Mathematics

Lecture 3: Discovered or Created

Math's effectiveness in explaining Nature

Overview

- Discovered or Created? Real or Non-real? Objective or Subjective?
- Math and the Natural World
- Why is Math so Effective? The Options
- Math and other fields

Discovered or Created? Real or Non-real?

- One of the key questions in any Area of Exploration (Math, Science, Social Science, Ethics, Aesthetics)
- General definition of Realism:
  "*a*, *b*, and *c* and so on exist, and the fact that they exist and have properties such as *F-ness*, *G-ness*, and *H-ness* is (apart from mundane empirical dependencies of the sort sometimes encountered in everyday life) independent of anyone's beliefs, linguistic practices, conceptual schemes, and so on." (SAP, "Realism")
- There are thus 2 general aspects to realism:
- 1) Existence – e.g. tables, rocks, the moon etc all exist, as well as facts about the table (it is square), rock (it being made of granite) and the moon (spherical)
- 2) Independence – e.g. the fact that the moon exists and is spherical is independent of anything anyone happens to say or think about the matter, as with the table being square
- E.g. A realist about the existence of tables is someone who believes that tables **exist independent** of human minds such that it has its properties regardless of what people think or say, i.e. **objective**
- On the other hand, a non-realist denies one or both dimensions of Realism.
- Non-realism can take many forms and anti-realism is a subset of non-realism

Realism and Non-realism about Math

- A realist about Math is someone who believes that mathematical objects exist independently of human minds and has its properties regardless of what people think or say
- A realist about Math thus thinks that Math is to be **discovered**, i.e. math is **objective**
- 2 main schools: Platonism (including Logicism) and Empiricism

- An **anti-realist** is someone who concedes the existence aspect of realism but denies the independence aspect, i.e. math is **created** and **subjective**
- So an intuitionist believes that mathematical objects do exist but only because they are **constructed** by the human mind and that is why it has the properties it has
- A **nominalist** (e.g. formalism) on the other hand is someone who denies the existence dimension; mathematical objects are mere vocal utterances

Discovered/Objective vs Created/Subjective

- So the question of Discovered/Real vs Constructed/Non-real is also a question of Objective vs Subjective
- Of course, there is a third possible option: Kantian view on Math, i.e. intuitionism
- Here, Math is objective not because of the independence dimension but because it is **universally** created due to a shared mental apparatus, i.e. Filters of Consciousness
- There is thus **no biasedness** simply because it is 'universally' biased
- Note: this notion of "Universality" for intuitionism/Kantian mathematics merely applies to *human* minds, NOT all minds
- In other words, it is not strictly universal in the traditional sense like what Platonism would hold
- So it is entirely possible that alien minds would construct math differently from us due to different mental apparatus

Do mathematical objects exist in the Natural World?

- Answer: No. An actual, i.e. mathematically correct, circle doesn't exist in the physical world. Coins, wheels, coin prata etc. are only *approximations* of actual circles.
- No matter how good a drawing you have, even by a computer, it is not a mathematically correct circle – or any other geometrical object for that matter.
- At some level of magnification, you can see that the 'circle' drawn by the computer is just a series of pixels that lie next to each other.
- Even something as simple as a line, defined as that which has length *but no breadth*, is impossible to draw correctly.
- In other words, mathematical objects do not exist in the natural world. Rather, they seem to exist in another realm (or have no existence whatsoever)
  - Platonists and Logicists: Platonic heavens (Discovery)
  - Intuitionists: in our minds (Construction).
  - Formalists: such mathematical objects have no existence per se; they are merely the rules of a manmade game.

Math and the Natural World

- Yet Math seems to be really effective at explaining the workings of the natural world and it has real practical uses! Here are some examples:
- i) Ellipses – 3rd C BC development by the Greeks. Had no practical purpose, was pursued purely for intellectual interest at that time BUT was put to use by Johannes Kepler to explain planetary motion in the 17th C AD! (Unreasonable Effectiveness, 6)
- ii) Calculus is also used to explain planetary motion
- iii) Euclidean geometry – used for construction and navigation
- iv) Riemannian geometry – developed by Riemann as a purely intellectual exercise but 30 years later, Einstein concluded that space conforms to Riemannian rather than Euclidean geometry
- v) The advancement of physics beyond classical mechanics to cover electricity, magnetism, sound and light waves were all made possible by corresponding developments in the theory of ordinary and partial differential equations.
- vi) And of course, so much of physics is captured in mathematical equations! E.g. $f=ma$, $p=iv$, $f=ke$, $e=mc^2$
- After all, if physics is the study of the laws of nature (LON) and LON are simply conditional statements of how the world works, to predict what is going to happen in the future, it is extremely helpful if these LON are formulated in mathematical language – **you get a level of precision that would not have been possible otherwise**.

A side note…

- Note 1: actually, these mathematical models merely *approximate* physical reality.
- E.g.: Planets don't move in perfect ellipses, the land is not perfectly flat like in Euclidean planar geometry
- Nonetheless, Newton's law of gravity was proved accurate to **less than a ten thousandth of a per cent** (Unreasonable effectiveness, 6)

- Note 2: the relationship between Physics and Math is intricate and mutual. It is symbiotic. Not only does math contribute to physics, physics does too.
- There are instances of developments in physics motivating new areas and results in math such as **symmetries in physics spurring the growth of group theory, brownian motion leading to functional integrals or the use of non-abelian gauge theory combined with supersymmetry in particle physics setting the stage for important work in modern math.** (Math and the Real world (364)

Math and the Natural World

- The question is “WHY?”
- Why is Math, an abstract, mental and intellectual exercise, with its own internalised and idealised logic that depends **not** on the natural world, so applicable to the real, i.e. physical and natural world?
- Or why is Math so “unreasonably effective” in its explanations of and application to the natural world?
- “How is it possible that mathematics, a product of human thought that is independent of experience, fits so excellently the objects of physical reality?” – Einstein
- How does it happen that a subject like mathematics, seemingly constructed and policed entirely by the ‘inner world’ of human minds, ends up being such a successful tool in describing and indeed harnessing the external physical world? Is it that the physical world has some intrinsic ‘mathematical order’, which then instilled in human brains the basic concepts of mathematics and logic through the evolutionary process? In other words, *did the human mind learn about mathematics from the external world rather than the other way around* [i.e. instead of trying to learn about the external world *from* mathematics]?

  R. Rajaraman, emphasis his
  Mathematics and the real world, 361

Why is math so effective?

- Several options are open to us:
- 1) The natural world itself is mathematical and we discover math through it, i.e. empiricism
- 2) The natural world is a ‘copy’ of the mental world where math resides as one of the Forms, i.e. platonism and logicism
- 3) The natural world is neither mathematical nor a copy of some mental world but simply *appears* to us to be mathematical as we **impose** our mathematical understanding onto the natural world, i.e. intuitionism
- 4) The natural world appears to us to be mathematical only because we study those aspects that are amenable to math, not because the world is actually mathematical; it is a happy coincidence (formalism?)

Option 1: the Natural world is mathematical (Discovered)

- i.e. Empiricism
- Nature seems to be mathematical – it has a curious preference for particular numbers and for spiral geometries (cf. Article F)
  - No. of petals in a flower follows the Fibonacci sequence (1,2,3,5,8,13,21…)
  - Pineapples have 8 rows of scales to the left and 13 to the right
  - Sunflower seeds are packed in 2 families of spirals (clockwise and counter) and the number of seeds for each series follows the Fibonacci numbers
  - More importantly, if they are packed at the golden angle (phi), then you get the most number of seeds, i.e. it's the most efficient arrangement of seeds
  - How plants branch also seem to follow a regular pattern – fractals, which are geometric shapes that are based on the Fibonacci sequence (e.g. the use of fractals allow us to draw realistic looking trees)
  - Underwater mystery circles made by puffer fish (article)
- Laws of Nature are also formulated in mathematical language.
  - Newton's Law of Universal Gravitation: $F = \frac{GM_1M_2}{d^2}$ where F= force due to gravity, m1= mass of 1st object, m2= mass of 2nd object, d=distance between their centers, G= gravitational Force Constant

Option 1: the Natural world is mathematical (Discovered)

- This position explains why a seemingly purely intellectual exercise of thought, i.e. math, is so applicable to physical reality
- This applicability lies in the fact that some of our math (elementary math) was **suggested** to us by physical reality. In other words, we developed math to deal with the natural world
- We **then** proceeded to play the 'game' of math, using our mental powers and producing abstract math like complex numbers, linear operators and Borel sets (advanced, 'pure' math)
  - E.g.: Euclidean geometry was first developed in response to practical problems (how to navigate, how to construct homes and other structures like pyramids etc) as early as the ancient Egyptians 5,000 years ago. It was only much later that Euclid formalised this into a system. Hence, not surprising that Euclidean geometry proves useful in describing physical reality.
  - E.g. Calculus was developed precisely to explain planetary motion

Option 1: the Natural world is mathematical (Discovered)

- This position also explains why comparing across different cultures, we have:
- **Differences** in math systems (e.g. notation system different for Roman and Arabic numbers, different base numbers, different placeholder systems) that is due to the different environments that each culture had (e.g. different medium of writing thus explaining different notation system)
- WHILE also explaining the great **similarities** across different cultures – since the LON hold true for all of us in this physical world (so far and only contingently so), no surprise that we discover the same kind of math
- BUT! This kind of similarity is a far cry from the Universality and Necessity that we typically associate with Math.
- So Empiricism accounts for the utility of Math at the expense of its certainty

Option 2: the Natural World as a 'copy' of the Forms (Discovered)

- i.e. the Platonist/Logicist view of Math
- Explains why across all cultures, there is great similarity between the different mathematical systems (place-value notation system, creation of abstract symbols to represent any physical objects, i.e. numbers, use of Round numbers for approximation)
- This position explains why the natural world seems to only **approximate** mathematical idealisations – because it is a physical and imperfect copy of the mental forms that exist in the Platonic Heavens
- But, as before, Platonism is unable to explain for how we discover math if math entities are truly non-causal and non-spatiotemporal.
- Also, if the natural world is an imperfect copy of the Platonic heavens, then why is it that Math is so effective in its explanations of and application to the natural world?
- E.g.: Newton's law of gravity was proved accurate to less than a ten thousandth of a per cent (Unreasonable effectiveness, 6)

Option 3: the Phenomenal World as mathematical (Created)

- i.e. the Intuitionist view of Math
- We have this primordial intuition of math/ mathematical filter that is universal to all humankind, i.e. we view the world through a mathematical lens.
- So our resulting math knowledge, even though it might seem totally irrelevant (because it is so abstract and not readily suggested to us by nature), is actually **generated from our experience of the world**.
- In other words, even abstract and 'pure' concepts like complex numbers are a result of our experience of the world through our mathematical lens.
- It is thus unsurprising that these concepts should prove to be relevant to physics (which is a study of the natural world) as this natural world is only ever revealed to us through our various filters of consciousness which includes a mathematical filter.
- Same too for the application of pi in population studies, differentiation in economics etc.

Option 4: Math is a study of certain aspects of the Natural World (formalism?)

- i.e. the natural world is not necessarily orderly or mathematical but the human mind chooses to study those aspects of the world that are amenable to math (Math and the real world, 362)
- In other words, options 1 through 3 are all wrong; we do not impose our math onto the world nor is the world mathematical nor a copy of a mathematical system.
- **At best,** only parts of the world is mathematical and the reason why our math seems so applicable to the natural world is that we **only** study those parts of the world while disregarding the other parts.
- At worst, it is merely **a happy coincidence**.
- Explains why there are aspects of human study (like the soft sciences, theology and humanities) that are not amenable to mathematical manipulation.
- And yet, there is truly so much of the natural world that is amenable to being formulated in mathematical language.
- If formalism is right, no good reason that math should be so effective and yet it is!

Math in other fields

- Math is now used beyond physics, in chemistry, biology and even the social sciences
  - E.g. statistical analysis of quantitative empirical data to substantiate and augment theories.
- Even for the 'softer' areas of social science like Grounded Theory in Psychology which, while using interviews, also utilise statistics to study the data by codifying the varied responses of interviewees.
- Advantage: promotes precision of thought that would not have been possible because Math forces one to think of the patterns inherent in relationships between variables.
  - E.g. F=ma (physics), the gradient of the MR curve is twice that of the AR curve (econs), statistical analysis (psychology)

Math IN other fields

- But!
- Such an approach of using math more and more, especially when used without discretion, can give the impression that a piece of work that is heavy with mathematical equations is more useful and/or more precise/rigorous than one that is not – and this isn't always the case!
- E.g. Human beings are unique individuals and while there are indeed similarities across the entire species (hence why we can even differentiate between homo sapiens and homo erectus), there are also unique qualities that defy categorisation and codification, and thus the ability of math to indicate relations and patterns between variables.
- To *force fit* such pieces of data into such categories would be to **misrepresent** the subject of the study.

Math VS other fields

- Regardless, of all the subjects that humankind has studied, there is no question that Math is the most certain (unless Empiricism is right…)
- This is due of course to the various characteristics of Math: A Priori, Necessary, Deductive, Universal (which Platonism and Intuitionism can account for but not so the Formalists)
- Conversely, the next most certain discipline, Science, is widely seen to be far less certain because of the nature and construction of its knowledge claims: Empirical, Inductive, Contingent
- The Social Sciences, being a study of human beings, i.e. beings with consciousness which are not amenable to neat categorisations like the natural world, are even less certain, i.e. there are a lot more exceptions in SS than in Science. This also means that any use of math in SS is a lot less than in Science (and less warranted too?).

Math VS other fields

- But what of other disciplines that do not (readily) admit of the empirical method (like SS and NS)? Like Ethics? Or the Humanities like Literature, Philosophy and Theology?
- Does the lack of empirical matter mean that one cannot use Math in these disciplines?
- If so, does this necessarily mean that these disciplines are inevitably less certain?
- E.g.: Descartes' Ontological Argument:
- 1. I have an idea of supremely perfect being, i.e. a being having all perfections.
- 2. Necessary existence is a perfection.
- 3. Therefore, a supremely perfect being exists (1,2)

Math vs Science

- Which is more fundamental to the study of the natural (or even human) world? Science or Math?
- In other words, can we do Science without Math?
- Seems possible!
- While formulating the various laws of nature in math does lend a degree of precision and rigour to physics (or any other science, even the social sciences), removing Math from the equation (hurhur) doesn't mean that Science cannot be done.
- After all, what *is* intrinsic and necessary to Science is not the language of Math that is used to represent the theory; it is the Scientific Method itself (i.e. the hypothesis, experimentation etc).
- E.g. Newton's 2nd law of motion can be formulated as F=ma or "the alteration of motion is ever proportional to the motive force impress'd; and is made in the direction of the right line in which that force is impress'd"
- Hartry Field (founder of Fictionalism) was apparently able to give a complete axiomatisation of Newton's mechanics without reference to numbers and functions at all, i.e. a science without math.

Homework

- Articles H (Math and the Natural World) and I (Unreasonable Effectiveness)