Name: _____ ( ) Class: _____

PRELIMINARY EXAMINATION
GENERAL CERTIFICATE OF EDUCATION ORDINARY LEVEL

ADDITIONAL MATHEMATICS 4047/02

Paper 2 **Monday 21 August 2017**

2 hours 30 minutes

Additional Materials: Answer Paper
Graph Paper

READ THESE INSTRUCTIONS FIRST

Write your name, class, and index number on all the work you hand in.
Write in dark blue or black pen on both sides of the paper.
You may use a pencil for any diagrams or graphs.
Do not use paper clips, highlighters, glue, or correction fluid.

Answer **all** the questions.
Write your answers on the separate Answer Paper provided
Give non-exact numerical answers correct to 3 significant figures, or 1 decimal place in the case of angles in degrees, unless a different level of accuracy is specified in the question.
The use of an approved scientific calculator is expected, where appropriate.
You are reminded of the need for clear presentation in your answers.

At the end of the examination, staple all your work together with this cover sheet.
The number of marks is given in brackets [ ] at the end of each question or part question.
The total number of marks for this paper is **100**.

FOR EXAMINER'S USE

Q1		Q5		Q9	
Q2		Q6		Q10	
Q3		Q7		Q11	
Q4		Q8			

100

This document consists of **6** printed pages.

Mathematical Formulae

1. ALGEBRA

Quadratic Equation

For the equation $ax^2 + bx + c = 0$,

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

Binomial Theorem

$$(a+b)^n = a^n + \binom{n}{1}a^{n-1}b + \binom{n}{2}a^{n-2}b^2 + ... + \binom{n}{r}a^{n-r}b^r + ... + b^n,$$

where n is a positive integer and $\binom{n}{r} = \frac{n!}{(n-r)!r!} = \frac{n(n-1).....(n-r+1)}{r!}$

2. TRIGONOMETRY

Identities

$$\sin^2 A + \cos^2 A = 1.$$

$$\sec^2 A = 1 + \tan^2 A.$$

$$\operatorname{cosec}^2 A = 1 + \cot^2 A.$$

$$\sin(A \pm B) = \sin A\cos B \pm \cos A\sin B$$

$$\cos(A \pm B) = \cos A\cos B \mp \sin A\sin B$$

$$\tan(A \pm B) = \frac{\tan A \pm \tan B}{1 \mp \tan A\tan B}$$

$$\sin 2A = 2\sin A\cos A$$

$$\cos 2A = \cos^2 A - \sin^2 A = 2\cos^2 A - 1 = 1 - 2\sin^2 A$$

$$\tan 2A = \frac{2\tan A}{1 - \tan^2 A}$$

Formulae for ΔABC

$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$

$$a^2 = b^2 + c^2 - 2bc\cos A$$

$$\Delta = \frac{1}{2}bc\sin A$$

1 The curve $y = \mathrm{f}(x)$ is such that $\mathrm{f}'(x) = 2x^2 - 4x + 5$.

(i) Explain why the curve $y = \mathrm{f}(x)$ has no stationary points. [2]

(ii) Given that the curve passes through the point (–1, –6), find an expression for $\mathrm{f}(x)$. [3]

2 The function f is defined by $\mathrm{f}(x) = x^4 - 2x^3 + kx^2 + 8$, where k is a constant.

It is given that $\mathrm{f}(x) = 0$ has a repeated root 2.

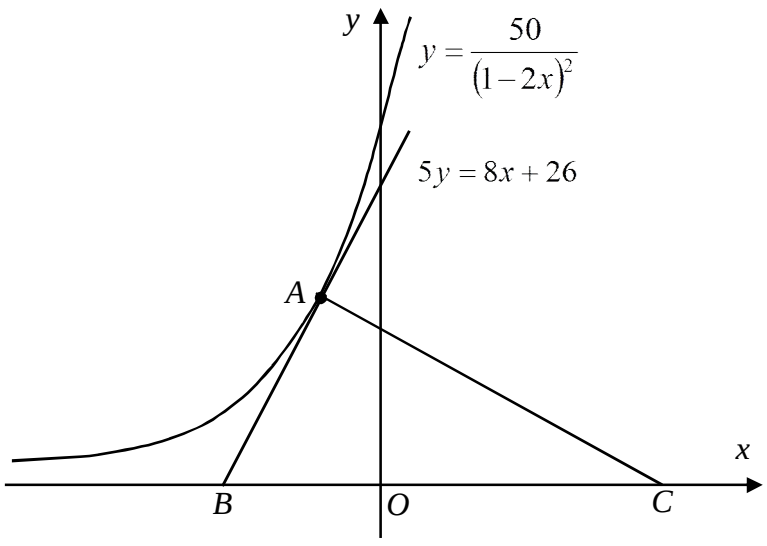
(i) Find the value of k, [2]

(ii) Determine, showing all necessary working, the number of real solution(s) of the equation $\mathrm{f}(x) = 0$. [4]

3 The roots of the equation $2x^2 - 4x + p = 0$, where p is a constant, are α and β.

The roots of the equation $6x^2 + qx + 9 = 0$, where q is a constant, are $\frac{\alpha^2}{\beta}$ and $\frac{\beta^2}{\alpha}$. Find the value of p and of q. [6]

4



The diagram shows part of the curve $y = \frac{50}{(1-2x)^2}$. The tangent $5y = 8x + 26$ at the point A on the curve cuts the x-axis at B. The normal at A cuts the x-axis at C. Find the area of triangle ABC. [9]

5 **(a)** **(i)** Write down the general term in the binomial expansion of $\left(\frac{x^2}{4}+\frac{2}{x}\right)^{18}$. [1]

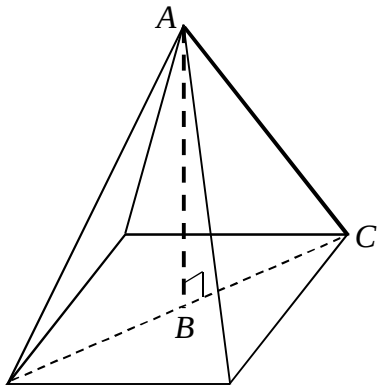
(ii) Write down the power of x in this general term. [1]

(iii) Hence, or otherwise, determine the term independent of x in the binomial expansion of $\left(\frac{x^2}{4}+\frac{2}{x}\right)^{18}$. [2]

(b) In the binomial expansion of $(2-kx)^n$, where $n \geq 3$ and k is a constant, the coefficients of x and x^2 are equal. Express k in terms of n. [4]

6 **Do not use a calculator in this question.**

(i) Express $\frac{26\sqrt{3}}{3\sqrt{3}+1}$ in the form $a+b\sqrt{3}$, where a and b are integers. [3]



A toy is modelled in the shape of a right pyramid with a square base as shown in the diagram. The vertical height AB of the pyramid is $\left(\sqrt{3}+4\right)$ cm. Given that the length of the slant edge AC is $\frac{26\sqrt{3}}{3\sqrt{3}+1}$ cm,

(ii) find an expression for BC^2 in the form $c+d\sqrt{3}$, where c and d are integers, [3]

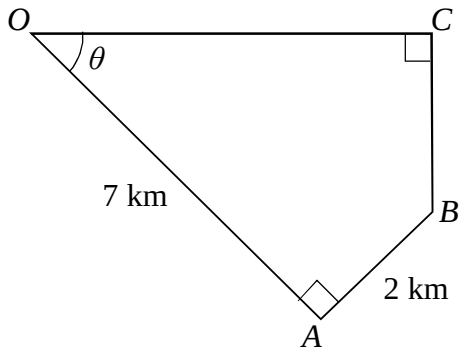
(iii) express the volume of the pyramid in the form $\frac{2}{3}\left(k-39\sqrt{3}\right)$ cm^3, where k is an integer. [2]

7 **(i)** Differentiate $\ln\left(2x^2+1\right)$ with respect to x. [2]

(ii) Express $\dfrac{4x^2+2x+1}{(x+1)\left(2x^2+1\right)}$ in partial fractions. [4]

(iii) Hence evaluate $\int_0^{\frac{1}{2}} \dfrac{4x^2+2x+1}{(x+1)\left(2x^2+1\right)}\,\mathrm{d}x$. [4]

8



The diagram shows a cycling circuit formed from four straight roads OA, AB, BC and CO. OA = 7 km, AB = 2 km, angle OAB = angle BCO = 90°, and angle $COA = \theta$ where $0° \le \theta \le 90°$. A cyclist cycled along the circuit $OABCO$.

(i) Show that $OA + AB + BC + CO = 9\sin\theta + 5\cos\theta + c$, where c is a constant to be found. [2]

(ii) Express $OA + AB + BC + CO$ in the form $R\cos(\theta - \alpha) + c$. [4]

(iii) Find the values of θ for which the cyclist cycled a total distance of 19 km. [3]

(iv) State the maximum possible value for the total distance of the circuit. [1]

9 The equation of a curve is $y = (1+2x)^3(3-2x)$.

(i) Find the coordinates of the stationary points of the curve. [6]

(ii) Determine the nature of these stationary points. [3]

(iii) Hence, sketch the curve. [2]

10 A circle, C_1, passes through the points A (–3, –25) and B (7, –25). The centre and radius of the circle are (a, b) and r respectively. The x-axis is a tangent to the circle.

(i) Find the value of a. [1]

(ii) Show that $b = -13$. [3]

(iii) Find the equation of the circle C_1. [1]

(iv) AT is a diameter of the circle. Find the equation of the tangent to the circle at T. [5]

(v) Given that a second circle, C_2, is a reflection of C_1 in the y-axis, find the equation of C_2. [2]

11 A student records the population, y, of a certain type of bacteria, x hours after the start of her experiment, in the table below. It is believed that an error was made in recording one of the values. The student wants to use the equation $y = ab^x$, where a and b are constants, to model the population of this type of bacteria.

x	1.5	3	4.5	6	7.5
y	8.37	13.8	22.8	47.5	62.0

(i) Plot lg y against x and hence determine which value of y in the table above is the incorrect recording, and estimate the correct value of y. [5]

(ii) Hence, estimate the value of each of the constants a and b. [5]

(iii) The population of a second type of bacteria is modelled by the equation $y = 2^x$. Using your values of a and b, calculate the value of x for which $ab^x = 2^x$. [2]

(iv) Explain how another straight line drawn on your diagram can lead to an estimate of the value of x for which the populations of the two types of bacteria are equal. Draw this line and hence verify your value of x found in part **(iii)**. [3]

End of Paper.