6}$$

Find the quadratic equation that Jamie is solving, giving your answer in the form $ax^2 + bx + c = 0$, where $a$, $b$ and $c$ are integers.

$-b = -9 \Rightarrow b = 9$

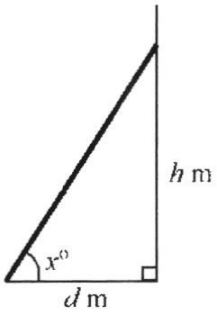
$2a = 6 \Rightarrow a = 3$

$-4(3)c = -48 \Rightarrow c = 4$ M1 for any 1 correct

*Answer* $3x^2 + 9x + 4 = 0$ [2]

---

**9.**




A ladder is leaning against a vertical wall as shown in the diagram.

The bottom of the ladder is $d$ metres from the wall.

The top of the ladder is $h$ metres above the ground.

The angle between the ladder and the ground is $x°$.

**(b)** Some safety instructions say it is safe to climb the ladder when $h = 3.5d$.

Determine the value of $x$ when $h = 3.5d$.

$\tan x = \frac{3.5d}{d}$

$x = \tan^{-1} 3.5$ M1 ($x = 74.0546041\ldots$)

*Answer* 74.1 [2]

**(b)** Some different safety instructions say the angle between the ladder and the ground should be $70°$. The ladder is then moved so that $x = 70°$.

Without further calculations, how does this change in angle affect the height, $h$ metres, of the top of the ladder above the ground?

The height will decrease

[1]

---

10. The table shows pairs of values of $x$ and $y$.

| $x$ | 4 | 5 |
|---|---|---|
| $y$ | 400 | 625 |

**(a)** Which of the following statement is correct?

Statement A: $y \propto x$

Statement B: $y \propto x^2$

Statement C: $y \propto x^3$

A: $\frac{400}{4} \neq \frac{625}{5} \Rightarrow y \neq kx$

B: $\frac{400}{4^2} = \frac{625}{5^2} \Rightarrow y = kx^2$

C: $\frac{400}{4^3} \neq \frac{625}{5^3} \Rightarrow y \neq kx^3$

*Answer* Statement B [1]

**(b)** Write a formula for $y$ in terms of $x$.

$k = 25$ (from part **(a)**) M1 (or B1) for 25

*Answer* $y = 25x^2$ [2]

---

**11.** Simplify.

**(a)** $3+p(3-p)$

*Answer* $-p^2+3p+3$ [1]

**(b)** $9x^{4} \div \frac{1}{2}x^{6}$

$\frac{9}{x^4} \times \frac{2}{x^6}$ M1 (for $\frac{9}{x^4}$)

*Answer* $\frac{18}{x^{10}}$ [2]

---

**12.** Solve the inequalities $x-3<\frac{3}{2}x-2\leq\frac{x+18}{6}$ and state the integer values of $x$ that satisfy them.

$x-3<\frac{3}{2}x-2$ $\quad$ $\frac{3}{2}x-2\leq\frac{x+18}{6}$

$2x-6<3x-4$ $\quad$ $9x-12\leq x+18$

$-2<x$ $\quad$ $8x\leq 30$

$x>-2$ $\quad$ $x\leq\frac{15}{4}$ or $\left(x\leq 3\frac{3}{4}\right)$

$-2<x\leq\frac{15}{4}$

*Answer* $-1, 0, 1, 2, 3$ [3]

---

**13.** A bag contains 12 blue marbles and 8 red marbles.

**(a)** A marble is chosen at random and then replaced.

What is the probability that it is a red marble?

*Answer* $\frac{2}{5}$ Or 0.4 [1]

**(b)** How many red marbles must be placed in the bag so that the probability of choosing a red marble would be $\frac{3}{5}$?

$\frac{8+x}{20+x}=\frac{3}{5}$ M1

$40+5x=60+3x$

$x=10$

*Answer* 10 [2]

---

**14.** Express as $\frac{3}{x-3}-\frac{4}{x^2-9}$ a single fraction in its simplest form.

$\frac{3(x+3)}{(x+3)(x-3)}-\frac{4}{(x+3)(x-3)}$ M1 factorise into $(x+3)(x-3)$

$=\frac{3x+9-4}{(x+3)(x-3)}$ M1

*Answer* $\frac{3x+5}{(x+3)(x-3)}$ [3]

---

**15.** A train 180 m long passes through a tunnel.

The average speed of the train is 42 km/h.

**(a)** Express 42 km/h in m/s.

$$\frac{42}{1}\times\frac{1000}{60\times60}$$

*Answer* 11.7 accept 11.67 m/s [1]

**(b)** The train passes through a 2.4 km tunnel.

Calculate the time for the train to completely pass through the tunnel.

Give your answer in minutes and seconds, to the nearest second.

$\frac{2400+180}{11.666667}$ M1 (for (2400 + 180)/*their* (a))

$= 221.142857...$ sec

$= 3.685714...$ min

$= 3$ min 41.14 sec

*Answer* 3 min 41 sec [2]

---

**16.** $ab^2+c=\frac{2c+b}{a}$

Rearrange the formula to make $c$ the subject.

$a^2b^2+ac=2c+b$ M1 no fraction

$ac-2c=b-a^2b^2$

$c(a-2)=b-a^2b^2$ M1 factorise

*Answer* $c=\frac{b-a^2b^2}{a-2}$ [3]

**17.** **(a)** Factorise completely $3ax+12by-9ay-4bx$.

$3ax-9ay+12by-4bx$

$=3a(x-3y)+4b(3y-x)$ M1 identifying $3a$ and $4b$

$=3a(x-3y)-4b(x-3y)$

*Answer* $(3a-4b)(x-3y)$ or $(4b-3a)(3y-x)$ [2]

**(b)** Show that $(3n+5)^2-7$ is a multiple of 3 for all integer values of $n$.

*Answer*

$9n^2+30n+25-7$

$=9n^2+30n+18$

$=3\times(3n^2+10n+6)$

Hence, $(3n+5)^2-7$ is a multiple of 3 for all integer values of $n$.

[2]

---

**18.** The timings of 16 students running a 100 metre race is recorded below.

| Time ($x$ sec) | $12\le x<13$ | $13\le x<14$ | $14\le x<15$ | $15\le x<16$ |
|---|---|---|---|---|
| Frequency | 3 | 5 | 6 | 2 |

**(a)** Calculate an estimate for

**(i)** the mean running time of the students,

*Answer* 13.9375 accept 13.9 sec [1]

**(ii)** the standard deviation of the running time.

0.93 – no marks

*Answer* 0.933 sec [2]

**(b)** It was found that there was a mechanical error in the stopwatch used to time the run.

An actual 10 seconds is measured as 9 seconds by the stopwatch.

Explain how the mean will change after the adjustment.

The mean time of the runners will drop.

(Do not accept if students go on to say drop by 10 seconds)

..... [1]

---

**28.** Express as a product of prime factors, $168 = 2^3 \times 3 \times 7$ and $A = 2^p \times 3^3 \times 5$.

Using the above information, find

**(a)** the smallest integers $m$ and $n$ such that $\frac{168 \times m}{\sqrt{n}}$ is a perfect square and $m > n$,

*Answer* $m =$ 21

$n =$ 4 [2]

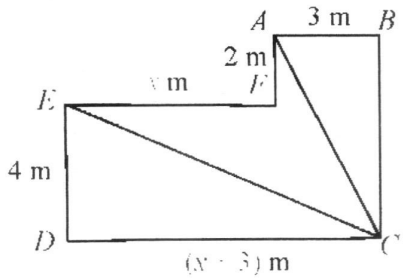
**(b)** the value of $p$ and of $A$ given that the highest common factor of 168 and $A$ is 12.

*Answer* $p =$ 2

$A =$ 540 [2]

---

19.




The diagram shows a shape $ABCDEF$.

All the corners of the shape are right angles.

$ED = 4$ m, $AF = 2$ m and $AB = 3$ m.

The perimeter of the shape is 38 m.

Find the area of the shaded part $ECAF$ of the diagram.

$3+6+(x+3)+4+x+2=38$

$x=10$

$\text{Area} = \frac{1}{2}(10)(4)+\frac{1}{2}(2)(3)$

23

*Answer* ..... $m^2$ [4]

**20.** The points $P$ and $Q$ have coordinates $(7, 2)$ and $(6, -3)$ respectively.

**(a)** Determine if the line $y + 5x = 4$ is parallel to $PQ$.

*Answer*

Gradient $PQ = \dfrac{2-(-3)}{7-6}$

$= 5 \neq -5$ (gradient of line)

Line is **not** parallel to $PQ$

[2]

**(b) (i)** Find the column vector $\overrightarrow{PQ}$.

$\begin{pmatrix} 6 \\ -3 \end{pmatrix} - \begin{pmatrix} 7 \\ 2 \end{pmatrix}$

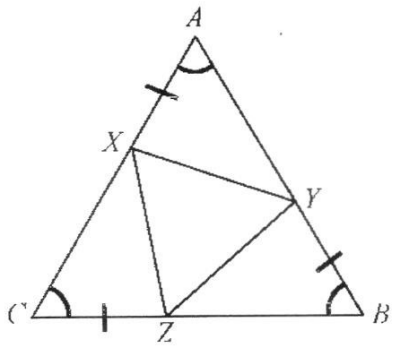
*Answer* $\begin{pmatrix} -1 \\ -5 \end{pmatrix}$ [1]

**(ii)** Find $\left|\overrightarrow{PQ}\right|$.

$\sqrt{(-1)^2 + (-5)^2}$

*Answer* 5.10 [2]

**21.**




In the diagram, triangle $ABC$ is an equilateral triangle.

$X$, $Y$ and $Z$ lie on lines $AC$, $AB$ and $BC$ respectively and $AX = CZ = BY$.

**(a)** Prove that triangle $AXY$ is congruent to triangle $CZX$.

*Answer*

Angle $XAY$ = Angle $ZCX$ (equilateral triangle $ABC$)

$AX = CZ$ (given)

$AY = AB - YB$

$CX = AC - AX$

Since $AB = AC$ and $AX = YB$ (given),

$CX = AY$

$\Delta AXY \equiv \Delta CZX$ (SAS)

[3]

**(b)** Determine if triangle $XYZ$ is an equilateral triangle.

*Answer*

$BZ = BC - CZ = AC - AX = CX$

$\Delta AXY \equiv \Delta CZX \equiv \Delta BYZ$ (SAS)

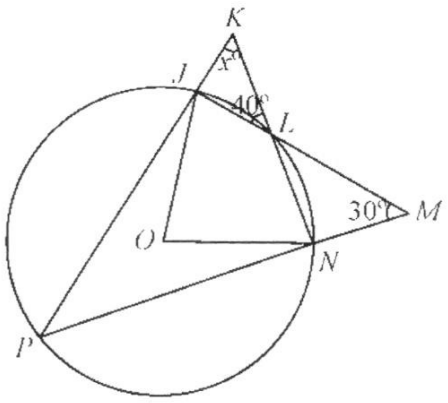
$\therefore XY = XZ = YZ$

$\therefore \Delta XYZ$ is equilateral

Or using angles (sum of angles in triangle)

[2]

**22.**




In the diagram, $O$ is the centre of the circle. $P$, $N$, $L$ and $J$ lie on the circumference of the circle. The lines $PJ$ and $NL$ are extended to meet at $K$. The lines $JL$ and $PN$ are extended to meet at $M$. Angle $JLK$ is $40^\circ$, angle $LMN$ is $30^\circ$ and angle $JKL$ is $x^\circ$.

Write down an expression, in terms of $x$, for

**(a)** **(i)** angle $PJM$,

*Answer* $x+40$ [1]

**(ii)** angle $PNL$.

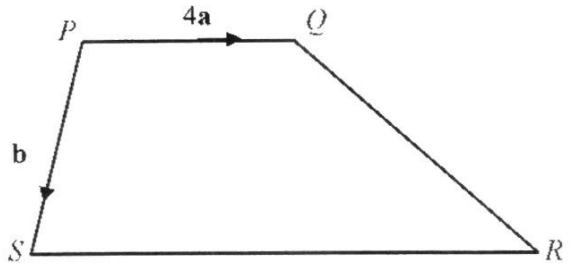
*Answer* $140-x$ [1]

**(b)** By considering triangle $JMP$, form an equation in $x$ and solve it.

$(x+40)+30+40=180$ M1 (angles $PJM+JMP+MPJ$)

*Answer* $x=$ $70$ [2]

23.




In the diagram, $PQRS$ is a trapezium where $\overrightarrow{PQ} = 4\mathbf{a}$ and $\overrightarrow{PS} = \mathbf{b}$.

A point $T$ lies inside the trapezium where $QT : TS = 4 : 5$.

$SR$ is twice the length of $PQ$.

No marks for wrong vector representation

**(a)** Express, as simply as possible, in terms of **a** and/or **b**,

**(i)** $\overrightarrow{QS}$,

$\mathbf{b} - 4\mathbf{a}$

*Answer* ..... [1]

**(ii)** $\overrightarrow{QT}$,

$\frac{4}{9}(\mathbf{b} - 4\mathbf{a})$

*Answer* ..... [1]

**(b)** Determine whether $P$, $T$ and $R$ lie on the same straight line.

*Answer*

$\overrightarrow{TR} = \frac{5}{9}\boldsymbol{b} + \frac{52}{9}\boldsymbol{a}$

$\overrightarrow{PR} = 8\mathbf{a} + \mathbf{b}$

$\overrightarrow{PT} = 4\mathbf{a} + \frac{4}{9}(\mathbf{b} - 4\mathbf{a})$

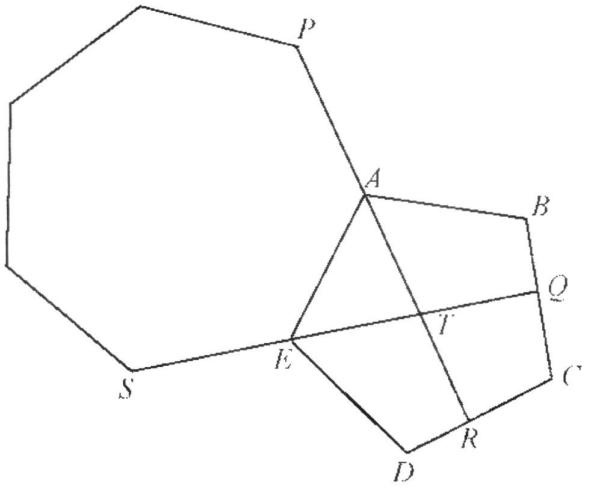
$\overrightarrow{PT} = \frac{20}{9}\mathbf{a} + \frac{4}{9}\mathbf{b}$

$\overrightarrow{PT} = \frac{4}{9}(5\mathbf{a} + \mathbf{b}) \neq k\overrightarrow{PR}$

Therefore, $P$, $T$ and $R$ do **not** lie on the same straight line

[3]

**24.**




In the diagram, a regular heptagon and regular pentagon are fitted together at the edge $AE$.

$PAR$ and $SEQ$ are straight lines.

Find angle $TRD$.

Angle $AET = \frac{360}{7} = 51.428...$

Angle $SEA = 180 - 51.428...7 = 128.571...$

Angle $DEA = \frac{(5-2)\times 180}{5} = 108$ (= angle $EDR$)

Angle $DET = 108 - 51.428... = 56.57142...$

Angle $ETR = 51.428... + 51.428... = 102.857...$

Angle $TRD = 360 - 56.57142... - 102.857.... - 108$

*Answer* $92.5714... = 92.6$ [5]

**25.** The table below shows the distribution of three types of apartments in three blocks of flats.

| | 3-room | 4-room | 5-room |
|---|---|---|---|
| Block 1 | 10 | 20 | 50 |
| Block 2 | 25 | 45 | 10 |
| Block 3 | 30 | 50 | 20 |

The information can be represented by the matrix $\mathbf{T} = \begin{pmatrix} 10 & 20 & 50 \\ 25 & 45 & 10 \\ 30 & 50 & 20 \end{pmatrix}$.

The floor areas of the 3-room, 4-room and 5-room apartments are 60 $m^2$, 90 $m^2$ and 110 $m^2$ respectively and this information can be represented by the matrix $\mathbf{A} = \begin{pmatrix} 60 \\ 90 \\ 110 \end{pmatrix}$

**(a)** Evaluate **TA**.

*Answer* $\begin{pmatrix} 7900 \\ 6650 \\ 8500 \end{pmatrix}$ ..... [2]

**(b)** Explain what the elements in **TA** represent.

Total floor areas in Block 1, Block 2 and Block 3 respectively.

..... [1]

**(c)** Maintenance fees are charged based on the floor area of the apartments in the three blocks of flats. The maintenance fee rates ($/$m^2$) for Block 1, 2 and 3 are \$3, \$4, and \$6 respectively. Write down a 1×3 matrix **C** to show the maintenance fee rates for the three blocks of flats.

*Answer* $\begin{pmatrix} 3 & 4 & 6 \end{pmatrix}$ ..... [1]

**(d)** Hence, using matrix multiplication, find the total estate management fee for these three blocks of flats.

$$\begin{pmatrix} 3 & 4 & 6 \end{pmatrix}\begin{pmatrix} 7900 \\ 6650 \\ 8500 \end{pmatrix}$$

$(101300)$

*Answer* ..... [2]

---

**26. (a)** The first four terms of a sequence are 5, 9, 13 and 17.

The sum of the first $n$ terms of this sequence is given by $an^2 + bn$.

**(i)** When $n = 1$, $a+b=5$.

Show that $4a+2b=14$.

*Answer*

When $n=2$, $a(2)^2+b(2)=5+9$

$4a+2b=14$

[1]

**(ii)** Solve $a+b=5$

$4a+2b=14$.

$4a+4b=20$

$2b=6 \Rightarrow b=3$

$a=2$

*Answer* $a=$ 2

$b=$ 3 [2]

**(b)** The $n$th term of another sequence is $n^2 + 4$.

Anthony says,

"The $n$th term of the sequence is always a prime number when $n$ is an odd number."

Anthony is wrong.

Give an example to show that Anthony is wrong.

$n = 1: (1)^2 + 4 = 5$

$n = 3: (3)^2 + 4 = 13$

$n = 5: (5)^2 + 4 = 29$

$n = 7: (7)^2 + 4 = 53$

$n = 9: (9)^2 + 4 = 85$ (Not prime, hence Anthony is wrong) A1 for one proof

(can be for $n$ =11, etc.)

Or any $n$ that is an odd number that does not give $n$th term a prime number

*Answer* ..... [2]

---

~ End of Paper ~

Mark Scheme Math Prelim P 2 2023

1 **(a)** Solve $\frac{1}{x-1}+\frac{2}{x-2}=\frac{3}{x-3}$.

| Question | Answer |
|---|---|
| 1a | $\frac{3x-4}{x^2-3x+2}=\frac{3}{x-3}$ |
| | $x=1\frac{1}{2}$ |

*Answer* $x=$..... [2]

**(b)** Simplify $\left(a^2b\right)^{\frac{1}{2}}\times\left(ab^{-3}\right)^{\frac{1}{5}}$.

| Question | Answer |
|---|---|
| 1b | $\frac{a^{\frac{1}{2}}}{b^{\frac{1}{2}}}$ |

*Answer* ..... [1]

**(c)** **(i)** Express $x^2+11x-15$ in the form $(x+a)^2+b$.

| Question | Answer |
|---|---|
| 1ci | $\left(x+5\frac{1}{2}\right)^2-45\frac{1}{4}$ |

*Answer* ..... [1]

**(ii)** Hence solve the equation $x^2+11x-15=0$, giving your answers correct to two decimal places.

| Question | Answer |
|---|---|
| 1cii | $\left(x+5\frac{1}{2}\right)^2=45\frac{1}{4}$<br>$x+5\frac{1}{2}=\pm\sqrt{45\frac{1}{4}}$<br>$x$= 1.23 or -12.23 |

*Answer* $x=$..... or ..... [3]



Mark Scheme Math Prelim P 2 2023

**(d)** Simplify $\frac{3a^2+12a-36}{ab-2b+a^2-2a}$.

| Question | Answer |
|---|---|
| 1d | $\frac{3(a+6)(a-2)}{b(a-2)+a(a-2)}$<br><br>$\frac{3a+18}{b+a}$ |

*Answer* ..... [3]

2 (a) Abigale has two possible options to invest a sum of money for 3 years.

Account A pays compound interest 6%, 8% and 10% for the 3 years respectively.

Account B pays compound interest 10%, 8% and 6% for the 3 years respectively.

Explain which, if any, is a better choice for Abigale's investment.

| Question | Answer |
|---|---|
| 2a | Both accounts pays the same compound interest.<br>A: $P\times1.06\times1.08\times1.1$<br>B: $P\times1.1\times1.08\times1.06$ |

.....

.....

..... [1]

(b) An amount of $3750 was placed in an account that pays simple interest.

After 3 years, the balance was $4620.

(i) Find the rate of interest

| Question | Answer |
|---|---|
| 2bi | Interest per yr $= \frac{4620-3750}{3} = \$290$<br>Interest rate $= \frac{290}{3750}\times100\% = 7\frac{11}{15}\%$ |

*Answer* .....% [2]

(ii) If the same interest is paid using compound interest, calculate the balance in the account after the same time period.

| Question | Answer |
|---|---|
| 2bii | $\text{Bal} = 3750\left(1+\frac{7\frac{11}{15}}{100}\right)^3$<br>=46898.01 |

*Answer* $..... [2]



(c) On 1 June 2022, Ben changed S$6200 into American dollars.
The exchange rate between American dollars (US$) and Singapore dollars (S$) was US$1 = S$1.37.

(i) Calculate the amount of American dollars he would receive.

| Question | Answer |
|---|---|
| 2ci | $\frac{6200}{1.37}$<br>=4525.55 |

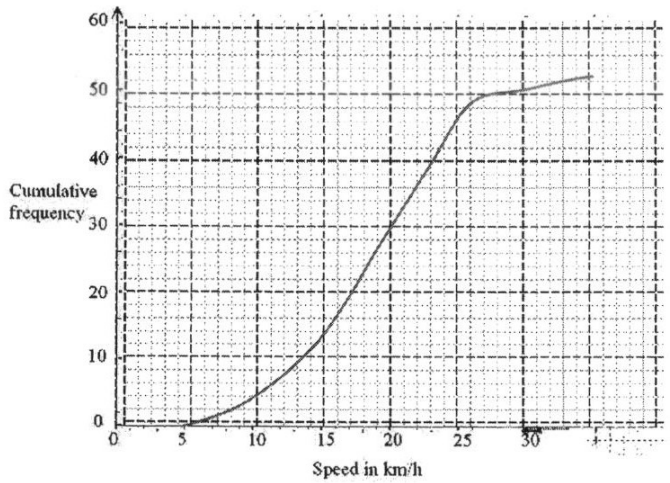
*Answer* $..... [2]

(ii) On a trip to the USA, Ben bought a tablet for US$1899 from a department store.
When Ben bought the tablet, the store made a profit of 8%.
Calculate how much the store paid for the tablet.

| Question | Answer |
|---|---|
| 2ci | $\frac{6200}{1.37}$<br>=4525.55 |

*Answer* $..... [2]

3 The cumulative frequency graph below shows speeds in km/h of cyclists passing a certain point during a race.




(a) Use the curve to estimate

(i) the median speed of the cyclists,

*Answer* 19 km/h [1]

(ii) the percentage of cyclists having speed more than 21 km/h

*Answer* 38.5 % [1]

(ii) the ninety-sixth percentile.

*Answer* 30 km/h [1]

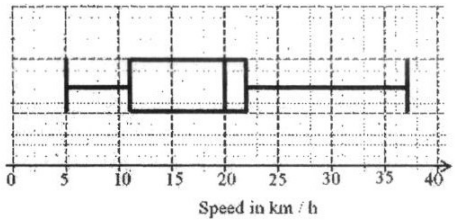
(iii) Complete the grouped frequency table of the speeds of the cyclists passing a certain point during the race.

| Speed ($x$ km/h) | $5 \le x < 10$ | $10 \le x < 15$ | $15 \le x < 20$ | $20 \le x < 25$ | $25 \le x < 30$ | $30 \le x < 35$ |
|---|---|---|---|---|---|---|
| Frequency | 4 | 9 | 16 | 16/17 | 5/4 | 2 |

[2]



(b) In the previous year, the same race had taken place.
The box-and-whisker plot shows the distribution of the speed of the cyclists.




Make two comparisons between the performances of the cyclists in the two years.
Use figures to support your answer.

| | |
|---|---|
| 3b | Median of previous year = 20, higher than median of current bear (19).<br>Better performance in previous year.<br><br>IQR of previous yr=11 which is higher than IQR of current yr (10)<br>Current yr's performance is more consistent |

.....
.....
.....
..... [3]

4 The variables $x$ and $y$ are connected by the equation

$$y=\frac{x^2}{5}+\frac{6}{x}.$$

The table below shows some values of $x$ and the corresponding values of $y$ correct to 1 decimal place.

**(a)** Complete the table.

| $x$ | 1 | 1.5 | 2 | 3 | 4 | 5 | 6 |
|---|---|---|---|---|---|---|---|
| $y$ | 6.2 | 4.5 | 3.8 | 3.8 | 4.7 | 6.2 | 8.2 |

[1]

**(b)** On the grid at the **next page**, draw the graph of $y=\frac{x^2}{5}+\frac{6}{x}$ for $1\le x\le 6$.

[3]

**(c)** Use your graph to find the value(s) of $x$ in the range of $1\le x\le 6$ for which

$$\frac{x^2}{5}+\frac{6}{x}-4=0.$$

| Question | Answer |
|---|---|
| 4c | 1.8 |
| | 3.3 |

*Answer* $x$ = .....[2]

**(d)** By drawing a tangent, find the gradient of the curve at point (4, 4.7).

| Question | Answer |
|---|---|
| 4d | Tangent line drawn |
| | Gradient = 1.225 ± 10% |

*Answer* ..... [2]

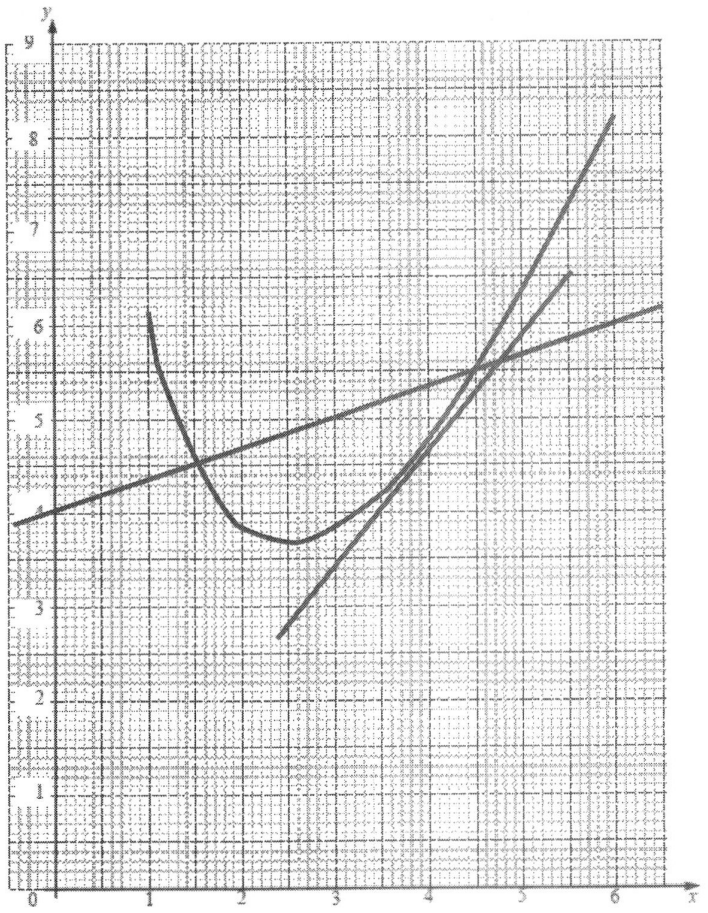
**(e)** By drawing the graph $y=\frac{1}{2}x+4$ on the grid, find the equation in the form

$2x^3+ax^2+bx+c=0$, which is satisfied by the $x$-coordinates of the points at which the two graphs intersect.

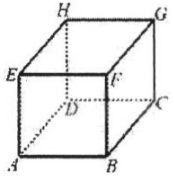
| Question | Answer |
|---|---|
| 4d | line drawn |
| | $\frac{x^2}{5}+\frac{6}{x}=\frac{1}{2}x+4$ |
| | $2x^3-5x^2-40x+60=0$ |

*Answer* ..... [4]





5 The diagram below shows a solid cube $ABCDEFGH$.




(a) The solid cube was produced after recasting a solid spherical toy with diameter 10 cm. Show that the length of a side of the cube is approximately 8.06 cm.

| Question | Answer |
|---|---|
| 5a | Length of cuboid $= \sqrt[3]{\frac{4}{3}\pi(5)^3}$<br>$= 8.0599$<br>$= 8.06$ cm |

*Answer* ..... shown cm [2]

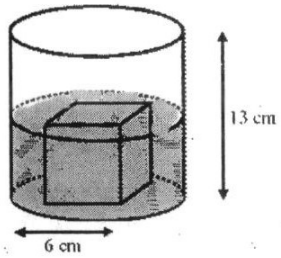
(b) An ant is crawling from the point $A$.
Find the shortest distance that it can take from point $A$ to point $G$.

| Question | Answer |
|---|---|
| 5b | Let $M$ be the midpoint of $EF$.<br>The shortest distance is obtained if the path taken is $AM$, followed by $MG$.<br>shortest distance $= AM + MG$<br>$= 2AM$ (since AM=MG)<br>$= 2\sqrt{AE^2 + EM^2}$<br>$= 2\sqrt{8.0599^2 + \left(\frac{1}{2}(8.0599)\right)^2}$<br>$= 18.0224$<br>$= 18.0$ cm $(3sf)$ |

*Answer* ..... cm [2]



(c) The solid cube is placed inside an open cylindrical container which has base radius 6 cm and height 13 cm.




$V$ cm$^3$ of water is poured into the container at a rate of 2.5 cm$^3$/s until it covers the cube as shown in the diagram above.

(i) Calculate the amount of time, in seconds, it takes for all the water to be poured in.

| Question | Answer |
|---|---|
| 5ci | $V$<br>$= \pi(6)^2(8.0599)$<br>$-\frac{4}{3}\pi(5)^3$<br>$= 387.95$ cm$^3$<br>Time required $= \frac{387.95}{2.5}$<br>$= 155$ s (3s.f.) |

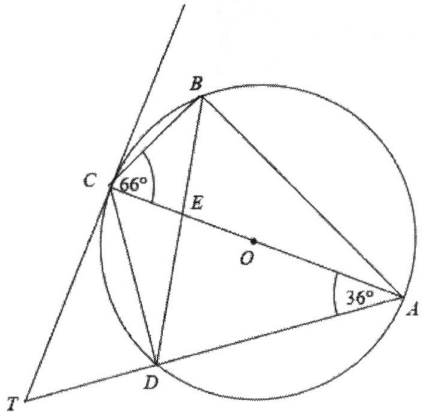
*Answer* ..... s [4]

(ii) When the cube is removed from the container, the water level drops by $x$ cm. Assuming that the water loss is negligible upon the removal of the cube, calculate the value of $x$.

| Question | Answer |
|---|---|
| 5ci | Let the current height be $h$ cm.<br>$387.95 = \pi(6)^2 h$<br>$h = 3.4302$<br>$x = 8.0599 - 3.4302$<br>$= 4.63$ |

*Answer* ..... cm [2]

6 In the diagram, $A$, $B$, $C$ and $D$ are points on the circle with centre $O$.
The lines $AC$ and $BD$ intersect at $E$.
The tangent to the circle at $C$ meets $AD$ produced at $T$.
$\angle ACB = 66°$, $\angle CAD = 36°$ and $AC = 10$ cm.




(a) Find the angle $DCT$.

| Question | Answer |
|---|---|
| 6a | Angle $DCA = 54°$<br><br>$36°$ |

*Answer* ..... [2]

(b) Show that the triangles $BCE$ and $ADE$ are similar.
Give a reason for each statement you make.

| Question | Answer |
|---|---|
| 6b | Angle $BEC$ = Angle $AED$ (vert. opp $\angle$)<br>Angle CBE = Angle DAE ($\angle$ in same seg)<br>triangles $BCE$ and $ADE$ are similar (AA similarity test) |

..... [2]



(c) Calculate $DE$.

| Question | Answer |
|---|---|
| 6c | $DA = 10\cos 36°$<br>$\angle DEA = 78°$<br>$\frac{DE}{\sin 36°} = \frac{10\cos 36°}{\sin 78°}$<br>$DE = 4.861518423$<br>$\approx 4.86$ |

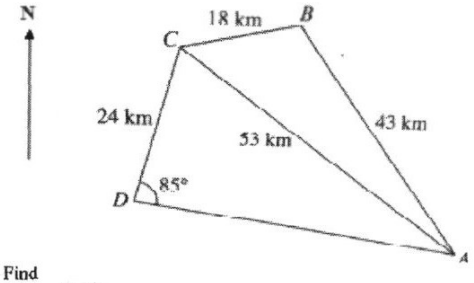
*Answer* ..... cm [4]

(d) Calculate the length of the minor arc $BC$.

| Question | Answer |
|---|---|
| 6d | $\angle BOC = 48°$<br>Arc length $= \frac{48}{360} \times 2\pi \times 5$<br>$= 4.18879$<br>$\approx 4.19$ cm |

*Answer* ..... cm [3]

7 A city tour starting at Hotel $A$ is planned. The coach travels 43 km in the **north-west** direction to $B$, the zoo , where it stops for 3 hours. After the visit to the zoo, the coach continues its journey to a popular seafood restaurant at $C$, 18 km away, for lunch. After lunch, the coach brings the tourists to visit a museum at $D$, 24 km away and $\angle CDA = 85°$.




**(a)** Find

**(i)** $\angle BAC$.

| Question | Answer |
|---|---|
| 7a | $\angle BAC = \cos^{-1}\left(\frac{53^2+43^2-18^2}{2(53)(43)}\right)$<br>$=18.0372 \approx 18.0°$ |

*Answer* ..... [2]

**(ii)** $\angle CAD$.

| Question | Answer |
|---|---|
| 7b | $\angle CAD = \sin^{-1}\left(\frac{24\sin 85°}{53}\right)$<br>$= 26.8°$ |

*Answer* ..... [2]



**(b)** Find the bearing of $C$ from $A$.

| Question | Answer |
|---|---|
| 7b | $360°-45°-18.0°$<br>$= 297.0°$ |

*Answer* ..... [2]

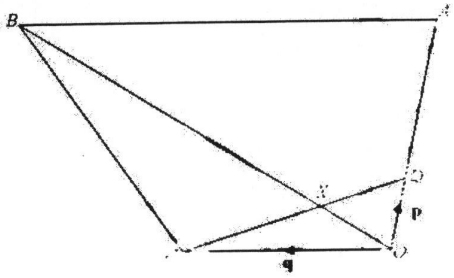
**(c)** Calculate the shortest distance from $D$ to $AC$.

| Question | Answer |
|---|---|
| 7b | Angle $DAC = 68.2°$<br>Shortest dist $= 24\times\sin 68.2°$<br>$=22.2836...$<br>$= 22.3$ |

*Answer* ..... km [3]

**8** **(a)** $OABC$ is a trapezium where $\overrightarrow{OD} = \mathbf{p}$ and $\overrightarrow{OC} = \mathbf{q}$. $OB$ and $CD$ meet at $X$. $AB = 2OC$ and $OD = \frac{1}{3}OA$.

5N ONLY




**(i)** Given that $\overrightarrow{CX} = h\overrightarrow{CD}$, show that $\overrightarrow{OX} = h\mathbf{p} + (1-h)\mathbf{q}$

| Question | Answer |
|---|---|
| 8ai | $\overrightarrow{CD} = -\mathbf{q} + \mathbf{p}$<br>$\overrightarrow{CX} = h(-\mathbf{q} + \mathbf{p}) = -h\mathbf{q} + h\mathbf{p}$<br>$\overrightarrow{OX} = \mathbf{q} + -h\mathbf{q} + h\mathbf{p}$ |

*Answer* shown [3]

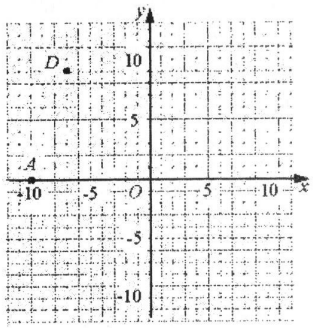
**(ii)** Given further that $\overrightarrow{OX} = k\overrightarrow{OB}$, find the value of $h$ and of $k$.

| Question | Answer |
|---|---|
| 8aii | $\overrightarrow{AB} = 2\mathbf{q}$<br>$\overrightarrow{OB} = 3\mathbf{p} + 2\mathbf{q}$<br>$\overrightarrow{OX} = h\mathbf{p} + (1-h)\mathbf{q} = 3k\mathbf{p} + 2k\mathbf{q}$<br>$k = \frac{1}{5}, \quad h = \frac{3}{5}$ |

*Answer* $h =$ ..... $k =$ ..... [4]



**(b)** The points $A$ and $D$ is shown in the grid below and $\overrightarrow{AB} = \begin{pmatrix} 10 \\ 5 \end{pmatrix}$.




**(i)** Find $\left|\overrightarrow{AB}\right|$.

| Question | Answer |
|---|---|
| 8bi | 11.2 |

*Answer* ..... [1]

**(ii)** Find the equation of the line $BD$.

| Question | Answer |
|---|---|
| 8bii | Gradient BD $= \frac{-4}{7}$<br>$y = -\frac{4}{7}x + 5$ |

*Answer* ..... [2]

**8/9** **(a)** $\xi$ = {integers $x$: $0 < x < 12$}
$P$ = {even numbers}
$Q$ = {prime numbers}

**(i)** List the elements in $P$.

| Question | Answer |
|---|---|
| 9ai | 2, 4, 6, 8, 10 |

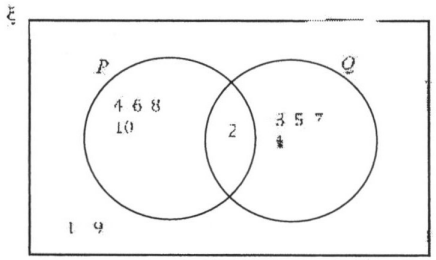
*Answer* ..... [1]

**(ii)** List the elements in $(P \cup Q)'$.

| Question | Answer |
|---|---|
| 9aii | 1, 9 |

*Answer* ..... [1]

**(iii)** Complete the Venn diagram showing all the elements of $\xi$.




[1]

**(iv)** Find the value of

**(a)** $n[(P \cap Q) \cup Q']$

| Question | Answer |
|---|---|
| 9aiva | 7 |

*Answer* ..... [1]

**(b)** $n[(P \cup Q) \cap (P \cap Q)']$

| Question | Answer |
|---|---|
| 9aivb | 8 |

*Answer* ..... [1]



**(b)** The table shows the number of cars owned by each of 25 families.

| | | | | |
|---|---|---|---|---|
| 1 | 3 | 0 | 2 | 1 |
| 3 | 1 | 6 | 0 | 1 |
| 4 | 2 | 1 | 3 | 2 |
| 0 | 1 | 3 | 2 | 4 |
| 2 | 0 | 2 | 1 | 3 |

**(i)** A family is chosen at random.
Find the probability that it owns 3 cars.

| Question | Answer |
|---|---|
| 9bi | $\frac{1}{5}$ |

*Answer* ..... [1]

**(ii)** Two families are chosen at random.
Find the probability that one family owns 2 cars and the other own more than 3 cars.

| Question | Answer |
|---|---|
| 9bii | $\frac{6}{25} \times \frac{5}{24} + \frac{5}{25} \times \frac{6}{24}$ $\frac{1}{10}$ |

*Answer* ..... [2]

**(iii)** A car is chosen at random.

Find the probability that it belongs to a family which owns 2 cars.

| Question | Answer |
|---|---|
| 9bii | $\frac{12}{48} = \frac{1}{4}$ |

*Answer* ..... [2]

**9/10** Air-conditioners are designed to control both the air temperature and humidity. Choosing the right cooling capacity for your air-conditioner is important for proper comfort and energy optimisation.

An oversized air-conditioning system is costly and may cost more to operate. It might also not properly dehumidify the space. This is because the air-conditioner cools the space too quickly and then powers down before the moisture level is reduced. On the other hand, an undersized air-conditioning system will not provide adequate cooling and dehumidification.

To properly size your air conditioning system, a useful rule of thumb is as follows:

**Cooling capacity required in kW* = Total aircon floor area ($m^2$) ÷ 5**
*to convert to Btu/hr, multiply the cooling capacity figure in kW by 3,412*

Mr Wong has recently purchased an apartment and would like to instal air-conditioning units to the Main-bedroom, Bedroom, and the Living room.

**(a)** Find the total floor area of the rooms to be air-conditioned in $m^2$.

| Question | Answer |
|---|---|
| 10a | Main Bedroom + Bedroom +Living Room<br>$=(4.2\times3.1)+(4.1\times3.1)+(6\times2.9)$<br>$= 13.02 + 12.71 + 17.4 = 43.13$ |

*Answer* ..... $m^2$ [2]

**(b)** Calculate the required Btu/hr for each of the rooms.

| Question | Answer |
|---|---|
| 10b | Main Bedroom = 8885<br>Bedroom = 8673<br>Living Room = 1190 |

*Answer* Main Bedroom ..... Bedroom ..... Living room ..... [1]



**(c)** Mr Wong's family use the air-conditioning in their bedrooms from 7pm to 6am and the living room from 5pm to 10pm daily. On Saturday and Sundays, Mr Wong will use the living room air-conditioning for an extra 3 hours before 5pm.

**(i)** Determine with explanation which of the packages is suitable for Mr Wong.

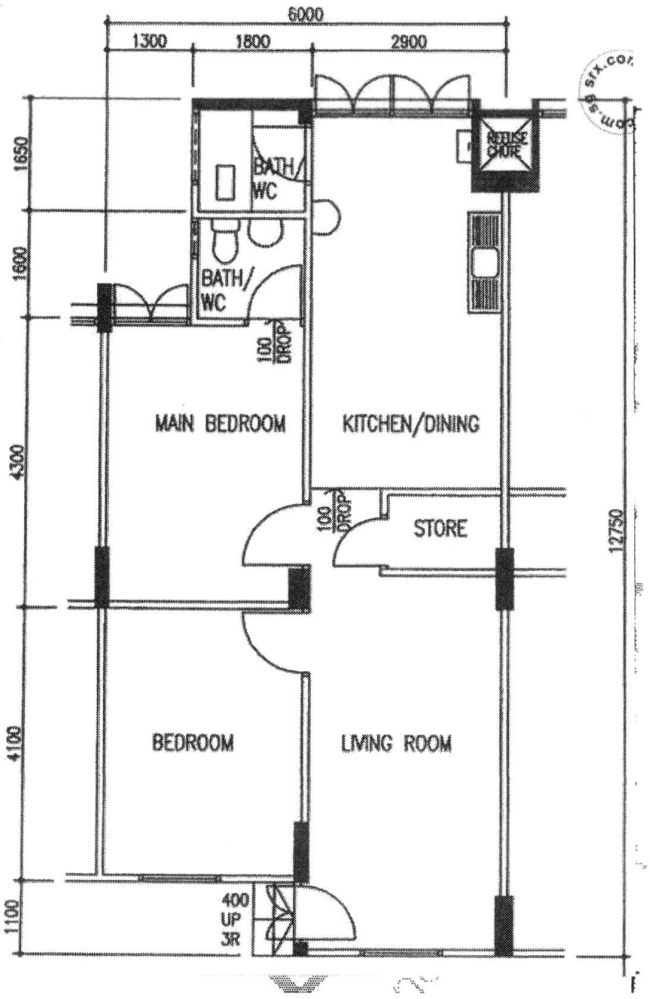
| Question | Answer |
|---|---|
| 10ci | B<br>Living room requires more than 9000 btu/hr |

*Answer* Package ..... because ..... [1]

**(ii)** Calculate the cost of electricity usage (inclusive of GST 8%) by Mr Wong's family for air-conditioning for the month of July 2023.

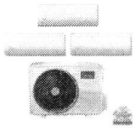
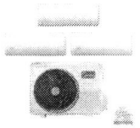
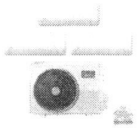
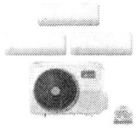
| Question | Answer |
|---|---|
| 10cii | **Bedrooms :**<br>20W × 2units × 11hr × 31 days<br>= 13640Whr = 13.64 kWhr<br><br>**Living Room**<br><br>Everyday : 24W × 5hr × 31 days<br>= 3720 kWhr = 3.72 kWhr<br><br>Weekend : 24W × 3hr × 10 days<br>= 720 kWhr = 0.72 kWhr<br><br>**Outdoor Unit** :<br><br>Everyday : 1420W × 13hr × 31 days<br>= 572260 kWhr = 572.26 kWhr<br><br>Weekend : 1420W × 3hr × 10 days<br>= 42600 kWhr = 42.6 kWhr<br><br>Total = 632.94 kWhr<br><br>Cost = 632.94 kWhr × 29.96 cents<br>= $ 189.628824<br>≈ $ 189.63 |

*Answer* $..... [7]




**Floorplan of apartment. All measurements are measured in mm.**



| Package A | Package B | Package C | Package D |
|---|---|---|---|
|  |  |  |  |
| MIDEA INVERTER SYSTEM 3 AIRCON MAE-3M21D / MSEID-09 X 1 + MSEID-12 X 2 (4 TICKS) INSTALLATION INCLUDED | MIDEA INVERTER SYSTEM 3 AIRCON MAE-3M21D / MSEID-09 X 2 + MSEID-12 X 1 (4 TICKS) INSTALLATION INCLUDED | MIDEA INVERTER SYSTEM 3 AIRCON MAE-3M21D / MSEID-09 X 3 (4 TICKS) INSTALLATION INCLUDED | MIDEA INVERTER SYSTEM 3 AIRCON MAE-3M21D / MSEID-12 X 3 (4 TICKS) INSTALLATION INCLUDED |
| S$2,199.00 ~~S$2,399.00~~ (save 5%) | S$2,149.00 ~~S$2,349.00~~ (save 2%) | S$2,099.00 ~~S$2,299.00~~ (save 9%) | S$3,249.00 ~~S$3,449.00~~ (save 5%) |
| ☆☆☆☆☆ | ☆☆☆☆☆ | ☆☆☆☆☆ | ☆☆☆☆☆ |
| ADD TO CART | ADD TO CART | ADD TO CART | ADD TO CART |
| Add to Wish List | Add to Wish List | Add to Wish List | Add to Wish List |

| Power consumption | 9000 btu/hr unit | 12000 btu/hr | Outdoor unit |
|---|---|---|---|
| | 20W | 24W | 1420W |

Electricity Consumption (kWh) = Power consumption(kW) × time (hours)

1kW = 1000W

29.96 cents/kWh

27.74 cents/kWh (w/o GST)
ELECTRICITY TARIFF
(wef 1 Jul - 30 Sep 23)

**July 2023**

| Sunday | Monday | Tuesday | Wednesday | Thursday | Friday | Saturday |
|---|---|---|---|---|---|---|
| | | | | | | 1 |
| 2 | 3 | 4 | 5 | 6 | 7 | 8 |
| 9 | 10 | 11 | 12 | 13 | 14 | 15 |
| 16 | 17 | 18 | 19 | 20 | 21 | 22 |
| 23 | 24 | 25 | 26 | 27 | 28 | 29 |
| 30 | 31 | | | | | |

End of Paper