*Mathematical Formulae*

## 1. ALGEBRA

*Quadratic Equation*

For the equation $ax^2 + bx + c = 0$,

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

*Binomial expansion*

$$(a+b)^n = a^n + \binom{n}{1} a^{n-1} b + \binom{n}{2} a^{n-2} b^2 + .... + \binom{n}{r} a^{n-r} b^r + \; .... + b^n,$$

where $n$ is a positive integer and $\binom{n}{r} = \frac{n!}{r!(n-r)!} = \frac{n(n-1)\ldots(n-r+1)}{r!}$

## 2. TRIGONOMETRY

*Identities*

$$\sin^2 A + \cos^2 A = 1$$

$$\sec^2 A = 1 + \tan^2 A$$

$$\operatorname{cosec}^2 A = 1 + \cot^2 A$$

$$\sin(A \pm B) = \sin A \cos B \pm \cos A \sin B$$

$$\cos(A \pm B) = \cos A \cos B \mp \sin A \sin B$$

$$\tan(A \pm B) = \frac{\tan A \pm \tan B}{1 \mp \tan A \tan B}$$

$$\sin 2A = 2 \sin A \cos A$$

$$\cos 2A = \cos^2 A - \sin^2 A = 2\cos^2 A - 1 = 1 - 2\sin^2 A$$

$$\tan 2A = \frac{2\tan A}{1 - \tan^2 A}$$

*Formulae for ΔABC*

$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$

$$a^2 = b^2 + c^2 - 2bc \cos A$$

$$\Delta = \frac{1}{2} bc \sin A$$

**1** Given that $\sin A = p$, where $A$ is obtuse, find

**(i)** $\cos A$, [2]

**(ii)** $\cot A$. [1]

**2** Find the range of values of $k$ for which $x^2 + 12x + 9$ is always greater than $4x + k$. [4]

**3** Given that $y = \dfrac{3e^{2x}}{2x+1}$, find the value of $k$ for which $\dfrac{dy}{dx} = \dfrac{kxy}{2x+1}$. [4]

**4** The radius of a circle increases at a rate of $3\,\text{cm}\,\text{s}^{-1}$. Find, in terms of $\pi$, the rate of increase of the area when

**(i)** the radius is 5 cm, [3]

**(ii)** the area is $4\pi\ \text{cm}^2$. [3]

**5** The function f is defined by $f(x) = 1 + 2\cos 2x$ for $0 \le x \le \pi$.

**(i)** State the amplitude and the period of f($x$). [2]

**(ii)** State the minimum and maximum values of f($x$). [2]

**(iii)** Sketch the graph of $y = f(x)$ for $0 \le x \le \pi$. [2]

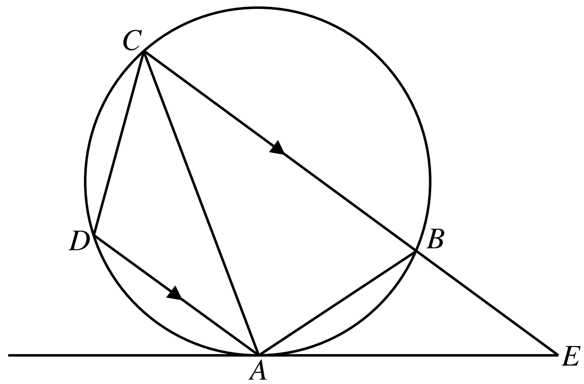
**6** A certain species of fish was introduced into a pond and its number was tracked daily. It was observed that $t$ days later, its population, $P$, was given by $P = 5000 + Ae^{kt}$, where $A$ and $k$ are constants. Initially, there were 3000 fishes in the pond. 10 days later, 3800 fishes were left.

**(i)** Show that $A = -2000$. [1]

**(ii)** Find the value of $k$, giving your answer correct to 3 significant figures. [3]

**(iii)** Explain why the number of fishes can never reach 5000. [1]

**7**




The diagram shows a quadrilateral $ABCD$ whose vertices lie on the circumference of the circle. The point $E$ lies on the extended line $CB$ such that $AE$ is a tangent to the circle. $CE$ and $DA$ are parallel lines.

**(i)** Explain why angle $BAE$ = angle $CAD$. [2]

**(ii)** Show that triangles $BAE$ and $DAC$ are similar. [2]

**(iii)** Given that $AB = BE$, explain why the line $AC$ bisects the angle $BCD$. [3]

**8** The first two non-zero terms in the expansion of $(1 + bx)(1 + ax)^6$ are 1 and $-\frac{21}{4}x^2$ .

Given that $a < b$, find the value of $a$ and of $b$. [7]

**9** **(a)** Solve the simultaneous equations.

$$(4^x)(8^y)=1$$

$$125^y \div \left(\sqrt[3]{5}\right)^x = \frac{1}{\sqrt{5}}$$ [5]

**(b)** A cylinder has a radius of $\left(\sqrt{2}-1\right)$ cm and a volume of $\left(12+4\sqrt{2}\right)\pi\ \text{cm}^3$.
Find, without using a calculator, the exact value of its height, *h*, in the form $a+b\sqrt{2}$, where *a* and *b* are integers. [4]

**10** Given that $y = x(2x-4)^3$,

**(i)** find the coordinates of the stationary points of the curve, [6]

**(ii)** determine the nature of each of these points. [4]

**11** A particle $P$ moves along a horizontal straight line so that $t$ seconds after motion has begun, its displacement, $s$ m, from a fixed point $O$, is given by $s = t^3 + 5t^2 - 2t + 4$.

**(i)** Express the velocity of $P$ in terms of $t$. [1]

A second particle $Q$ starts its motion at the same instant as $P$ and moves along the same horizontal line as $P$. Its acceleration, $a$ m/s$^2$, $t$ seconds after motion has begun, is given by $a = 6t + 4$.
$Q$ has a velocity of 4 m/s when $t = 1$, and an initial displacement of 6 m.

**(ii)** Obtain expressions, in terms of $t$, for the velocity and for the displacement of $Q$. [4]

**(iii)** Find the value of $t$ at the instant when $P$ and $Q$ collide and determine whether $P$ and $Q$ are travelling in the same direction at this instant. [4]

**12** **(a)** **(i)** Show that $x^2 - 10x + 21$ can never be smaller than $-4$. [2]

**(ii)** Given that $y = x^2 - 10x + 21$, write down the coordinates of the turning point and state whether the point is a maximum or minimum. [2]

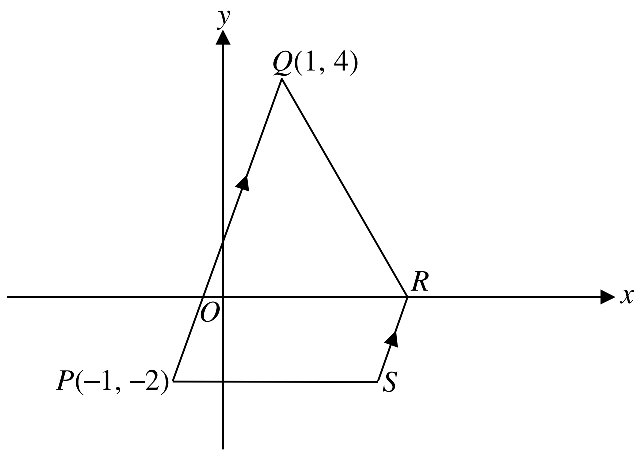
**(iii)** Solve the equation $x^2 - 10x + 21 = 0$. [2]

**(iv)** Hence, sketch the graph of $y = x^2 - 10x + 21$. [2]

**[Turn over**

**(b)** Given that $6x^2 + \frac{x}{a} - \frac{1}{b^2} = 0$, where $a, b \neq 0$, show that there are no values of $a$ and $b$ for which the equation has equal roots. [2]

**13 Solutions to this question by accurate drawing will not be accepted.**




The diagram shows a trapezium $PQRS$ in which $PQ$ is parallel to $SR$.
The point $P$ is $(-1,-2)$ and the point $Q$ is $(1, 4)$.

**(i)** Find the equation of the perpendicular bisector of $PQ$. [4]

The perpendicular bisector of $PQ$ meets the $x$-axis at point $R$.

**(ii)** Find the coordinates of $R$. [2]

**[Turn over**

**(iii)** Given that the equation of *RS* is $y = 3x - 9$ and *PS* is parallel to the *x*-axis, find the coordinates of *S*. [2]

**(iv)** Calculate the area of the trapezium *PQRS*. [2]

**End of Paper**

## Answer Key

| | |
|---|---|
| 1i | $-\sqrt{1-p^2}$ |
| 1ii | $\dfrac{-\sqrt{1-p^2}}{p}$ |
| 2 | $k<7$ |
| 3 | $k=4$ |
| 4i | $30\pi\ \text{cm}^2\ \text{s}^{-1}$ |
| 4ii | $12\pi\ \text{cm}^2\ \text{s}^{-1}$ |
| 5i | Amplitude = 2, Period = $\pi$ |
| 5ii | Minimum value = −1<br>Maximum value = 3 |
| 6ii | $=-0.0511$ |
| 8 | $a=-\dfrac{1}{2}, b=3$ |
| 9a | $\therefore y=-\dfrac{1}{7}$ |
| 9b | $h=52+36\sqrt{2}$ |
| 10i | $(2,0)$ and $\left(\dfrac{1}{2},-13\dfrac{1}{2}\right)$ |
| 10ii | (2, 0) is a point of inflexion.<br>$\left(\dfrac{1}{2},-13\dfrac{1}{2}\right)$ is a minimum point |
| 11i | $v=\dfrac{ds}{dt}$<br>$=3t^2+10t-2$ |
| 11ii | $\therefore s=t^3+2t^2-3t+6$ |
| 11iii | $P$ and $Q$ are travelling in ***<u>same</u>*** direction |
| 12aii | $(5,-4)$, Minimum |
| 12aiii | $x=7$ or $x=3$ |
| 13i | $y=-\dfrac{1}{3}x+1$ |
| 13ii | (3, 0) |
| 13iii | $\left(\dfrac{7}{3},-2\right)$ |
| 13iv | $\dfrac{40}{3}$ units$^2$ |

[Turn over