1 Measurement

Main concept(s)

1. Physical quantities & SI units
2. Uses of SI base units
3. Estimation
4. Errors and Uncertainties
5. Vectors

Learning Outcome(s)

Candidates should be able to:

(a) recall the following base quantities and their units: mass (kg), length (m), time (s), current (A), temperature (K), amount of substance (mol).
(b) state that one mole of any substance contains 6.02×10^{23} particles and use the Avogadro number NA = 6.02 × 1023 mol^{-1}
(c) express derived units as products or quotients of the base units and use the named units listed in 'Summary of Key Quantities, Symbols and Units' as appropriate.
(d) use SI base units to check homogeneity of physical equations.
(e) show an understanding of and use the conventions for labelling graph axes and table columns as set out in the ASE publication Signs, Symbols and Systematics (The ASE Companion to 16–19 Science, 2000)
(f) use the following prefixes and their symbols to indicate decimal sub-multiples or multiples of both base and derived units: pico (p), nano (n), micro (μ), milli (m), centi (c), deci (d), kilo (k), mega (M), giga (G), tera (T).
(g) make reasonable estimates of physical quantities included within the syllabus.
(h) distinguish between scalar and vector quantities, and give examples of each
(i) add and subtract coplanar vectors
(j) represent a vector as two perpendicular components
(k) show an understanding of the distinction between systematic errors (including zero errors) and random errors.
(l) show an understanding of the distinction between precision and accuracy.
(m) assess the uncertainty in a derived quantity by simple addition of actual, fractional or percentage uncertainties or by simple numerical substitution (a rigorous statistical treatment is not required).

Introduction

Physics is an experimental science in which measurements are made. Evidence from measurements is the ultimate authority in establishing physics theories. Precise measurements enable the checking of experimental data against theoretical prediction. Accurate measurements allow us to overcome the unreliability of our human senses and for the purpose of standardisation. Precise measurements made possible many applications e.g. study of the structure of matter, from geophysics to astronomy, in the medical industry where development of measuring instruments provides data that are crucial in informing clinical decisions. In this chapter, you will not only learn concepts associated with measurements, but also vectors to represent physics quantities, and enable us to compute quantities and solve problems.

Essential Questions

- Why is there a need for standards in measurements and how are the standards of measurements defined or established?
- Why are uncertainties inherent in all measurements and how can uncertainties be reduced?
- Why do we need to know the uncertainty of a measurement?
- How is the skill of making approximations of particular physical quantities useful in science?

1.1 QUANTITIES AND UNITS

1.1.1 Physical Quantities

- Quantities which can be measured.
- Examples: length, density, time, force, pressure, velocity, energy, temperature, magnetic field strength, wavelength, frequency, etc.
- Stated with a numerical value, and, its unit. For example, mass of man is 65 kg.
- The standard system of measure adopted in Physics is the *International System of Units (*SI). [Be familiar with the 'Summary of Key Quantities, Symbols and Units' listed in the Appendix at the end of this Module Notes.]

1.1.2 Base Quantities & Base Units

- A set of quantities chosen by scientists, which, by convention, cannot be defined in terms of any other base quantities.
- These are the quantities by which all other physical quantities are defined.
- There are seven base quantities:

Base quantity	SI Base unit	Symbol
Length	metre	m
Mass	kilogram	kg
Time	second	s
Temperature	kelvin	K
Electric current	ampere	A

Amount of substance	mole	mol
Luminous intensity	candela	cd

Quiz
Spot the odd one out:

kilogram | mass | length | time | temperature

1.1.3 The mole & Avogadro's Number

One mole is defined as the amount of substance that contains as many elementary particles as there are atoms in 0.012 kg (12 g) of carbon-12 isotope.

The Avogadro constant N_A is the number of atoms in 0.012 kg of carbon-12 isotope, and has been determined to have a value of 6.02×10^{23} mol^{-1}.

Hence, one mole of a substance contains 6.02×10^{23} particles.

The mass of 1 mole of substance is known as the **molar mass** of the substance expressed in g mol^{-1}.

Since there are N_A particles in 1 mole of any element, the mass of an atom for a given element is

$$m_{atom} = \frac{\text{molar mass}}{N_A}$$

As such, *molar* quantities refer to physical quantities associated with 1 mole of a substance. As an example, the molar volume of CO_2 means the volume occupied by 1 mole of CO_2 and so on.

Example 1
The Hope Diamond and the Rosser Reeves Ruby
The Hope diamond is almost pure carbon which has a mass of 44.5 carats. The Rosser Reeves ruby is primarily aluminium oxide (Al_2O_3) which has a mass of 138 carats.

One carat is equal to a mass of 0.200 g.

Determine
(i) the number of Al_2O_3 molecules in the ruby, and
(ii) the number of moles of carbon atoms in the diamond.

Solution:
(i)
Mass of the Rosser Reeves ruby = 138 × 0.2 = 27.6 g
Molar mass of Al_2O_3 molecules = 2 × 27 + 3 ×16 = 102 g mol^{-1}
No. of moles of aluminium oxide, $n = \frac{\text{mass}}{\text{molar mass}} = \frac{27.6}{102} = 0.271$ mol
No. of aluminium oxide molecules in the Rosser Reeves ruby, N = nN_A
= 0.271 × 6.02 × 10^{23}
= 1.63 × 10^{23} molecules

(ii)***Try it yourself!***
Mass of the Hope diamond = 44.5 x 0.200 g = 8.90 g
Molar mass of carbon atoms = 12 g mol^{-1}
No. of moles of carbon atoms, $n = \frac{\text{mass of sample}}{\text{molar mass}}$ = 0.742 mol

No. of carbon atoms in the Hope diamond = N
= nN_A
= (0.742) x (6.02 x 10^{23})
= 4.47 × 10^{23} atoms

1.1.4 Derived Quantities & Derived Units

- Derived quantities are all other physical quantities other than the base quantities.
- Derived units are obtained by using the defining equation of that derived quantity.
- Derived units are called SI units if they are constructed from the 7 base units.
- In calculations, square brackets, "[ ]", denote "units of".

Example 2

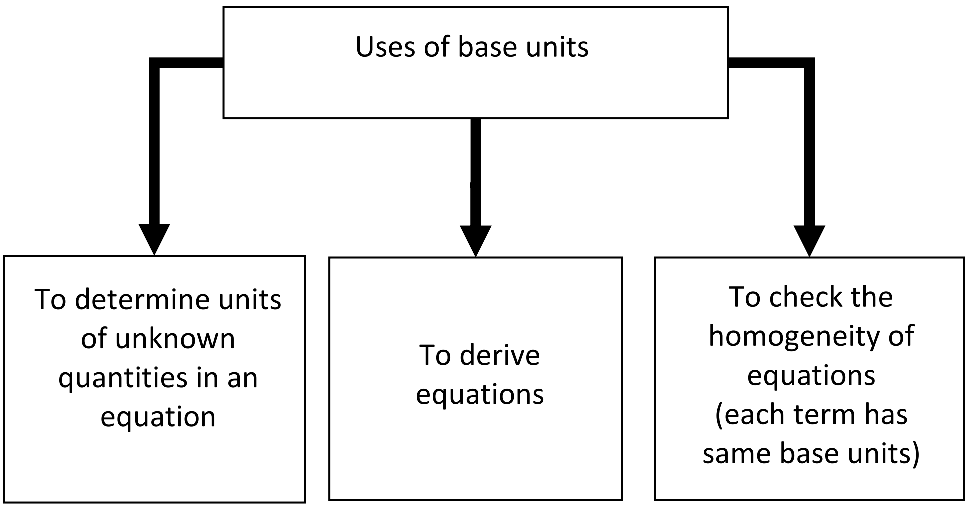
Derived quantity	Defining equation	How the base units are found	SI units	SI base units
Velocity	Velocity = displacement moved / time v = s / t	[v] = [s] / [t] = m / s = m s^{-1}	m s^{-1}	m s^{-1}
Density	Density = mass / volume ρ = m / V	[ρ] = [m] / [V] = kg / m^3 = kg m^{-3}	kg m^{-3}	kg m^{-3}
Force	Force = mass x acceleration F = ma	[F] = [m] x [a] = kg x m s^{-2} = kg m s^{-2}	N (newton) or kg m s^{-2}	kg m s^{-2}
Work	Work = force x displacement moved in the direction of force W = F x d	[W] = [F] x [d] = [ma] x [d] = kg x m s^{-2} x m = kg m^2 s^{-2}	J (joule) or kg m^2 s^{-2}	kg m^2 s^{-2}
Electric charge	Charge = current x time Q = I x t	[Q] = [I] x [t] = A x s = A s	C(coulomb) or A s	A s
Power			J s^{-1} or W (watt) or kg m^2 s^{-3}	kg m^2 s^{-3}

Note: 'newton', 'joule', 'coulomb', 'joule per second', 'watt' are SI units but not SI base units.

1.1.5 Decimal multiples & submultiples of SI base and derived units

Prefix	Multiple	Symbol
tera	10^{12}	T
giga	10^{9}	G
mega	10^{6}	M
kilo	10^{3}	k
Prefix	**Submultiple**	**Symbol**
deci	10^{-1}	d
centi	10^{-2}	c
milli	10^{-3}	m
micro	10^{-6}	μ
nano	10^{-9}	n
pico	10^{-12}	p

1.2 USES OF SI BASE UNITS



(i) Only terms with the same units can be added or subtracted.
(ii) Units on both sides of an equation must be the same.
(iii) The exponent of a term has no units. E.g. e^x, 10^x → x has no units.
(iv) The logarithm of a quantity has no units. E.g. lg (t), where t may or may not have units but [lg (t)] has no units.

Example 3 (Determining units of unknown quantities in an equation)
The relation between the pressure P of a sample of gas at a certain temperature T and the volume V available to it is given to a good approximation, by the Van der Waals' relation

$$\left(P+\frac{a}{V^2}\right).(V-b)=kT$$

where a, b and k are constants. Find the units of a, b and k.

Solution:
Thinking questions:
1) *What are the terms in the equation? What are their base units?*
2) *What are the rules that can be applied to derive the unknown units?*

Let [ ] denote "units of".

$\left[\frac{a}{V^2}\right]$ = [P]

[a] = [P] x [V^2]
= (kg m^{-1} s^{-2}) (m^6)
= kg m^5 s^{-2}

Also,
[b] = [V] = m^3

Recall rule (i): "*Only terms with the same units can be added or subtracted.*"

Thus the terms P and $\frac{a}{V^2}$ have the same units.

The terms V and b also have the same units.

Comparing units on both sides of the equation, we have:

$$\left[P+\frac{a}{V^2}\right].[V-b]\quad=[k][T]$$

Therefore,

$$[k]=\left[P+\frac{a}{V^2}\right].[V-b][T^{-1}]\;=(\text{kg m}^{-1}\text{ s}^{-2})(\text{m}^3)\text{K}^{-1}$$
$$=\text{kg m}^2\text{ s}^{-2}\text{K}^{-1}$$

Recall rule (ii), *"Units on both sides of an equation must be the same."*

Thus we compare the units on both sides.

This whole term, $\left(P+\frac{a}{V^2}\right)$ has the units of (kg m^{-1} s^{-2}) and the term (V-b) has the units of m^3. Thus units on the left side of equation is (kg m^{-1} s^{-2})(m^3).

Example 4 (Deriving equations)
The square of the speed of an object undergoing uniform acceleration a depends on a and the displacement s, according to the expression

$$v^2 = ka^x s^y$$

where k is a dimensionless constant. What are x and y?

<u>Solution:</u>
Thinking questions:

1) *What does dimensionless mean? (Answer: This means k has no units.)*
2) *Are all constants dimensionless? (Answer: No. An example is Avogadro's constant.)*

Units of v^2 = units of $(ka^x s^y)$

Since k is dimensionless → k does not have any units

Units of v^2 = units of $(a^x s^y)$
= units of (a^x) x units of (s^y)
= (units of a)^x x (units of s)^y

$(\text{m s}^{-1})^2 = (\text{m s}^{-2})^x (\text{m})^y$

$\text{m}^2\ \text{s}^{-2} = \text{m}^{x+y}\ \text{s}^{-2x}$

Since both sides of the equation has the same units,
Comparing powers of m, $x + y = 2$ ----- (1)
Comparing powers of s, $-2x = -2$ ----- (2)

Solving, x = 1 and y = 1

Example 5 (Checking the homogeneity of equations)
The period of a simple pendulum is given by,

$$T = 2\pi\sqrt{\frac{L}{g}}$$

where *L* is the length of the pendulum and *g* is the acceleration of free fall. Is this equation dimensionally homogeneous?

<u>Solution:</u>
Thinking question:

1) *What does dimensionally homogeneous mean? [Answer: This means that each term in the equation has the same base units. E.g. v = u + at; [at] = m s^{-1}.]*

Units on the Right Hand Side (RHS) of equation: $\left[\sqrt{\frac{L}{g}}\right] = \sqrt{\frac{m}{m\ s^{-2}}} = s$

which is the same as the units on the LHS of equation.
Hence, the equation is dimensionally homogeneous.

<u>Is it possible that an equation is dimensionally homogeneous and yet physically wrong[1]?</u>

Yes, because of:

i. <u>incorrect coefficient which has no units</u>, e.g. v = u + 4at [correct equation: v = u + at]
ii. <u>extra term having the same units</u>, e.g. $v = u + at + \sqrt{as}$ [correct equation: v = u + at]
iii. <u>missing term having the same units</u>, e.g. v = at [correct equation: v = u + at]
iv. <u>wrong quantities having the same units</u>, e.g. pressure = force / area [correct equation: force normal to an area / area]; e.g. work done and moment of a force have similar base units although their definitions are different]

The physical correctness of an equation must ultimately be <u>verified experimentally</u>.

1.3 ESTIMATES OF PHYSICAL QUANTITIES

- A good physics student should have an appreciation of the magnitude of the quantity he/she is measuring.
- In general, a rough estimate is made <u>base on everyday information and experience</u>, and, rounding off all values to <u>1 significant figure</u>.
- Usually the <u>order of magnitude would suffice</u>, there is no need to know the exact value.

Examples:

- Mass of an apple: ~ 100 g
- Power of a hair dryer: ~ 1000 W
- Resistance of a domestic filament lamp: ~ 1000 Ω
- Temperature of a red-hot ring on an electric cooker: ~ 800 ˚C
  In normal lighting conditions, a metal would not be seen to glow until it is over 700 ˚C.
- Average speed of a good sprinter:
  Facts : 100 m in 10.2 s to 9.5 s
  Speed = 100 / 9.8 = 10 m s^{-1} (1 sig. fig.)
- Height of a double-decker bus:
  Each deck is slightly greater than most persons' height: ~ 2 m
  Height of a double-decker bus: ~ 4 m
- Kinetic energy of a bus travelling on the expressway (~10^6 J)
  Working: use ½ m v^2
  Estimate mass of bus = 10,000 kg
  Estimate speed on expressway = 100 km/h
  Hence, $\frac{1}{2}mv^2 = \frac{1}{2}(10{,}000)\left(\frac{100 \times 1000}{3600}\right)^2 = 3.8\,\text{MJ} \approx 4\,\text{MJ}$ (1 sig. fig.)

[1] 'physically wrong' means does not follow the law of Physics.

- Volume of water in an Olympic-sized swimming pool (~10^3 m^3)
  Working: Volume = length x breadth x depth
  Length of pool = 50 m (Standard Olympic swimming pool)
  Estimate width of pool = 8 lanes x 2 m per lane = 16 m
  Estimate depth of pool = average of 2 m deep throughout the pool
  Hence, volume = 50 x 16 x 2 = 2000 m^3 (1 sig. fig.)
- Energy required to bring a kettleful of water to boil:
  Working: use $Q = mc\Delta\theta$
  Mass of water = density of water x volume of a kettle
  = 1 g cm^{-3} x 2000 cm^{-3} = 2 kg
  Specific heat capacity of water = 4200 J kg^{-1} $^\circ C^{-1}$
  $\Delta\theta$ = 100˚C – room temperature = 100˚C – 30˚C = 70˚C
  Thermal energy required = 2 x 4200 x 70 = 600 kJ

Example 6

Estimate the mass of a single sheet of A4 paper.

A 0.01 g **B** 0.3 g **C** 1 g **D** 30 g

Solution:

MINI-TEST 1

1. What are the SI base units of k so that the following equation is homogeneous?

   velocity = k x density

2. Convert 1.3 g cm^{-3} to SI base units.

3. Which is a reasonable estimate for the volume of a metre rule found in a school laboratory?

   A 1.5 cm^3 **B** 15 cm^3 **C** 150 cm^3 **D** 1500 cm^3

<u>My solution:</u>

1.4 ERRORS AND UNCERTAINTIES

An **error** is the difference between a measured quantity and its true value.
If the true value is not known, then it is more correct to refer to an uncertainty.

An **uncertainty** is the range of values on both sides of a measurement in which the true value of the measurement is expected to lie.
A measurement quoted as (36.4±0.3) cm implies that the most likely value is 36.4 cm with an uncertainty of ± 0.3 cm. The actual value is likely to lie between 36.1 cm and 36.7 cm.

Absolute uncertainty is an estimate of the maximum likely difference between a value obtained from measurement, and, the average value. For example, the height of a person measured using a measuring tape could be expressed as (170.0 ± 0.1) cm. This means the *average* of repeated measurements is 170.0 cm, and, the *absolute uncertainty* is 0.1 cm.

1.4.1 Estimation of Absolute Uncertainty for DIRECT measurements

Factors that determine the absolute uncertainty in a measurement include:
1) Precision of measuring instrument
2) Experimental procedure
3) Factors of the physical environment

Absolute uncertainty of a quantity that can be measured directly by an instrument may be determined from the smallest division of the instrument, usually taken to be half or a whole of the smallest division of the measuring scale.

However, if factors 2 or 3 above are more significant compared to factor 1, then the maximum uncertainty is quoted.

1.4.2 Expressing Uncertainty

- We quote the measurement with its absolute uncertainty as ($X \pm \Delta X$) (units).

- The fractional uncertainty of the measurement is given by $\Delta X / X$.

- The percentage uncertainty of the measurement is given by ($\Delta X / X$) x 100%.

- Uncertainty is just an estimation; hence absolute uncertainty ΔX is always rounded off to 1 significant figure.

- X is always expressed to the same order of precision as ΔX.
  E.g. (120 ± 20) g, (121 ± 5) g, (121.3 ± 0.1) g, (121.35 ± 0.01) g

Rationale: It doesn't make sense if the calculated quantity is more precise than the uncertainty.

Example 7

A thermometer has small scale divisions of 1 ^{O}C and the temperature measured is twenty-five degree centigrade.

i) Estimate the absolute error of the measurement of T.

ii) How should the reading be expressed?

iii) Hence calculate the fractional and percentage error of the temperature measured.

Solution:

i) Absolute Uncertainty of the measurement of T, ΔT = 0.5 ^{O}C (half of a division)

ii) T = (25.0 ± 0.5) ^{O}C
(ΔT must be in 1 s.f. T must be expressed to the same number of d.p. as ΔT.)

iii) Fractional error of T, $\Delta T / T = 0.5 / 25.0 = 0.02$ (1 s.f.)
Percentage error of T = Fractional error of T x 100 % = 2 % (1 s.f.)

1.4.3 Estimating uncertainties of DERIVED quantities

- Many physical quantities cannot be measured directly. Instead, they are derived from measurement of certain basic quantities. For example, the value of the acceleration due to gravity (g) can be estimated from an experiment involving measuring the length and period of oscillations of a pendulum. 'g' is the derived quantity; while the length and timings performed are the basic quantities.
- Uncertainties in the measurement of these individual basic quantities will then combine, resulting in a *larger* uncertainty in the derived quantity.
- Uncertainties always add up, they do not cancel each other.

(a) Addition/Subtraction rule

If A = x + y, then $\boxed{\Delta A = \Delta x + \Delta y}$

If B = x – y, then $\boxed{\Delta B = \Delta x + \Delta y}$

(b) Product/Quotient rule

For a quantity that is expressed by multiplying or dividing other quantities, its absolute error CANNOT be found directly, we need to first find its fractional error:

1. If A = xyz

fractional error in A is $$\boxed{\frac{\Delta A}{A} = \frac{\Delta x}{x} + \frac{\Delta y}{y} + \frac{\Delta z}{z}}$$

absolute error in A is $$\boxed{\Delta A = \left(\frac{\Delta x}{x} + \frac{\Delta y}{y} + \frac{\Delta z}{z}\right).A}$$

2. If $A = \frac{x}{y}$

<u>fractional error</u> in A is $$\frac{\Delta A}{A} = \frac{\Delta x}{x} + \frac{\Delta y}{y}$$

<u>absolute error</u> in A is $$\Delta A = \left(\frac{\Delta x}{x} + \frac{\Delta y}{y}\right).A$$

(c) Multiply with a constant

If A = kx, where k is a constant

<u>absolute error</u> in A is $$\Delta A = k\Delta x$$

<u>fractional error</u> in A is $$\frac{\Delta A}{A} = \frac{\Delta x}{x}$$

(d) Constant power

If $A = x^p$

<u>fractional error</u> in A is $$\frac{\Delta A}{A} = |p|\frac{\Delta x}{x}$$

(e) General case

If $A = \frac{kx^p y^q}{z^r}$

<u>fractional error</u> in A is $$\frac{\Delta A}{A} = |p|\frac{\Delta x}{x} + |q|\frac{\Delta y}{y} + |r|\frac{\Delta z}{z}$$

(c) *to* ***(e)*** *may in fact be derived from* ***(a)*** *and* ***(b)****.*

(f) Combination of Addition, Subtraction, Product, Quotient, Trigonometry and/or Logarithms (Extreme values method)

When faced with such an equation, for example:

$$A = a + \frac{b}{c\,(\cos\theta)}$$

It is simpler to use the ***extreme values method***.

1. Calculate the maximum and minimum values of the subject of the equation (in the case above, A), by substituting the highest or lowest values of the quantities in the equation.
2. The absolute uncertainty will be equivalent to :

$$\Delta A = \frac{\text{maximum value of A - minimum value of A}}{2}$$

The extreme values method is also applied when the <u>same quantity</u> appears in both the numerator and denominator, for example:

$$B = \frac{mn}{m + n}$$

If the product/quotient rule is applied, the uncertainty in m and n will have varying effects on the numerator and denominator and thus cannot be applied.

Example 8

The diameter of a wire is (0.56 ± 0.01) mm. Express the cross-sectional area of the wire with its uncertainty.

<u>Solution:</u>

$$\text{Cross-sectional area of wire, } A = \pi r^2 = \pi \frac{d^2}{4}$$

$$= \pi \times \frac{0.56^2}{4}$$

$$= 0.2464 \text{ mm}^2$$

$$\frac{\Delta A}{A} = 2\frac{\Delta d}{d} \qquad (\text{since } A = \pi \frac{d^2}{4})$$

$$= 2 \times \frac{0.01}{0.56}$$

$$= 0.0357$$

$$\therefore \Delta A = 0.0357 \times A$$

$$= 0.0357 \times 0.2464$$

$$= 0.0088 \text{ mm}^2$$

$$= 0.009 \text{ mm}^2 \text{ (1 sig. fig.)}$$

Hence cross-sectional area of wire (A ± ΔA) = (0.246 ± 0.009) mm^2.

(Note: A is expressed to the same number of decimal places as ΔA.)

*****Recall: The uncertainty is expressed to <u>1 significant figure</u> and the calculated value should have the <u>same order of precision</u> as the uncertainty.***

Example 9

The energy of a mass is given by: $E = mgh + \frac{1}{2}mv^2$

where measurements show that
m = (2.500 ± 0.002) kg, h = (3.00 ± 0.01) m, v = (1.00 ± 0.01) m s^{-1}

Determine E and its uncertainty using the value of g as 9.81 m s^{-2}.

Solution:

Highest value of E = 2.502(9.81 x 3.01 + ½ x 1.01^2) = 75.155 J
Lowest value of E = 2.498(9.81 x 2.99 + ½ x 0.99^2) = 74.495 J
Absolute error in E, $\Delta E = \frac{75.155 - 74.495}{2}$ = 0.330 = 0.3 J (1 s.f.)
Average value of E = $\frac{75.155 + 74.495}{2}$ = 74.82 J =74.8 J (*to the same number of d.p.* as ΔE)
Hence, E = (74.8 ± 0.3) J

1.4.4 What is random error?

A random error is an error which gives a **scatter** of readings about an **average value.** It has an **equal chance** of being **positive or negative** and can be **reduced by averaging**.

1. Random error is revealed by repeated measurements of a particular quantity.

2. Common sources of random errors are
   i. fluctuating factors in the physical environment – e.g. room temperature
   ii. errors of judgement or inconsistent procedural errors – e.g. the observer's estimation of a fraction of the smallest division of measuring instrument used may vary in repeated measurements; parallax error may be inconsistent in repeated measurements
   iii. disturbances – e.g. mechanical vibrations from nearby rotating machinery
   iv. imperfections in the objects being measured – e.g. diameter of a wire (wire is not uniform)

3. The effects of random error may be reduced by
   i. taking the average of repeated measurements
   ii. graphical means (drawing the line of best-fit)[2]

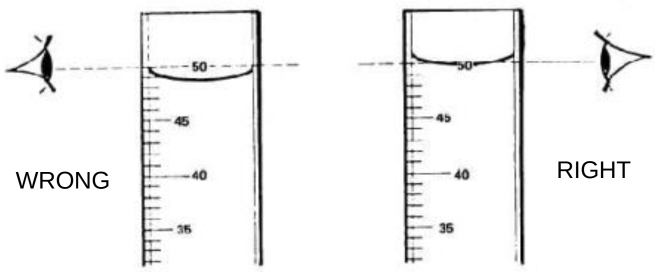
[2]Further discussion in Practical

1.4.5 What is systematic error?

A systematic error is a **constant deviation** of readings **from the true value** in **one direction**. **All measurements will be too high or too low by the same amount**. It can be **reduced by correct laboratory practice**.

1. Systematic errors are **not** revealed by repeated measurements, and **cannot** be reduced by taking the average or drawing the line of best-fit.

2. Common sources of systematic errors are
   i. an instrument having a zero error – instrument shows non-zero readings when nothing is being measured
   ii. an instrument being incorrectly calibrated (e.g. a slow running stopclock).
   iii. end error (e.g. due to wear and tear at the ends of a ruler after being used for many years).
   iv. experimental conditions – an instrument used under experimental conditions (e.g. temperature and pressure) different from those for which it was calibrated.
   v. background measurement (e.g. when using the Geiger-Muller counter and forgetting to subtract the background radiation count-rate; note that background radiation contributes to *both* systematic and random errors due to the random nature of radioactivity.)
   vi. consistent procedural errors (e.g. consistently reading off the volume of water in a measuring cylinder from the highest point of its meniscus).



The diagram on the left shows the observer reading the water level from the highest point of the meniscus, which is incorrect.

3. Systematic errors can be eliminated or corrected by
   i. taking into account zero errors of the instrument to determine the actual value of the measurements.
   ii. checking the instruments used against other similar instruments and recalibrate the instrument, if necessary, before using.
   iii. ensuring that instrument is used in the same experimental conditions (e.g. temperature and pressure) which it is calibrated for.
   iv. ensuring correct techniques or procedures are used to take measurements.

4. Systematic errors can be revealed by comparing the measurements with the true value or by graphical means. On a graph, a systematic error would manifest itself as an intercept on the y-axis other than the expected y-intercept.
   For example, in a pendulum experiment to investigate *T* (Period) against *l* (length) all the lengths measured may be too small because the student did not add the radius of the bob. A straight line graph passing through the origin is expected to be obtained by plotting T^2 against *l*. Because every measured value of *l* is too short by the radius, a straight line would still be obtained but the line will not pass through the origin, which is the expected y-intercept. In such cases, the y-intercept of the plotted straight line graph revealed the systematic error.

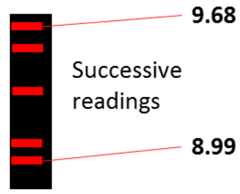
1.4.6 Precision vs Accuracy

Precision (of an instrument):	a term used to describe the level of uncertainty in an instrument's scale. High precision instruments have small scale divisions.
Accuracy (of an instrument):	is the closeness of a reading on an instrument to the true value of the quantity being measured. An accurate instrument will give readings close to the true values.
Precision (of a set of readings):	is the degree of closeness of the readings with one another. It is associated with small random error.
Accuracy (of a set of readings):	is the degree of closeness of the average value of the readings to the true value. It is associated with small systematic error.

Further notes: Precision vs Accuracy (of a set of readings):

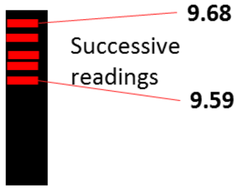
1. If an experiment has small systematic errors, it is said to have high accuracy.
2. If an experiment has small random errors, it is said to have high precision.
3. The figure below illustrates the meaning of 'accurate readings' and 'precise readings':

Experiment	1	2	3	4	5	Ave	(Max – Min)
g / m s^{-2}	8.99	9.36	9.68	9.06	9.52	9.32	0.69



Large Random Error

Experiment	1	2	3	4	5	Ave	(Max – Min)
g / m s^{-2}	9.59	9.61	9.68	9.62	9.65	9.63	0.09

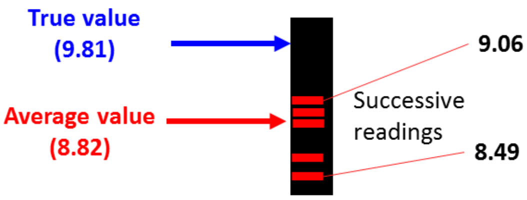


Small Random Error

***Good Precision* – Readings are close to each other**

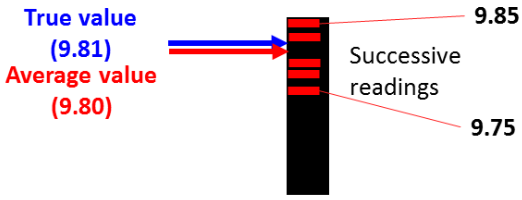
The scatter of readings shows the presence of random errors.
Compare (Max – Min) to determine the magnitude of random errors.

Experiment	1	2	3	4	5	Ave	(Max – Min)
g / m s^{-2}	8.49	8.99	8.55	9.06	9.02	8.82	0.57



Large Systematic Error

Experiment	1	2	3	4	5	Ave	(Max – Min)
g / m s^{-2}	9.75	9.83	9.79	9.78	9.85	9.80	0.10



Small Systematic Error

***Good Accuracy* – Average value is close to True value**

Systematic errors are deviations from the true value.
Compare Average (or Mean) value with the True value to determine the magnitude of the systematic error.

Example 10

Four students made a series of measurements of the acceleration of free fall g where g = 9.81 ms^{-2}. The table shows the results obtained. Which student obtained a set of results that could be described as precise but not accurate? (N96/1/3)

Student	Results g / m s^{-2}			
A	9.81	9.79	9.84	9.83
B	9.81	10.12	9.89	8.94
C	9.45	9.21	8.99	8.76
D	8.45	8.46	8.50	8.41

Solution:

Thinking questions:

1) *What should we be looking at if we need to determine if the data collected is* ***precise****?*
2) *What should we be looking at if we need to determine if the data collected is* ***accurate****?*

determines accuracy

determines precision

Student	Results g / m s^{-2}				***Range***	***Mean value***
A	9.81	9.79	9.84	9.83	***0.05***	***9.82***
B	9.81	10.12	9.89	8.94	***1.18***	***9.69***
C	9.45	9.21	8.99	8.76	***0.69***	***9.10***
D	8.45	8.46	8.50	8.41	***0.09***	***8.46***

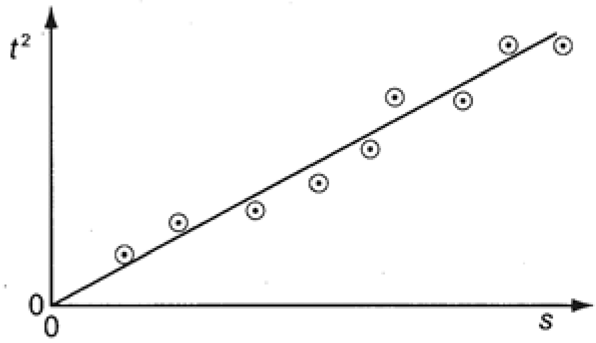
From the 'Mean value' and 'Range', we can conclude that Student D's result is precise but not accurate because:

1. The mean value of Student D is the furthest to the true value of 9.81 m s^{-2}.
2. The range of readings of Student D is the 2nd smallest value in comparison to the rest of the students.

Answer: D

Example 11

An object falls freely from rest and travels a distance *s* in time *t*. A graph of t^2 against *s* is plotted and used to determine the acceleration of free fall g.



The gradient of the graph is found to be 0.218 $s^2\ m^{-1}$ and g is determined by the equation:

$$g = \frac{2}{\text{gradient of graph}}$$

Which statement about the value obtained for g is correct?

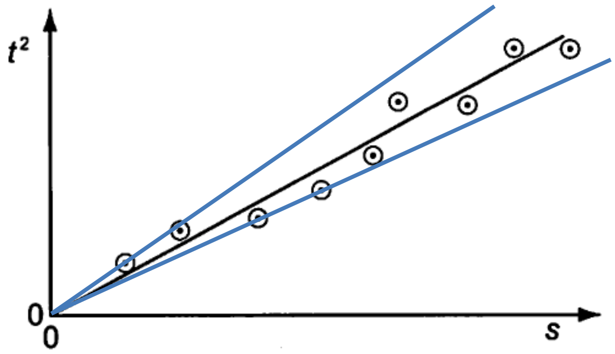
A It is accurate but not precise.
B It is both precise and accurate.
C It is neither precise nor accurate.
D It is precise but not accurate.

Solution:

1) How to determine 'accuracy' from the graph?
   - Compare the 'average value of the readings', which is given by the gradient of the line of best fit
   - with the true value of g, which is 9.81 m s^{-2}

By calculation, average value of g = $\frac{2}{0.218}$ = 9.17 m s^{-2}. This is not an accurate value.

2) How to determine 'precision' from the graph?
   - A quick way, not always clear-cut, is to look at how much scattering of data points there is about the line of best fit. We see here there is hardly any point that lies on the line.
   - A more rigorous way is to draw 2 'extreme lines' by pivoting about the expected/theoretical/accurate coordinate (0, 0), since at t=0, s=0. The gradient of these 2 'extreme lines' gives the maximum and minimum values of g obtained from each experimental reading when taken separately. Without having to calculate the gradients of these 2 'extreme lines', simply judging the difference in gradients between them, one can get a sense of the range of values of g obtained from these experiment data.



Since the deviation of gradient between the 2 'extreme lines' is not small, the value of g is not precise.

Answer: C

1.5 VECTORS

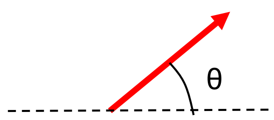
1.5.1 SCALARS & VECTORS

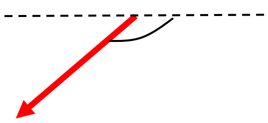
A **scalar** quantity is a quantity that has magnitude only, not direction.
e.g. speed, distance, volume, mass, energy, power, temperature.

A **vector** quantity is a quantity that has both magnitude and direction.
e.g. displacement, velocity, acceleration, momentum, force, gravitational field strength, electric field strength and magnetic flux density.

1. Arrows are used to represent vectors. The direction of the arrow gives the direction of the vector. The length of a vector arrow is proportional to the magnitude of the vector.

2. Negative vector is one with the same magnitude but its direction is reversed.





Note: Vector $\vec{A}$ has the same magnitude as vector $-\vec{A}$, but vector $\vec{A}$ is at an angle θ above the horizontal and vector $-\vec{A}$ is (180°– θ) below the horizontal. The angle of the vector made with the horizontal is the direction of vectors.

3. To describe a vector, we must specify both its magnitude and its direction. For example, if Vector $\vec{A}$ is displacement, we can write

Magnitude of $\vec{A}$ = 200 m
Direction of $\vec{A}$ = 30° anticlockwise above the horizontal

1.5.2 Vector Addition & Subtraction - Finding Resultant of vectors

If you earn $20 on Friday and $40 on Saturday, your net income for 2 days is the sum of $20 and $40. With numbers, the word net implies addition. The same is true with vectors in that vectors can be added (vector addition). The sum of two vectors is called the **resultant vector**.

If vector $\vec{B}$ is added to vector $\vec{A}$, we can write the resultant vector $\vec{R}$ of the two vectors:

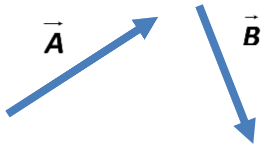
$$\vec{R} = \vec{A} + \vec{B}$$

Vectors have magnitudes and directions, so different rules apply in vector addition to determine the resultant vector.

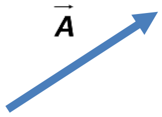
You can add the two vectors $\vec{A}$ and $\vec{B}$ with the three-step procedure (*head-to-tail method*) to determine the resultant vector $\vec{R}$, where $\vec{R} = \vec{A} + \vec{B}$, as show below:

Vector addition

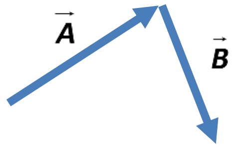
To add $\vec{B}$ to $\vec{A}$



1. Draw $\vec{A}$.

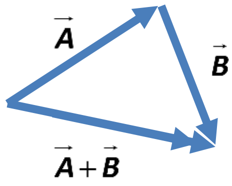


2. Place the *tail* of $\vec{B}$ at the *head* of $\vec{A}$.



3. Draw an arrow from the *tail* of $\vec{A}$ to the the *head* of $\vec{B}$.

   This is resultant vector $\vec{A} + \vec{B}$



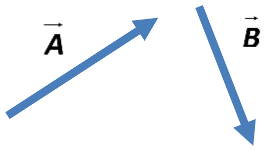
If Vector $\vec{B}$ is *subtracted* from Vector $\vec{A}$, this vector subtraction is a special case of vector addition:

$$\vec{A} - \vec{B} = \vec{A} + \left(-\vec{B}\right)$$

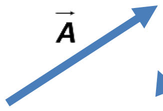
Since the minus sign means that the direction of a vector is opposite that of the one with a plus sign, Vector $-\vec{B}$ has the same magnitude as the Vector $\vec{B}$ but in the opposite direction.

Vector subtraction

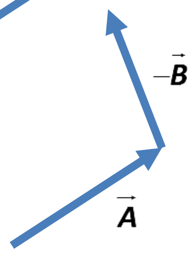
To *subtract* $\vec{B}$ to $\vec{A}$



1. Draw $\vec{A}$.

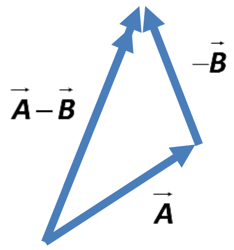


2. Place the *tail* of $-\vec{B}$ at the *head* of $\vec{A}$.

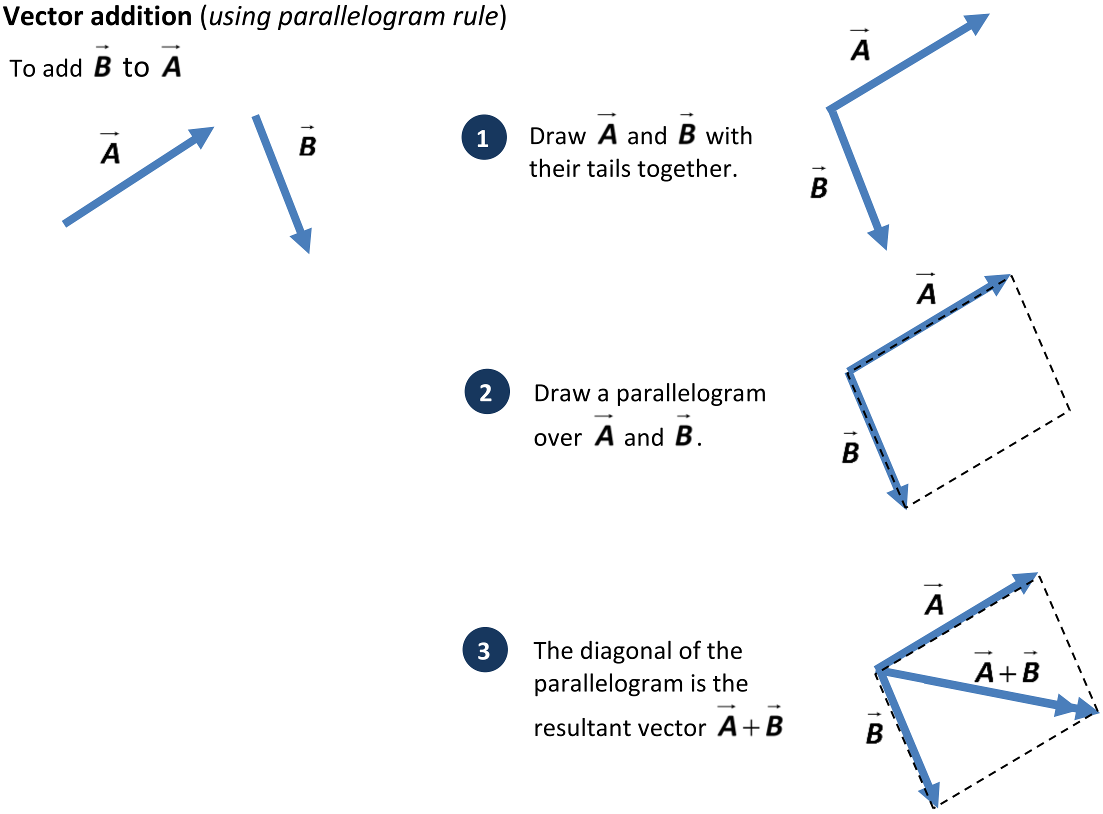


3. Draw an arrow from the *tail* of $\vec{A}$ to the the *head* of $-\vec{B}$.

   This is resultant vector $\vec{A} - \vec{B}$



A second method to add two vectors is to use the *parallelogram rule* of vector addition as shown below:

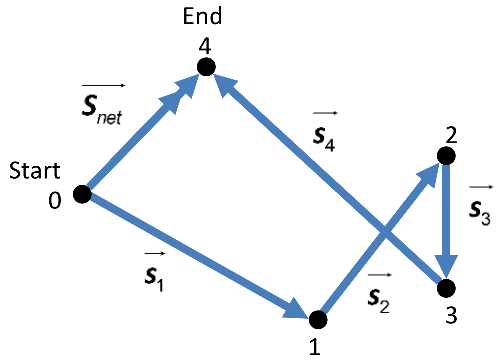


Vector addition can be easily extended to more than two vectors. The diagram below shows the path of a man moving from initial position 0 to position 1, then position 2, then position 3, and finally at position 4. These four segments are described by displacement vectors $\vec{s_1}$, $\vec{s_2}$, $\vec{s_3}$ and $\vec{s_4}$. The man's resultant displacement, an arrow from position 0 to position 4, the vector $\overrightarrow{S_{net}}$:

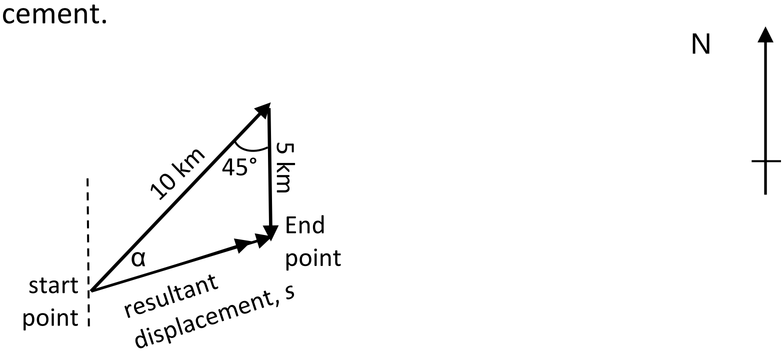
$$\overrightarrow{S_{net}} = \vec{s_1} + \vec{s_2} + \vec{s_3} + \vec{s_4}$$

The vector sum is found by using the head-to-tail method three times in succession.

Resultant displacement of 4 individual displacements



Example 12 (Resultant Displacement)
A car travels 10 km in a North-East (NE) direction, followed by 5 km towards the South. Determine the resultant displacement.



(3 cm represents 10 km; 1.5 cm represents 5 km)

We can use a scale drawing of a vector diagram to determine the resultant displacement. A protractor is used to draw the angles accurately. The resultant displacement is determined by drawing a vector *s*, from start point to end point. The vector *s* is drawn from the TAIL of the first vector representing 10 km to the HEAD of the last vector representing 5 km.

Since the length of arrow *s* is proportional to the magnitude of vector *s*, the resultant displacement *s* is then determined through measurement using a ruler and multiplying it with the scaling factor.

For example, the measurement of the resultant displacement *s* using a ruler is is 2.2 cm. Therefore, the actual represented distance is (10 ÷ 3) x 2.2 = 7.3 km. *(Note: The method of using scale diagram is not recommended. Only use it if the question specify for it.)*

Using cosine rule to determine the resultant displacement:
$s^2 = 10^2 + 5^2 - 2\,(10)\,(5)\cos 45°$
s = 7.4 km

Using sine rule to find the angle α,

$$\frac{\sin 45}{7.4} = \frac{\sin \alpha}{5}$$

α = 28.6°

(90° – (45°+28.6°)) = 16.4°
The resultant displacement is at an angle 16.4° above the horizontal.

Example 13 (Resultant Velocity)
A boat is crossing a river perpendicular to the river bank at a speed of 4.0 m s^{-1}. Water flows parallel to the river bank at 3.0 m s^{-1}. Determine the resultant velocity of the boat (i.e., the velocity relative to an observer on the shore).

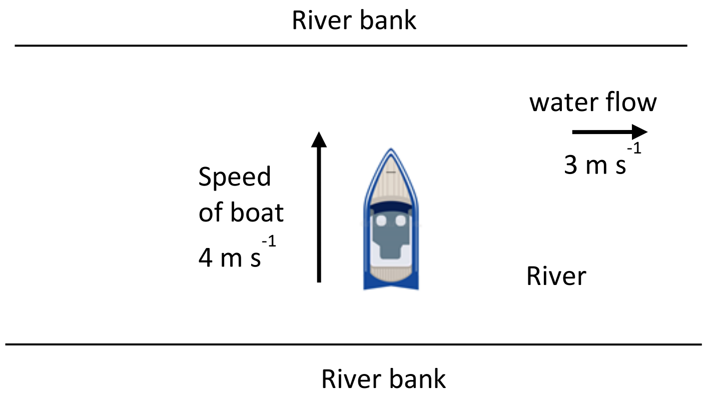


Diagram above shows top view of boat crossing river

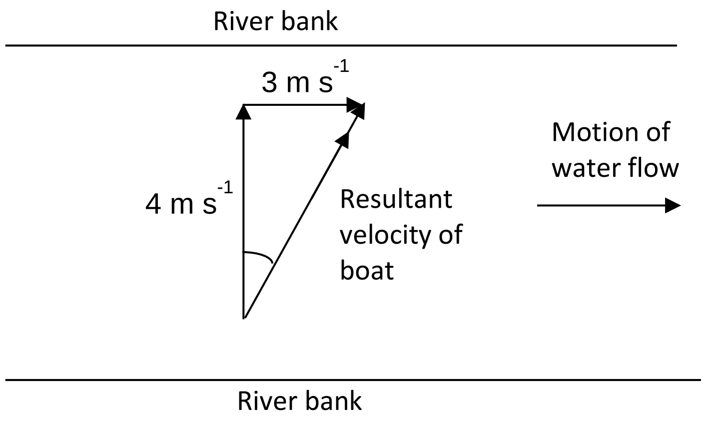


Diagram above shows vector diagram for velocity of boat, velocity for water flow and resultant velocity of boat

The resultant velocity of the boat can be determined by drawing a vector arrow joining the TAIL of the velocity vector of the boat to the HEAD of the velocity vector of the water flow. We can use Pythagoras theorem to solve for the resultant velocity as the vectors form a right-angle triangle.

Magnitude of resultant velocity = $\sqrt{(3^2 + 4^2)}$= 5 m s^{-1}

Direction of resultant velocity = $\tan^{-1}\left(\frac{3}{4}\right) = 36.9^{o}$

Resultant Velocity of the boat is 5 m s^{-1} at 36.9^{O} clockwise from the direction of boat's velocity.

Which direction should the boat be steered if we want the boat to travel the shortest distance (i.e. in a direction perpendicular to the banks)?

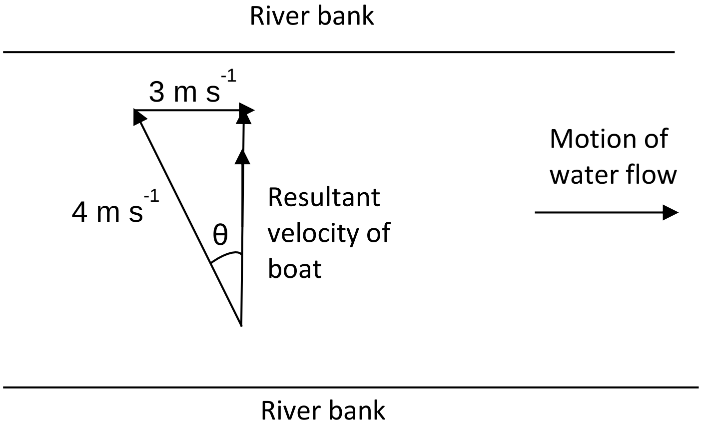


Diagram above shows vector diagram of velocity for boat, water flow and resultant velocity.

The vector diagram is now modified such that the resultant velocity is in a direction perpendicular to the river bank as that is the direction we want the boat to be steered in. We need to find the angle θ.

$$\sin\theta = \frac{3}{4}$$
$$\theta = 48.6°$$

The boat should be steered in a direction 48.6° counter-clockwise from the boat's resultant velocity.

1.5.3 Application of Vector Subtraction

In this subtopic, we consider how to use vector subtraction to determine
1. Change in velocity
2. Relative velocity

1. Finding CHANGE in velocity

If an object moves with an initial velocity and changes its velocity, we can determine the **CHANGE in velocity** of the object. Since velocity is vector, the **CHANGE in velocity** of the object is also a vector.

Therefore, **CHANGE in velocity** can be written, using vector subtraction, as follows:

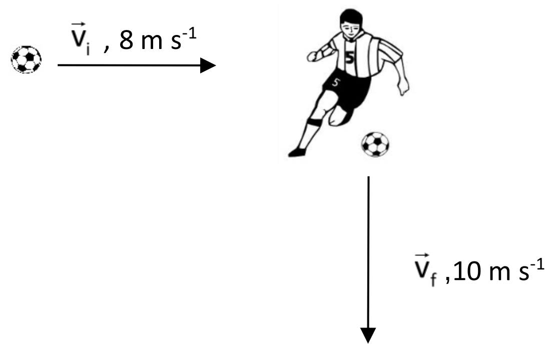
Change in velocity = final velocity – initial velocity

$$\Delta\vec{v} = \vec{v}_f - \vec{v}_i$$

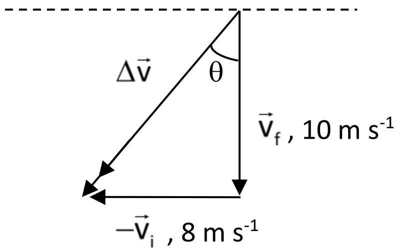
where,
$\Delta\vec{v}$ is the vector representing **change in velocity**,
$\vec{v}_f$ is the vector representing the **final velocity** and
$\vec{v}_i$ is the vector representing the **initial velocity**

Example 14
A football approaches a player at 8 m s^{-1} in a straight line. After the player kicks the ball, find the **change in velocity** of the football if it is deflected through 90^o and moves away from the player at 10 m s^{-1}.

Solution:



Approach to solving the question:
1) Write down the equation: **Change in velocity = final velocity – initial velocity**
$$\Delta\vec{v} = \vec{v}_f - \vec{v}_i$$
2) Convert the equation so that a vector addition can be done: $\Delta\vec{v} = \vec{v}_f + \left(-\vec{v}_i\right)$
Recall that putting a minus in front of v_i reverses its direction.
3) Draw the vector diagram to help in solving. The change in velocity vector is drawn by connecting the TAIL of $\vec{v}_f$ to the HEAD of $-\vec{v}_i$.



4) Since the vector diagram is a right angle triangle, use Pythagoras theorem to solve. (If it is not a right angle triangle, use cosine or sine rule to solve.)

$|\Delta\vec{v}| = \sqrt{(10^2 + 8^2)} = 12.8 \text{ m s}^{-1}$

5) Since change in velocity is a vector quantity, we will need to determine and specify the direction.

$\theta = \tan^{-1}\left(\frac{8}{10}\right) = 38.7^{\circ}$

Change in velocity is 12.8 m s^{-1} at 38.7° clockwise from the final velocity of the incident ball.

2. Finding Relative velocity

The velocity of **object B relative to object A** is the velocity which B appears to have, to an observer who is moving with A. Using vector subtraction,

Velocity of **B relative to A**, $\vec{v}_{BA} = \vec{v}_B - \vec{v}_A$

where,

$\vec{v}_A$ is the vector representing the velocity of object **A** and

$\vec{v}_B$ is the vector representing the velocity object **B**

Similarly, the velocity of **object A relative to object B** is the velocity which A appears to have, to an observer who is moving with B:

Velocity of **A relative to B**, $\vec{v}_{AB} = \vec{v}_A - \vec{v}_B$

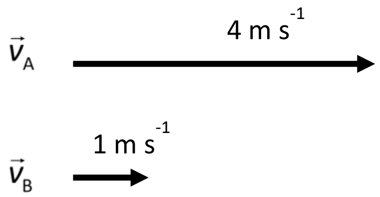
Example 15

Object A moves at 4 m s^{-1} while object B moves at 1 m s^{-1} in the same direction.

a) What is the velocity of A relative to B?

b) What is the velocity of B relative to A?

Solution:



Take direction of A, $\vec{v}_A$, as positive,

a) $\vec{v}_{AB} = \vec{v}_A - \vec{v}_B = \vec{v}_A + (-\vec{v}_B) = 4 + (-1) = 3$ m s^{-1}

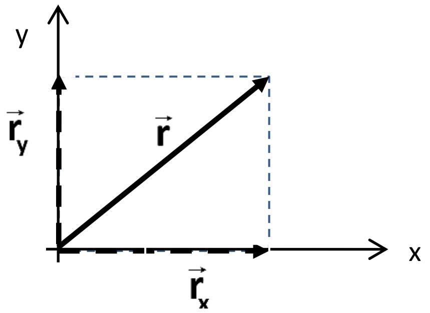
The relative velocity of A with respect to B is 3 m s^{-1} along the direction of A.[3]

b) $\vec{v}_{BA} = \vec{v}_B - \vec{v}_A = \vec{v}_B + (-\vec{v}_A) = 1 + (-4) = -3$ m s^{-1}

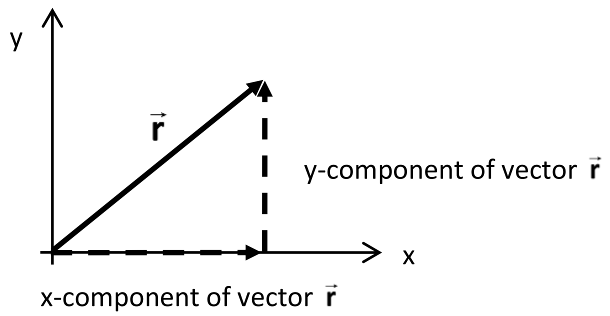
The relative velocity of B with respect to A is 3 m s^{-1} opposite to the direction of A.

[3]Whenever you are asked to determine a vector, specify both magnitude & direction.

1.5.4 Vector Resolution & Component vectors



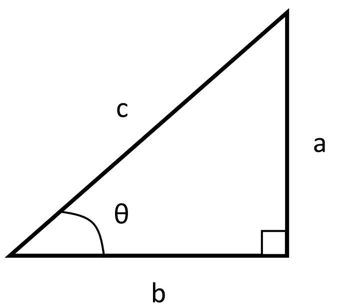
- Any vector can be thought of as an addition of two other vectors in different directions.
- *Resolving* a vector in two dimensions means re-representing the vector by two other vectors at right angles to each other, such that the summation of these two vectors, $\vec{r}_x$ and $\vec{r}_y$ give the original vector $\left(\vec{r} = \vec{r}_x + \vec{r}_y\right)$.
- Each of the two-dimensional vector is known as a component of $\vec{r}$. $\vec{r}_x$ is the x-component of $\vec{r}$. $\vec{r}_y$ is the y-component of $\vec{r}$.



Steps in resolving a vector:

1. Choose two *convenient* perpendicular directions to resolve the vector in. (e.g. x and y directions in the above example).

2. Use trigonometry to obtain the magnitude of the components. In trigonometry:

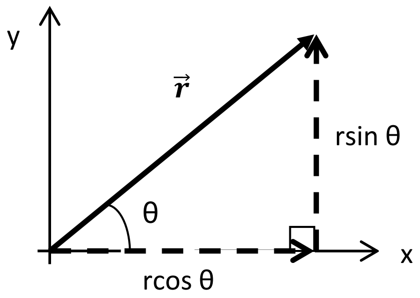


$$\sin\theta = \frac{a}{c} \quad \therefore a = c\sin\theta$$
$$\cos\theta = \frac{b}{c} \quad \therefore b = c\cos\theta$$

3. Let θ be the angle vector $\vec{\mathbf{r}}$ makes with the x-axis.

   x-component of $\vec{\mathbf{r}}$ has magnitude equal to <u>rcosθ</u>.

   y-component of $\vec{\mathbf{r}}$ has magnitude equal to <u>rsinθ</u>.



Therefore, using Pythagoras Theorem,

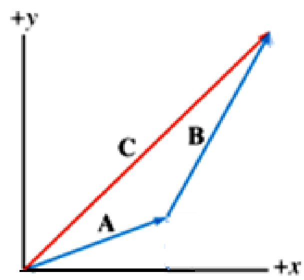
$$|r_x|^2 + |r_y|^2 = |r\cos\theta|^2 + |r\sin\theta|^2$$

$$|r_x|^2 + |r_y|^2 = |r|^2$$

Usefulness of resolving a vector (a property of vectors that are perpendicular to each other):
The resolved components are independent of each other (i.e. a change in one component does not alter the other component).

Addition using vector resolution method

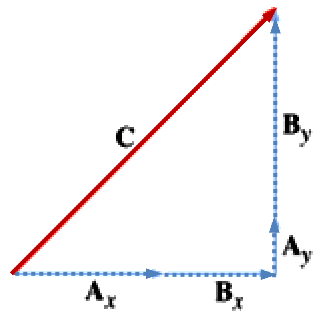
Goal: Find the resultant vector $\vec{\mathbf{C}}$ where $\vec{\mathbf{C}}$ is the addition of $\vec{\mathbf{A}}$ and $\vec{\mathbf{B}}$. i.e, $\vec{\mathbf{C}} = \vec{\mathbf{A}} + \vec{\mathbf{B}}$



<u>Step 1: Resolve the components of $\vec{\mathbf{A}}$ and $\vec{\mathbf{B}}$</u>

Draw the resolved components of $\vec{\mathbf{A}}$, $\vec{\mathbf{A}} = \vec{\mathbf{A}}_x + \vec{\mathbf{A}}_y$

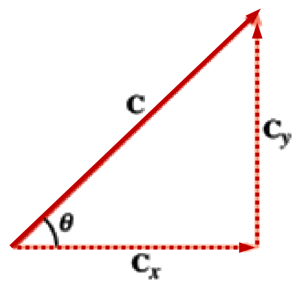
Draw the resolved components of $\vec{\mathbf{B}}$, $\vec{\mathbf{B}} = \vec{\mathbf{B}}_x + \vec{\mathbf{B}}_y$



Step 2: Sum up the resolved components along the two perpendicular directions

Along x-direction: $\vec{\mathbf{C}}_x = \vec{\mathbf{A}}_x + \vec{\mathbf{B}}_x$

Along y-direction: $\vec{\mathbf{C}}_y = \vec{\mathbf{A}}_y + \vec{\mathbf{B}}_y$



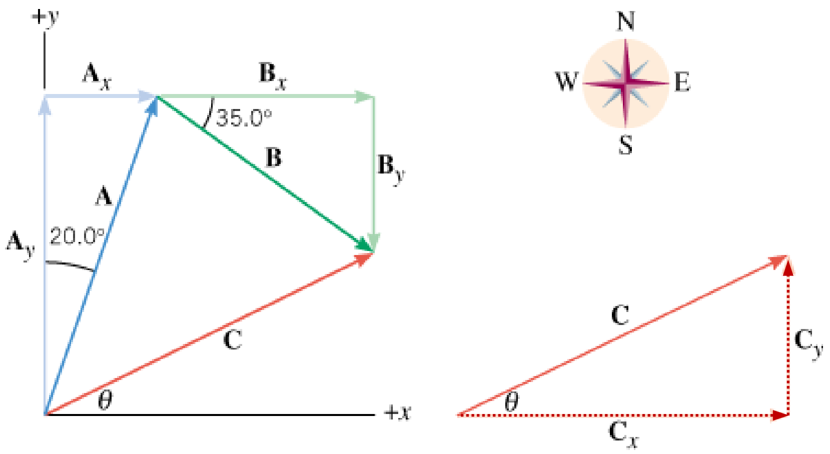
Step 3: Calculate the magnitude and direction of $\vec{\mathbf{C}}$

Magnitude of $\vec{\mathbf{C}}$, $C = \sqrt{C_X^2 + C_Y^2}$ (Pythagoras theorem)

Direction of $\vec{\mathbf{C}}$, $\theta = \tan^{-1}\left(\frac{C_Y}{C_X}\right)$

Example 16

A jogger runs 145 m in a direction 20.0° east of north (displacement $\vec{A}$) and then 105 m in a direction 35.0° south of east (displacement vector $\vec{B}$). Determine the magnitude and direction of the resultant displacement $\vec{C}$ for these two displacements.



Resolve vectors $\vec{A}$ and $\vec{B}$ along the x and y directions. Then determine the ***resultant*** displacement in each of these directions, i.e. $\vec{C}_x$ and $\vec{C}_y$.

Vector	x -component	y -component
$\vec{A}$	$\mathbf{A_x}$ = 145 sin 20° = 50 m	$\mathbf{A_y}$ = 145 cos 20° = 136 m
$\vec{B}$	$\mathbf{B_x}$ = 105 cos 35° = 86 m	$\mathbf{B_y}$ = - 105 sin 35° = - 60 m
$\vec{C}$	$\mathbf{C_x}$ = 50 + 86 = 136 m	$\mathbf{C_y}$ = 136 - 60 = 76 m

Magnitude of $\vec{C} = \sqrt{(136^2 + 76^2)} = 156$ m (Pythagoras theorem)

Direction of $\vec{C} = \tan^{-1}\left(\dfrac{76}{136}\right) = 29.2^\circ$

$\vec{C}$ is 156 m in the direction 29.2° North of East

MINI-TEST 2

1. The acceleration of free fall g was to be determined by measuring the period of oscillation T and the length *l* of a simple pendulum, and using the formula

$$g = \frac{4\pi^2 l}{T^2}$$

In the experiment, the uncertainties in measuring *l* and T were estimated to be 4% and 1% respectively. If the value of g is experimentally found as 9.697 m s^{-2}, determine the percentage uncertainty in g and express the value g with its uncertainty.

2. Errors in measurement may either be systematic or random.
Which of the following involves random error?
A Not allowing for zero error on a moving-coil voltmeter
B Using an incorrectly calibrated balance to weigh objects
C Stopping a stopwatch at the end of a race
D Using the value of g as 10 N kg^{-1} when calculating weight from mass

3. For the vectors shown in Fig. 3.1 and Fig. 3.2, draw the vector diagram for
(a) $\vec{A} + \vec{B}$
(b) $\vec{A} - \vec{B}$

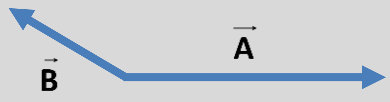


Fig. 3.1

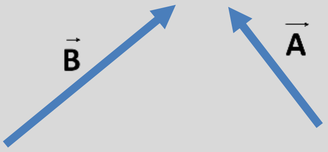


Fig. 3.2

4. Fig. 4.1 shows vectors $\vec{C}$ and $\vec{D}$.
(a) Use geometry and trigonometry to determine the magnitude and direction of $\vec{E} = \vec{C} + \vec{D}$.
(b) Use vector resolution to determine the magnitude and direction of $\vec{E} = \vec{C} + \vec{D}$.

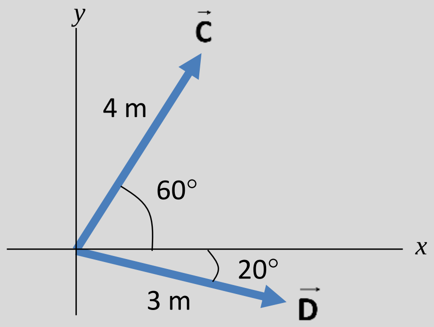


Fig. 4.1

<u>My Solution:</u>

CHAPTER SUMMARY / SELF-CHECK

1. What are physical quantities?
2. What are the base quantities and SI base units?
3. What are derived quantities? How are their units determined?
4. What is the condition for the terms in a physical equation to add or subtract?
5. How can the concept of SI base units help us determine the correctness of equations? What are the limitations?
6. Can you recall what the following prefixes represent: pico (p), nano (n), micro (μ), milli (m), centi (c), deci (d), kilo (k), mega (M), giga (G), tera (T) ?
7. Am I able to make reasonable estimation of various physical quantities?
8. Why is there a need to state the uncertainty in a measurement?
9. Distinguish between 'absolute uncertainty', 'fractional uncertainty' and 'percentage uncertainty'.
10. How many s.f. must be used to express absolute uncertainty?
11. What are the differences between random error and systematic error?
12. How can random errors and systematic errors be reduced?
13. How are 'random error' and 'systematic error' related to 'accuracy' and 'precision'?
14. What are some significant features of a graph to pay attention to when analysing or drawing a graph?
15. What are the different modes of representations in the study of Physics?
16. What is the difference between a scalar and a vector?
17. How do you:
    i) resolve a vector?
    ii) add two vectors?
    iii) subtract two vectors?
18. After resolving a vector into two components, what is the angle between the directions of the resolved components?

Measurement Tutorial

Quantities and units

Q1 J96/I/1

Which list of SI units contains only base units?

A kelvin, metre, mole, ampere, kilogram

B kilogram, metre, second, ohm, mole

C kilogram, newton, metre, ampere, ohm

D newton, kelvin, second, volt, mole

Q2 N98/I/1

What are the SI base units of pressure?

A kg m s^{-1} **B** kg m^{-1} s^{-2} **C** kg m^{2} s^{-2} **D** kg m^{-2} s^{-1}

Q3 N16/I/1

Which combination of lengths gives the largest rectangular area?

A 1 centimetre x 1 decimetre

B 1 kilometre x 1 millimetre

C 1 megametre x 1 nanometre

D 1 picometre x 1 micrometre

Q4 N99/I/1

The relationship between four physical quantities is given by the equation P = Q – RS. Given that the equation is homogeneous, which of the following statements must be correct?

A P, Q, R and S all have the same units.

B P, Q, R and S are all scalar quantities.

C The product RS has the same units as P and Q.

D The product RS is numerically equal to (Q – P)

Q5 (Longman)

The mass of a solid cube of iron is 50.4 x 10^{-2} kg. 1 mole of iron atoms has a mass of 5.6 x 10^{-2} kg.

Determine the,

(a) the number of moles of iron atoms in the cube,

(b) the mass of an iron atom (^{56}Fe) and

(c) the number of iron atoms in the cube.

Q6 J79/II/29 (modified)
The experimental measurement of the heat capacity C of a solid as a function of temperature T is to be fitted to the expression

$$C = \alpha T + \beta T^3$$

What are the possible units of α and β?
[Note: Heat capacity is the amount of heat required to raise 1 K of temperature of the substance.]

	α	β
A	J	J K^{-2}
B	J K^2	J
C	J K	J K^3
D	J K^{-2}	J K^{-4}
E	J	J

Estimation

Q7 Estimate the number of times a human heart beats in a lifetime. [~3 x 10^9] [2]

Q8 Which of the following statements is/are reasonable?

a. Joe is 180 cm tall.

b. I rode my bike to campus at a speed of 50 m s^{-1}.

c. I can throw a ball a distance of 2 km.

d. Joan's newborn baby has a mass of 33 kg.

e. I can throw a ball at a speed of 50 km/hr.

f. The atmospheric pressure is 100 Pa.

g. The power of a LED light bulb is 200 W.

Uncertainty and Error

Q9 The specification of a digital voltmeter is: "accuracy ± 1%".
The meter reads 3.924 V. What should be the recorded reading together with its uncertainty?

A (3.92±0.039) V **B** (3.90±0.04) V **C** (3.92±0.04) V **D** (3.924±0.039) V

Q10 N94/I/2 (modified)
A resistor is marked as having a value of 4.7 Ω ± 2%. The power P dissipated in the resistor, when connected in a simple electrical circuit, was to be calculated from the current in the resistor, which measured as (2.50 ± 0.05) mA.

What is the percentage uncertainty in the calculated value of P? (note: $P = I^2R$)

A 2% **B** 4% **C** 6% **D** 8%

Q11 N10/I/3

A wire of uniform circular cross-section has diameter *d* and length *L*. A potential difference *V* between the ends of the wire gives rise to a current I in the wire.

The resistivity ρ of the material of the wire is given by the expression

$$\rho = \frac{\pi d^2 V}{4LI}$$

In one particular experiment, the following measurements are made.

d = 1.20 ± 0.01 cm
I = 1.50 ± 0.05 A
L = 100 ± 1 cm
V = 5.0 ± 0.1 V

Which measurement gives rise to the least uncertainty in the value for the resistivity?

A d **B** I **C** L **D** V

Q12 N12/I/2

The equation connecting object distance u, image distance v and focal length f for a lens is:

$$\frac{1}{f} = \frac{1}{u} + \frac{1}{v}$$

A student measures values of u and v, with their associated uncertainties. These are:

u = 50mm ± 3mm,
v = 200mm ±5mm.

He calculates the value of f as 40 mm. What is the uncertainty in the value of f? [3]

[±2 mm]

Q13 N07/II/1 (modified)

A student times the fall of a small metal ball. Data for the time t taken for the ball to fall a vertical distance h from rest are given below.

h = (266 ± 1) cm
t = (0.740 ± 0.005) s

The relationship between h, g and t is

$$h = \frac{1}{2}gt^2$$

where g is acceleration of free fall.

(a) Use these data to determine
 (i) a value, to three significant figures, of the acceleration of free fall g, [2]
 (ii) the percentage uncertainty, to two significant figures of
  1. the distance h, [2]
  2. the time t. [2]

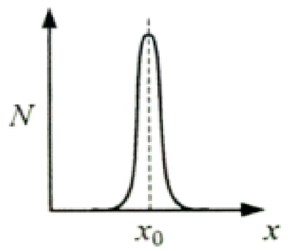
(b) Use your answers in (a) to determine the actual uncertainty in the value g. Hence give a statement of g, with its uncertainty, to an appropriate number of significant figures. [3]

(c) Suggest two reasons, why, in this experiment, although the value of t is precise, it may not be accurate. [2]

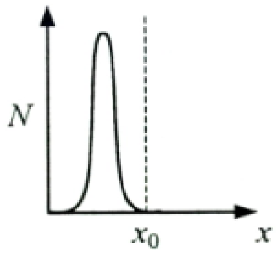
[9.72 m s^{-2}, 0.38%, 0.68%, (9.7 ± 0.2) m s^{-2}]

Q14 The true value of a quantity x is x_0. In an experiment, the quantity is measured many times and the number N of readings giving a value x is plotted against x.
Which of the following graphs best shows measurements that are precise but not accurate?

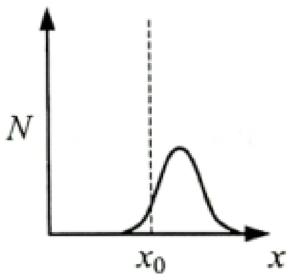
A



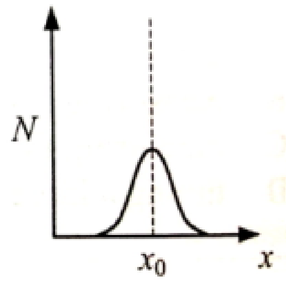
B



C



D



Q15 N02/I/2
Four balances were used to measure the mass of a 1.000 kg weight. The reading was taken five times at each balance. The values obtained and the means are shown in the given table. Which balance is not very precise but has the smallest systematic error?

Balance	Reading / kg					Mean / kg
	1	2	3	4	5	
A	1.000	1.000	1.002	1.001	1.002	1.001
B	1.011	0.999	1.001	0.989	0.995	0.999
C	1.012	1.013	1.012	1.014	1.014	1.013
D	0.993	0.987	1.002	1.000	0.983	0.993

Vectors and Vector Addition

Q16 2016/1/3

An aircraft flies with an airspeed of 700 km h^{-1} through a 250 km h^{-1} jet-stream wind from the west. The pilot wishes to fly directly north from Australia towards Changi airport in Singapore. To achieve this, the pilot points the aircraft away from the north direction. What is the speed of the aircraft in the direction of north relative to the ground?

A 450 km h^{-1} **B** 650 km h^{-1} **C** 740 km h^{-1} **D** 950 km h^{-1}

Q17 J89/II/8 (part)

(a) Two vectors **A** and **B** are at right angles to each other. Draw a vector diagram to show how the sum of the vectors could be found. [2]

(b) A car changes its velocity from 30 m s^{-1} due east to 25 m s^{-1} due south.

(i) Draw a vector diagram to show the initial and final velocities and the change in velocity. [2]

(ii) Calculate the change in speed. [1]

(iii) Calculate the change in velocity. [2]

(iv) "The answers in (bii) and (biii) should be the same," a student said. With reference to your answers, comment on the student's statement. [2]

Assignment Questions

A1 H1 N07/I/2

When a beam of light is incident on a surface, it delivers energy to the surface. The intensity of the beam is defined as the energy delivered per unit area unit time.

What is the unit of intensity, expressed in SI base units?

A kg m^{-2} s^{-1} **B** kg m^2 s^{-3} **C** kg s^{-2} **D** kg s^{-3}

A2 The length of a piece of paper is measured as (297 ±1) mm. The width is (209 ±1) mm.

(a) What is the fractional uncertainty in its length? [2]

(b) What is the percentage uncertainty in its length? [2]

(c) What is the area of one side of the piece of paper? State your answer with its uncertainty. [3]

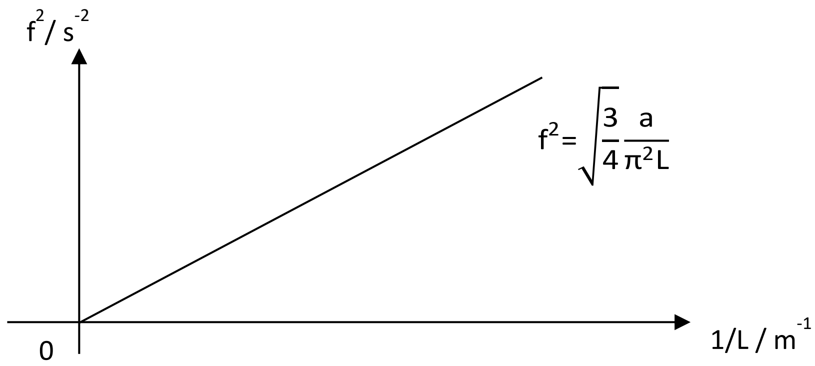
(d) Estimate the number of atoms in the piece of paper. [4]

[0.003 , 0.3% ,(62100 ± 500) mm^2 or (6.21±0.05) x 10^4 mm^2, 6 x 10^{21}]

A3 A student obtained the following graph from a school-based physics practical. The equation that relates the two quantities, f and L is given as

$$f^2 = \sqrt{\frac{3}{4}\frac{a}{\pi^2 L}}$$

where a and π are constant



(a) From the information given in the graph, derive the unit for the constant a. [2]

(b) Has the student obtained a large or small systematic error? Justify your answer. [2]

(c) Why must a line of best-fit always be drawn in all Physics practical? [1]

[s^{-2} m, small]

Supplementary Questions

S1 How many significant figures does each of the following numbers have?

a. 6.21 e. 0.0621 i. 600
b. 62.1 f. 0.620 j. 6.21×10^3
c. 6210 g. 0.62 k. 6.21×10^{-3}
d. 6210.0 h. 1.062 l. 62.1×10^3

S2 Compute the following numbers, leave your answer to the appropriate number of significant figures.

a. 33.3 x 25.4 e. (4.32 x 1.23) – 5.1
b. 33.3 – 25.4 f. 33.3 ÷ 45.1
c. 33 x 25.4 g. $\sqrt{33.3}$
d. 33.3^2 h. 2.345 x 3.321

S3 Express the following numbers and computed results in scientific notation.

a. 9827 d. 32014 x 47
b. 0.000000550 e. 0.059 ÷ 2304
c. 3200000 f. 320.0 x 0.050

Quantities and units

S4 (a) List the seven base quantities and their units. [3]
(b) Express the units of the following physical quantities in terms of base units:
(i) Energy, [2]
(ii) Magnetic flux density, B using the expression F = BIL where F is the force, L is the length of the conductor carrying a current I. [2]

[kg m^2 s^{-2}, kg A^{-1} s^{-2}]

S5 N01/II/3
The unit of a physical quantity may be shown with a prefix. For example, the prefix micro (μ) has the decimal equivalent 10^{-6}, so that 1 microamp (1μA) can be written as 10^{-6} A. Complete the table below, to show each prefix with its corresponding decimal equivalent.

Prefix	Decimal equivalent
Pico	
Micro	10^{-6}
Giga	
	10^{12}

[3]

[10^{-12}, 10^9, tera]

S6 Convert the following to SI units. Use scientific notation and apply the proper use of significant figures. .

a. 9.12 μs d. 55 cm / ms g. 50.9 mm^3
b. 3.42 km e. 80 km/ hr h. 70.1 g mm^{-3}
c. 44 cm f. 60.5 cm^2 i. 30 years

Estimation

S7 N08/I/2

Which estimate is realistic?

A The kinetic energy of a bus travelling on an expressway is 30 000 J.

B The power of a domestic light is 300 W.

C The temperature of a hot oven is 300 K.

D The volume of air in a car tyre is 0.03 m^3.

Uncertainty and Error

S8 N17/I/2

After taking measurements of the quantities in the expression $\frac{xy^2}{z}$, the total uncertainty is calculated as 6%.

Which individual percentage uncertainties of x, y and z when combined give this total of 6%?

	x/%	y/%	z/%
A	1	1	4
B	2	1	2
C	3	2	2
D	4	1	1

S9 A physical equation is of the form

$$Q = k\frac{r^3(P_1 - P_2)}{L}$$

where k is a constant, r = (1.55 $\pm$ 0.03) mm, P_1 = (125 $\pm$ 1) kPa, P_2 = (100 $\pm$ 1) kPa, L = (120 $\pm$ 5) m.

What is the fractional uncertainty of Q? [3]

[0.2]

S10 The external diameter d_1 and the internal diameter d_2 of a metal tube were quoted as (64±2) mm and (47±1) mm respectively.

What is the percentage error in (d_1 - d_2)?

A 0.3% **B** 1% **C** 6% **D** 18%

S11 N18/I/1

The period *T* of a simple pendulum of length *L* is given by the following equation

$$T = 2\pi\sqrt{\frac{L}{g}}$$

where *g* is the acceleration of free fall.

In an experiment to measure the acceleration *g*, the length *L* is measured as (6.25 ± 0.05) cm. What is the maximum percentage uncertainty in T such that g can be determined with a maximum uncertainty of ±2%?

A 0.6 % **B** 1.2 % **C** 1.4 % **D** 2.8 %

S12 N11/I/2 (modified)

In an experiment to measure the viscosity η of a liquid, the following equation was used:

$$\eta = \frac{kr^2}{v}$$

where r = (0.83 ± 0.01) mm
v = (0.065 ± 0.002) m s^{-1}
and k is a constant of value 93.7 N m^{-3}.

How should the value of η be expressed? [4]

[(9.9 ± 0.5) x 10^{-4} N m^{-2} s]

S13 J03/I/4

Which one of the following techniques could reduce or eliminate the systematic error of the quantity being measured?

A adjusting an ammeter to remove its zero error before measuring a current
B measuring several internodal distances on a standing wave to find the mean intermodal distance
C measuring the diameter of a wire repeatedly and calculating the average
D timing a large number of oscillations to find a period

S14 H1 N07/I/1

A voltmeter connected across a resistor in a circuit gives readings which have high precision but low accuracy.

Which of the following best describes the likely error in readings taken with this voltmeter?

	random error	systematic error
A	high	high
B	high	low
C	low	high
D	low	low

S15 Several pairs of readings are plotted on a graph and the gradient of the best fit line is used to determine the physical quantity. Which of the following statements about this method is incorrect?

A It reduces the effects of random errors.
B It may be possible to identify "poor" readings.
C It may be possible to identify and avoid random errors.
D It may be possible to identify and avoid systematic errors.

Vectors and Vector Addition

S16 A stone attached to a string is whirled round a horizontal circle with a constant speed of 10 m s^{-1}. Calculate the difference in velocity when the stone is

(i) at opposite ends of a diameter [2]

(ii) in two positions A and B where AOB is 90° and O is the centre of the circle [3]

[20 m s^{-1} in the direction of $\vec{v}_{final}$, 14.1 m s^{-1}, 45° counter-clockwise of $\vec{v}_{final}$]