West Spring Secondary School
PRELIMINARY EXAMINATION 2022

Mathematics **4048/02**

Paper 2

Secondary 4 EXPRESS / 4 NORMAL (ACADEMIC) OOS /
5 NORMAL (ACADEMIC)

Name _____ ( ) **Date** 30 August 2022

Class _____ **Duration** 2 hours 30 minutes

Candidates answer on the Question Paper.

No Additional Materials are required.

READ THESE INSTRUCTIONS FIRST

Write your name, index number and class in the spaces at the top of this page.
Write in dark blue or black pen on both sides of the paper.
You may use an HB pencil for any diagrams or graphs.
Do not use staples, paper clips, glue or correction fluids.

Answer **all** questions on the Question Paper.
If working is needed for any question it must be shown with the answer.
Omission of essential working will result in loss of marks.
The use of an approved scientific calculator is expected, where appropriate.
If the degree of accuracy is not specified in the question, and if the answer is not exact, give the answer to three significant figures. Give answers in degrees to one decimal place.
For π, use either your calculator value or 3.142, unless the question requires the answer in terms of π.

At the end of the examination, fasten all your work securely together.
The number of marks is given in brackets [ ] at the end of each question or part question.
The total number of marks for this paper is 100.

FOR EXAMINER'S USE	
	/100

Mathematical Formulae

Compound interest

$$\text{Total amount} = P\left(1+\frac{r}{100}\right)^n$$

Mensuration

$$\text{Curved surface area of a cone} = \pi rl$$

$$\text{Surface area of a sphere} = 4\pi r^2$$

$$\text{Volume of a cone} = \frac{1}{3}\pi r^2 h$$

$$\text{Volume of a sphere} = \frac{4}{3}\pi r^3$$

$$\text{Area of triangle } ABC = \frac{1}{2}ab\sin C$$

$$\text{Arc length} = r\vartheta, \text{ where } \vartheta \text{ is in radians}$$

$$\text{Sector area} = \frac{1}{2}r^2\vartheta, \text{ where } \vartheta \text{ is in radians}$$

Trigonometry

$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$

$$a^2 = b^2 + c^2 - 2bc\cos A$$

Statistics

$$\text{Mean} = \frac{\sum fx}{\sum f}$$

$$\text{Standard deviation} = \sqrt{\frac{\sum fx^2}{\sum f} - \left(\frac{\sum fx}{\sum f}\right)^2}$$

Answer **all** questions

1 The table below shows the amount of flour, butter and sugar needed in making a sponge cake and a butter cake respectively.

	Flour	**Butter**	**Sugar**
Sponge Cake	300 g	300 g	250 g
Butter Cake	450 g	250 g	200 g

(a) Represent the data in the table in a 2×3 column matrix **A**.

Answer **A** = [1]

The cost of 1 g of flour is 0.16 cents, 1 g of butter is 0.48 cents and 1 g of sugar is 0.18 cents.

(b) Evaluate the matrix $\mathbf{P} = \mathbf{A}\begin{pmatrix} 0.16 \\ 0.48 \\ 0.18 \end{pmatrix}$.

Answer **P** = [2]

(c) State what each element of matrix **P** represents.

.....

..... [1]

(d) The total cost of making both sponge cake and butter cake in dollars can be represented in a 1×1 column matrix **M**.

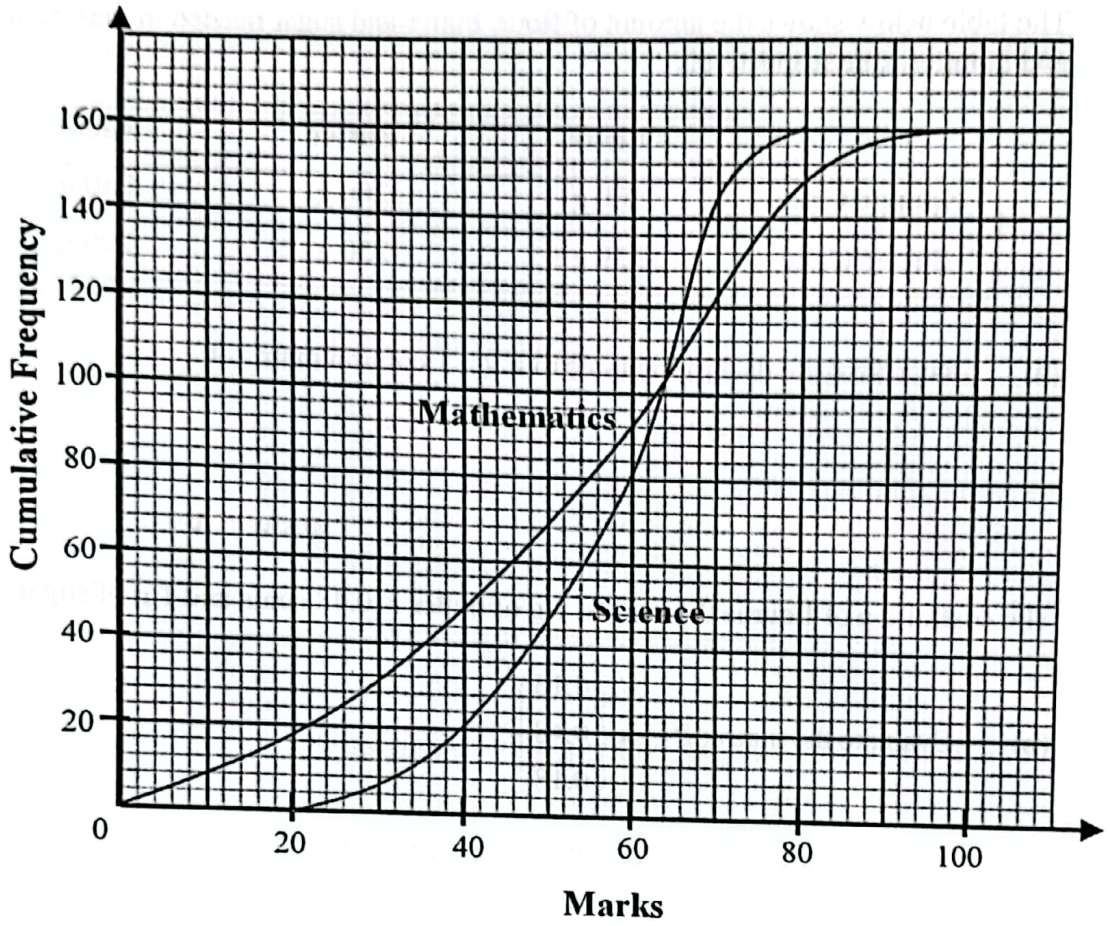
The matrix **M** is defined to be $\frac{1}{100}\begin{pmatrix} a & b \end{pmatrix}\mathbf{P}$, where a and b are integers.

Write down the value of a and the value of b.

Answer a = ...

b = ... [2]

2 The cumulative frequency curve shows the results of 160 students in the Mathematics and Science examination.



(a) A student is chosen at random.

Find the probability of choosing a student who scored more than 60 marks but not greater than 80 marks in the Mathematics examination.

Answer ..... [2]

(b) Given that 50% of the students scored more than x marks in the Mathematics examination, find the value of x.

Answer $x =$ ..... [1]

(c) The 90th percentile of the distribution of the Science marks is 70 marks.

Explain what this tells us about the distribution of the Science marks.

.....

.....

..... [1]

(d) Use the graph to estimate the interquartile range for the Science examination.

Answer ..... marks [2]

(e) Vanessa commented that the students perform better in the Mathematics examination than in the Science examination because the highest marks for Mathematics is higher than that for Science.

Do you agree?
Give a reason for your answer.

.....

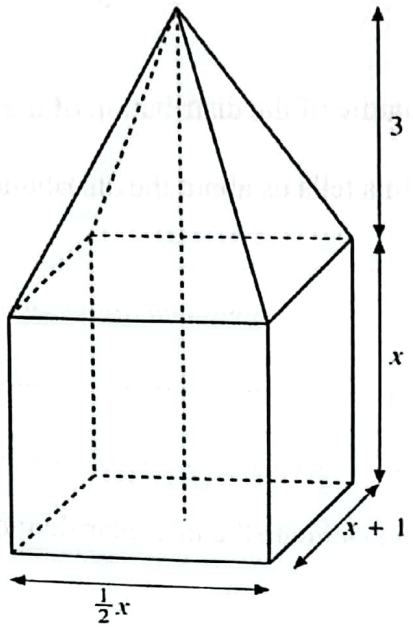
.....

.....

.....

..... [2]

3



The solid consists of a cuboid of dimensions $(\frac{1}{2}x)$ cm by $(x + 1)$ cm by x cm and a pyramid that is mounted on top of it.

The height of the pyramid is 3 cm.

(a) If the volume of the solid is y cm^3, show that $y = \frac{1}{2}x^3 + x^2 + \frac{1}{2}x$.

Answer

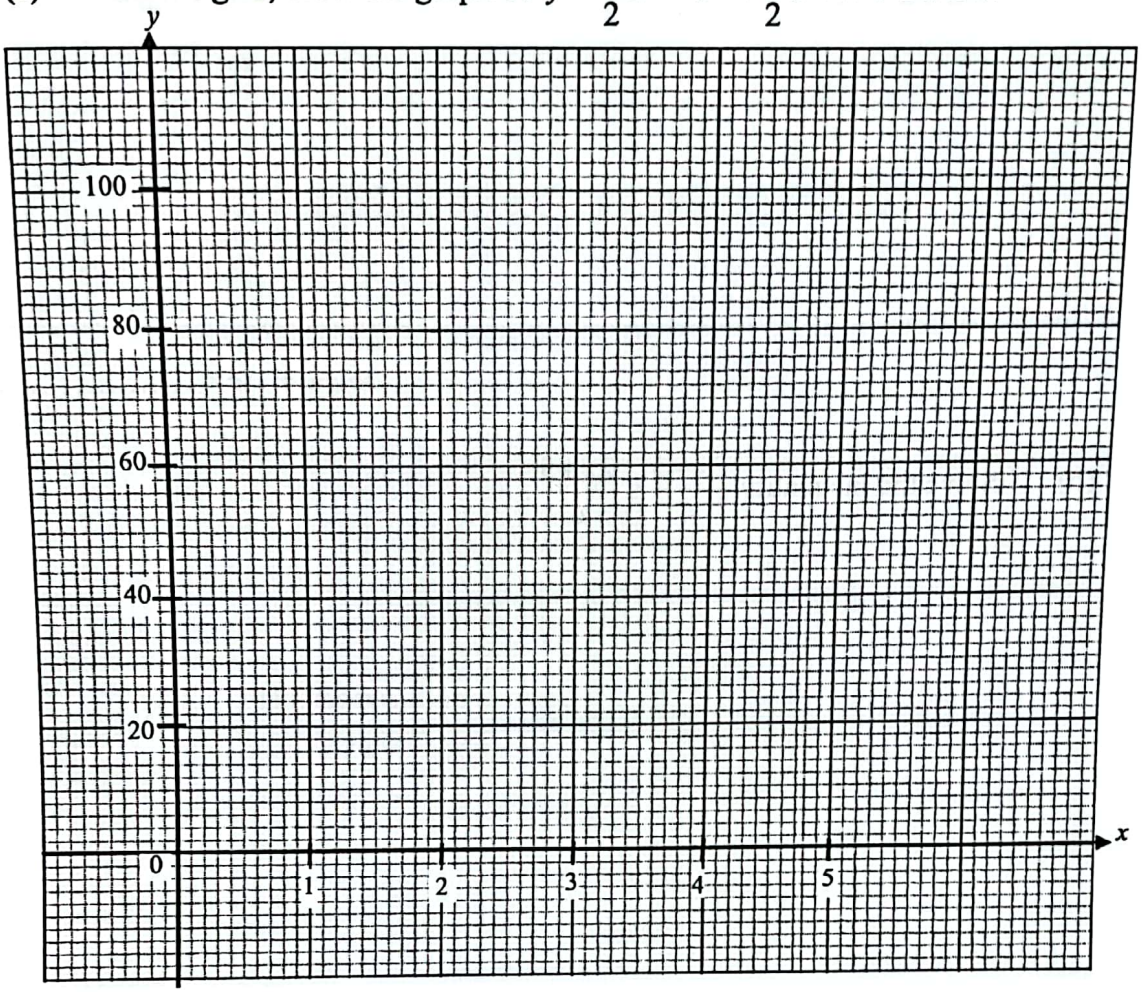
[2]

(b) Complete the table of values for $y = \frac{1}{2}x^3 + x^2 + \frac{1}{2}x$.

x	1	2	3	4	5
y	2	9		50	90

[1]

(c) On the grid, draw the graph of $y = \frac{1}{2}x^3 + x^2 + \frac{1}{2}x$ for $1 \leq x \leq 5$.



[3]

(d) Use your graph to estimate the volume of the solid if the height of the solid is 5.5 cm.

Answer .....cm^3 [1]

(e) Another cuboid of dimensions 5 cm by 4 cm by $(5 - x)$ cm can be formed by reshaping the solid.

Estimate the value of x by drawing a suitable straight line on your graph.

Answer $x =$ ..... [3]

(a) Solve the equation $\frac{-5}{w+2}=5$.

Answer $w =$ ..... [2]

(b) Solve the inequality $\frac{3x-5}{4} \leq \frac{2x+5}{6}$.

Answer ..... [2]

(c) It is given that $a=\frac{7b+6c}{10-b}$.

(i) Find a when $b=6$ and $c=-1$.

Answer $a =$ ..... [1]

(ii) Express b in terms of a and c.

Answer $b =$ ..... [2]

5 (a) $ABCD$ is a parallelogram, such that the point A is (1, 7), point C is (–2, 9) and $\overrightarrow{AB} = \begin{pmatrix} 4 \\ 3 \end{pmatrix}$.

(i) Write down the column vector $\overrightarrow{AC}$.

Answer $\overrightarrow{AC} = \begin{pmatrix} \quad \\ \quad \end{pmatrix}$ [1]

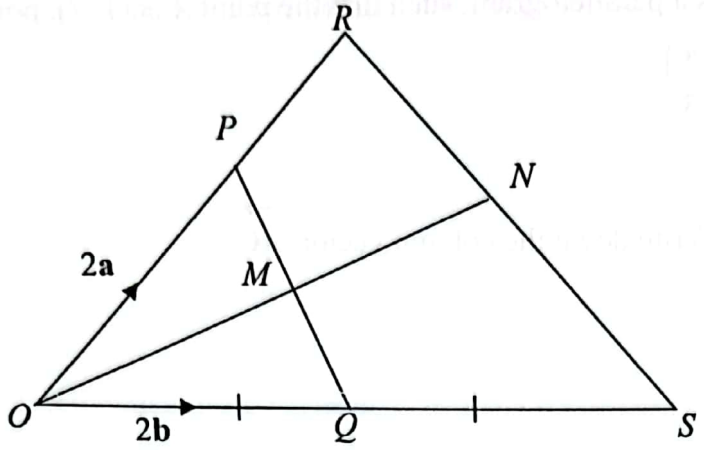
(ii) Find the coordinates of B.

Answer (....., ...) [1]

(iii) Find the coordinates of D.

Answer (....., ...) [2]

(b)



In triangle ORS, point P is on $\overrightarrow{OR}$ is such that $\overrightarrow{OP} = \frac{2}{3}\overrightarrow{OR}$.
M is the midpoint of $\overrightarrow{PQ}$ and Q is the midpoint of $\overrightarrow{OS}$
N is the point on RS such that $RN : NS = 3 : 4$.
$\overrightarrow{OP} = \mathbf{2a}$ and $\overrightarrow{OQ} = \mathbf{2b}$.

(i) Express $\overrightarrow{OM}$, as simply as possible, in terms of **a** and **b**.

Answer $\overrightarrow{OM}$ = ... [1]

(ii) Show that $\overrightarrow{MN} = \frac{5}{7}(\mathbf{a}+\mathbf{b})$.

Answer

[3]

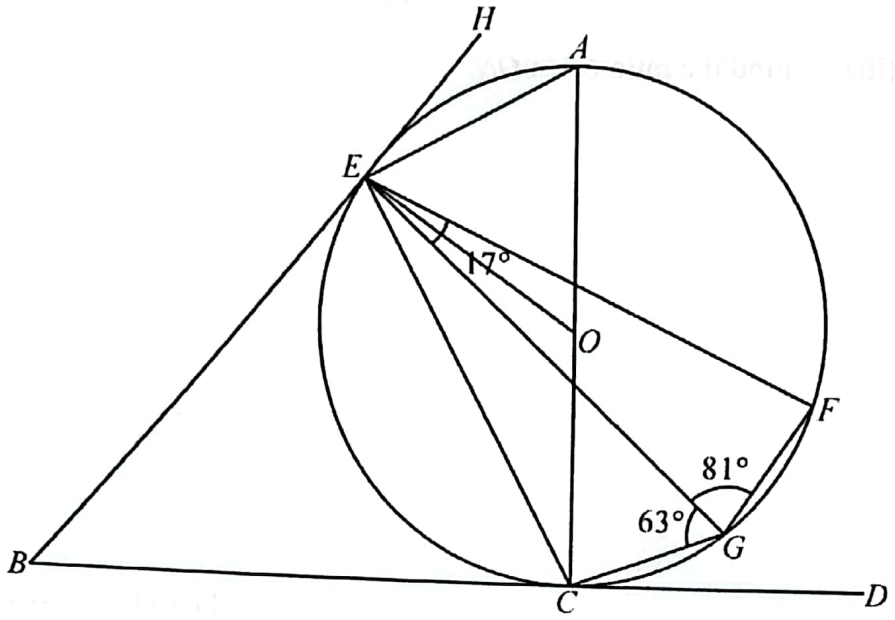
(iii) Find the ratio $OM : ON$.

Answer ..... : ..... [2]

(iv) Find $\dfrac{\text{area of } \Delta PRM}{\text{area of } \Delta ORM}$.

Answer ..... [1]

6



AOC is the diameter of the circle with centre O.
BCD and BEH are tangents to the circle at C and E respectively.
Angle $FEG = 17°$, angle $EGC = 63°$ and angle $EGF = 81°$.

Find, giving a reason for each step of your working,

(a) angle EFG,

Answer Angle EFG = ... [1]

(b) angle CAE,

Answer Angle CAE = ... [1]

(c) angle *CEG*,

Answer Angle *CEG* = ... [1]

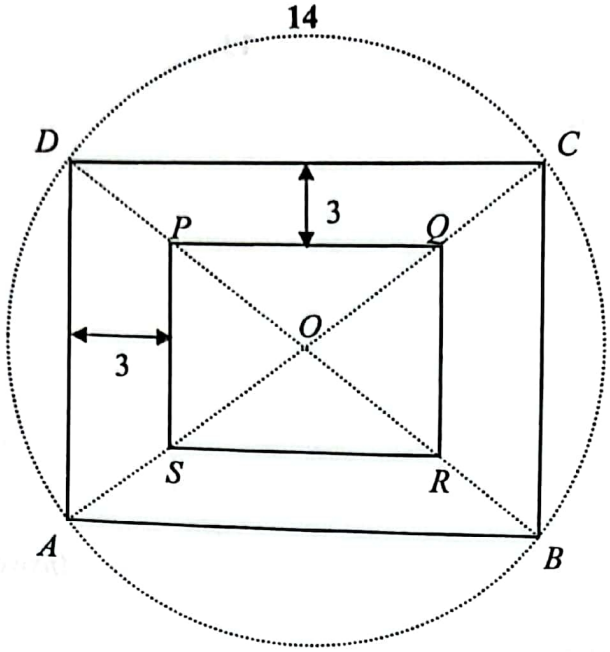
(d) angle *COE*,

Answer Angle *COE* = ... [1]

(e) angle *BCE*.

Answer Angle *BCE* = ... [2]

7



The diagram shows a rectangular lawn $PQRS$ which has an area of 180 m^2.
The lawn is surrounded by a flower bed of 3 metres wide.
A circular sprinkler, O is installed at the centre of the lawn.
It is used to water the entire rectangle $ABCD$.

(a) It is given that PQ is x metres.

(i) Write down an expression, in terms of x, for the length of AB.

Answer $AB =$ .....m [1]

(ii) Write down an expression, in terms of x, for the length of BC.

Answer $BC =$ .....m [1]

(b) The flower bed alone has an area of 198 m^2.

Write down an equation to represent this information and show that it simplifies to $x^2 - 27x + 180 = 0$.

Answer

[3]

(c) Solve the equation $x^2 - 27x + 180 = 0$.

Answer $x =$..... or ..... [3]

(d) Explain why one of the solutions in **part (c)** must be rejected.

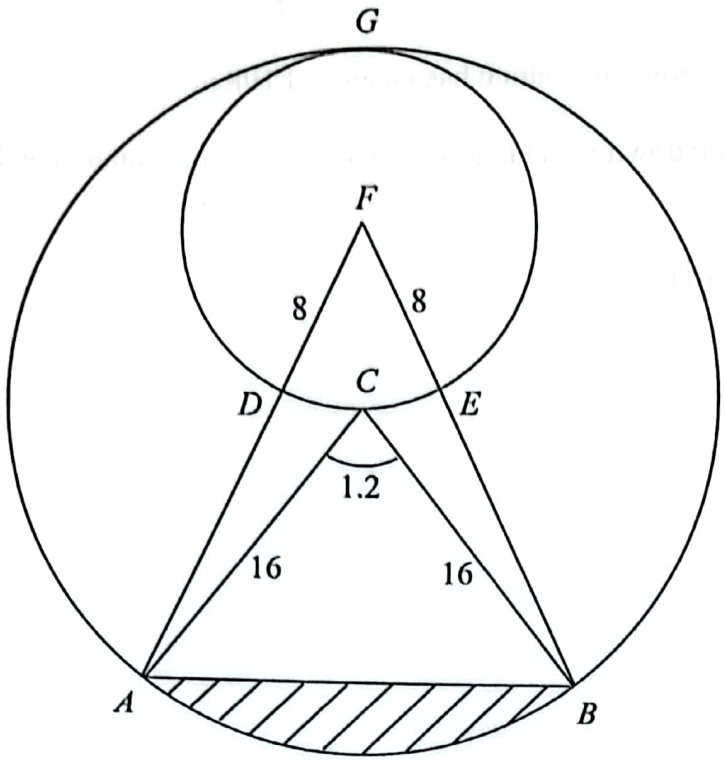
.....

..... [1]

(e) Hence, find the spraying radius of the sprinkler.

Answer .....m [3]

8



Points A and B lie on the larger circle, centre C.
$AC = BC = 16$ cm and angle $ACB = 1.2$ radians.
A smaller circle, centre F, passes through C and touches the larger circle at G.
ADF and BEF are straight lines.
The radius of the smaller circle is 8 cm and angle $DAC = 0.234$ radians.

(a) **(i)** Calculate the length of arc AB.

Answer .....cm [1]

(ii) Find the area of sector ACB.

Answer .....cm^2 [1]

(iii) Calculate the area of the shaded region.

Answer .....cm^2 [2]

(b) **(i)** Show that angle $DFE = 0.732$ rad, correct to 3 significant figures.

Answer

[1]

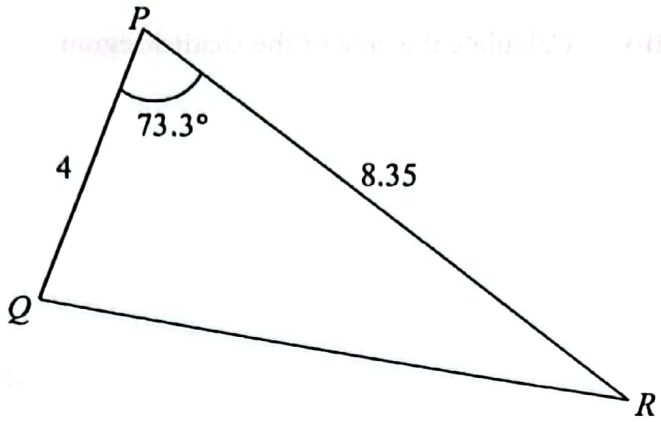
(ii) Calculate the perimeter of sector FDE.

Answer .....cm [2]

(iii) Calculate AD.

Answer .....cm [3]

9



The diagram shows a horizontal field with three points, P, Q and R marked on it.
A rope is used to join the three points to form a triangle PQR.
$PQ = 4$ m, $PR = 8.35$ m and angle $QPR = 73.3°$.

(a) **(i)** Find the area of triangle PQR.

Answer .....m^2 [2]

(ii) Calculate the total length of the rope used to form triangle PQR.

Answer ..... m [3]

(b) A marking, M is made along PQ such that RM is the shortest distance from R to PQ.

(i) Find the length of RM.

Answer ..... m [2]

(ii) Jensen standing at R, is flying a kite.
The kite, G is vertically above M and $GM = 6.8$ m.

Find the angle of elevation of G when viewed from P.

Answer ..... [3]

(iii) Jensen's mother would like to watch Jensen fly the kite.
She wants to stand at a point whereby she would take the least effort to view the kite.

Suggest one point, either P or Q, that she should stand at.
Justify your answer with reason.

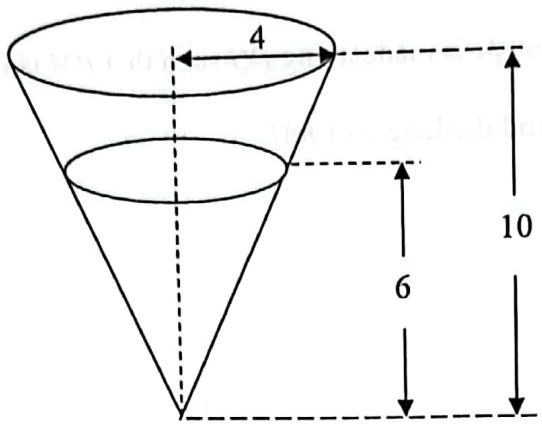
She should stand at ..... because .....

.....

.....

..... [2]

10



The diagram shows a cone with height 10 cm and radius 4 cm.
The cone is filled with water to a height of 6 cm.

(a) Calculate the volume of the cone.

Answer .....cm^3 [2]

(b) The cone has a label wrapped round it, which covers exactly its curved surface.
Calculate the area of paper needed for the label.

Answer .....cm^2 [2]

(c) Calculate the volume of the empty space above the water.

Answer .....cm^3 [2]

(d) Subsequently, Henry adds some spherical drops of oil, of diameter 2 cm into the water. He carefully adds the drops of oil one by one until the oil overflows.

Find the number of drops of oil he can add before the oil overflows.

Answer ..... [2]

11 Individual income tax is imposed on an individual's annual income, which includes the annual salary, bonus, and any other profits made in the assessment year.

An individual's chargeable income is obtained by subtracting the total relief he or she is eligible from the individual's annual income.

The income tax payable is determined according to an individual's chargeable income in the assessment year.

The table below gives information on the tax rate for year 2021.

Chargeable Income (S$)	Income Tax Rate (%)	Gross Tax Payable (S$)
On the first 20,000	0	0
On the next 10,000	2	200
On the first 30,000	-	200
On the next 10,000	3.50	350
On the first 40,000	-	550
On the next 40,000	7	2,800
On the first 80,000	-	3,350
On the next 40,000	11.5	4,600
On the first 120,000	-	7,950
On the next 40,000	15	6,000
On the first 160,000	-	13,950
On the next 40,000	18	7,200
On the first 200,000	-	21,150
On the next 40,000	19	7,600
On the first 240,000	-	28,750
On the next 40,000	19.5	7,800
On the first 280,000	-	36,550
On the next 40,000	20	8,000
On the first 320,000	-	44,550
On the next 320,000	22	

(a) Mrs Lee's annual income is \$102 000 in the assessment year 2021.
She is entitled to a total relief of \$30 400.

Show that she must pay a total of \$2 762 for her income tax.

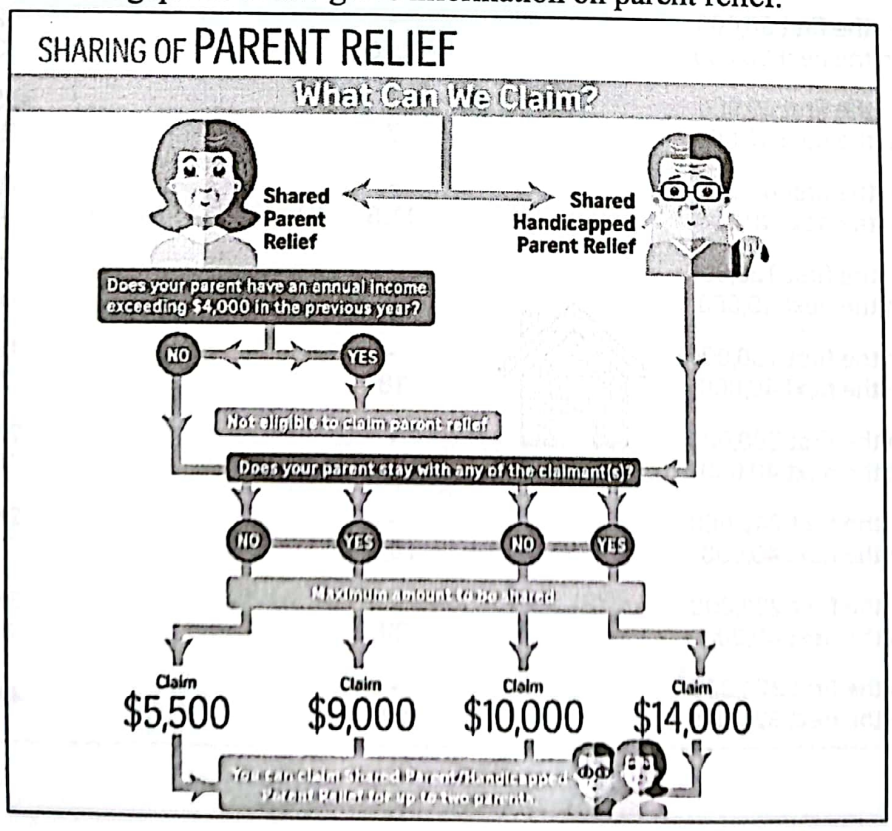
Answer

(b) Mr Bay paid a total of \$4 155 income tax in the assessment year 2021.

Calculate how much his chargeable income was.

Answer \$.....

The infogaphics below gives information on parent relief.



(c) Amy and John are siblings.

Amy is married with one child and is a working mother.

Amy's annual income is $105 000 in the assessment year 2021.

John is not married, and his annual income is $90 000 in the assessment year 2021.

Both Amy and John do not stay with their parents, who are retirees.

Their mother helps to look after Amy's child when she works.

The table below shows the types of relief Amy and John are eligible for.

	Amy	John
NS (Self/ Wife) relief	$ 750	$3 000
Earned income relief	$1 000	$1 000
Grandparent Caregiver Relief	$3 000	-
Working Mother's Child Relief	15% of annual income	-

Both Amy and John are eligible to claim for parent relief, which include their father and mother.

The total amount claimed for parent relief for both parents are shared between Amy and John.

John feels that his chargeable income is higher than that of Amy since he is not eligible for the grandparent caregiver relief and working mother's child relief.

John told Amy that he should claim the parent relief for both parents so that the total amount of tax payable by both will be the least.

Do you agree with his statement?

Justify the decision you make and show your calculations clearly.

.....

..... [6]

END OF PAPER 2