Anglo-Chinese School (Barker Road)

Marking Scheme
Secondary 4 Express / 5 Normal (Academic)
Additional Mathematics Paper 1
Preliminary Examination 2023

| | | | | | |
|---|---|---|---|---|---|
| 1 | | Height = $\dfrac{45\sqrt{5}-38\sqrt{7}}{6-\sqrt{35}} \times \dfrac{6+\sqrt{35}}{6+\sqrt{35}}$ | | | |
| | | $= \dfrac{270\sqrt{5}+45\sqrt{175}-228\sqrt{7}-38\sqrt{245}}{1}$ | | | |
| | | $= 270\sqrt{5}+225\sqrt{7}-228\sqrt{7}-266\sqrt{5}$ | | | |
| | | $= 4\sqrt{5}-3\sqrt{7}$ | | | |
| | | | | | |
| 2 | (a) | $\mathrm{f}(x)=x^3-(2h+1)x^2-(k-2h)x+k$ | | | |
| | | $\mathrm{f}(1)=(1)^3-(2h+1)(1)^2-(k-2h)(1)+k$ | | | |
| | | $1-2h-1-k+2h+k=0$ (shown) | | | |
| | | By factor theorem, $x-1$ is a factor of f($x$) | | | |
| | | | | | |
| | (b) | $x^3-(2h+1)x^2-(k-2h)x+k=(x-1)(x^2-2hx-k)$ | | | |
| | | $\mathrm{f}(x)=(x-1)(x^2-2hx-k)$ | | | |
| | | Since f($x$) has three real roots when f($x$) = 0,<br>$x^2-2hx-k=0$ has two real roots. | | | |
| | | $b^2-4ac\geq 0$ | | | |
| | | $(-2h)^2-4(1)(-k)\geq 0$ | | | |
| | | $4h^2+4k\geq 0$ | | | |
| | | $h^2+k\geq 0$ | | | |
| | | | | | |
| 3 | (a) | $T=30°\mathrm{C}$ | | | |
| | | | | | |
| | (b) | $1000=2030-2000e^{-k(1.5)}$ | | | |
| | | $2000e^{-k(1.5)}=1030$ | | | |
| | | $k=0.442392$ | | | |
| | | $1300=2030-2000e^{-0.442392t}$ | | | |
| | | $2000e^{-0.442392t}=730$ | | | |
| | | $\ln e^{-0.442392t}=\ln\dfrac{730}{2000}$ | | | |
| | | $t=2.28h$ | | | |
| | | | | | |
| | (c) | $y=2030$ | | | |

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| 4 | (a) | $\mathrm{f}''(x)=\dfrac{18}{(1-2x)^3}$ | | | |
| | | $\mathrm{f}'(x)=\dfrac{18(1-2x)^{-2}}{-2\times-2}+c_1$ | | | |
| | | $=\dfrac{9(1-2x)^{-2}}{2}+c_1$ | | | |
| | | $2\mathrm{f}'(-1)=1$ | | | |
| | | $2\left(\dfrac{9}{2(1-2(-1))^2}+c_1\right)=1$ | | | |
| | | $\dfrac{1}{2}+c_1=\dfrac{1}{2}$ | | | |
| | | $c_1=0$ | | | |
| | | $\mathrm{f}'(x)=\dfrac{9}{2(1-2x)^2}$ | | | |
| | | $\mathrm{f}(x)=\dfrac{9}{4(1-2x)}+c_2$ | | | |
| | | Sub (1,0) | | | |
| | | $0=\dfrac{9}{4(1-2(1))}+c_2$ | | | |
| | | $c_2=\dfrac{9}{4}$ | | | |
| | | $y=\dfrac{9}{4(1-2x)}+\dfrac{9}{4}$ | | | |
| | | | | | |
| 5 | (a) | $\mathrm{f}'(x)=\dfrac{(p-x)[2(x+2)(1)]-(x+2)^2(-1)}{(p-x)^2}$ | | | |
| | | $=\dfrac{2(p-x)(x+2)+(x+2)^2}{(p-x)^2}$ | | | |
| | | $=\dfrac{(x+2)(2p+2-x)}{(p-x)^2}$ | | | |
| | | | | | |
| | (b) | For $\mathrm{f}(x)$ is an increasing function, $\mathrm{f}'(x)>0$ | | | |
| | | $(x+2)(2p+2-x)>0$ | | | |

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|---|---|---|---|---|---|
| | | $-x^2+2px+4p+4>0$ | | | |
| | | $x^2-2px-4p-4<0$ ---(1) | | | |
| | | $(x+2)(x-8)<0$ | | | |
| | | $x^2-6x-16<0$ ----(2) | | | |
| | | Comparing (1) & (2) | | | |
| | | $-2p=-6$ | | | |
| | | $p=3$ | | | |
| | | Alternative method | | | |
| | | Comparing this with $(x+2)(2p+2-x)>0$ in (a),<br>$(x+2)(x-8)<0$<br>$(x+2)(8-x)>0$<br>$2p+2=8$<br>$p=3$ | | | |
| | | | | | |
| | (c) | $\frac{(x+2)^2}{3-x}=0 \Rightarrow x=-2$ | | | |
| | | $f'(x)=\frac{(-2+2)(2(4)+2+2)}{(3+2)^2}$<br>$=0$ (gradient of tangent) | | | |
| | | Gradient of normal is undefined (vertical line)<br>$\therefore$ Eqn. of normal : $x=-2$ | | | |
| | | | | | |
| 6 | (a) | $5\sin^2 A-3\cos^2 A=7\sin 2A$ | | | |
| | | $5\sin^2 A-3\cos^2 A=7(2\sin A\cos A)$ | | | |
| | | $5\sin^2 A-14\sin A\cos A-3\cos^2 A=0$ | | | |
| | | $(5\sin x+\cos x)(\sin x-3\cos x)=0$ | | | |
| | | $\frac{\sin A}{\cos A}=-\frac{1}{5}$ or $\frac{\sin A}{\cos A}=3$ | | | |
| | | $\tan A=-\frac{1}{5}$ (reject, $A$ is acute) or $\tan A=3$ (shown) | | | |
| | | | | | |
| | (b) | $\cos(60^\circ+A)=\cos 60^\circ\cos A-\sin 60^\circ\sin A$ | | | |
| | | $=\left(\frac{1}{2}\right)\left(\frac{1}{\sqrt{10}}\right)-\left(\frac{\sqrt{3}}{2}\right)\left(\frac{3}{\sqrt{10}}\right)$ | | | |

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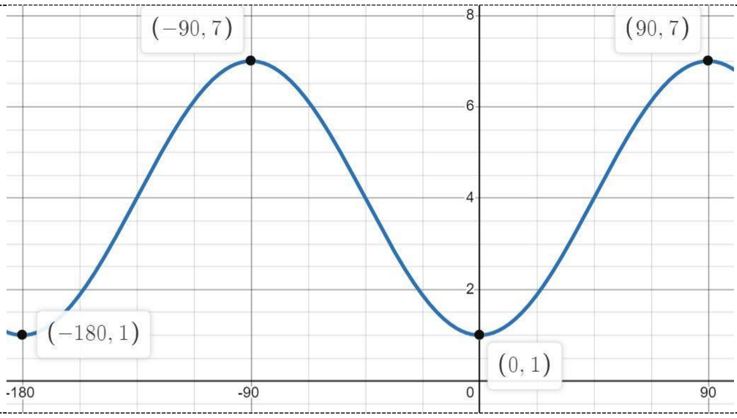
| | | $=\dfrac{1-3\sqrt{3}}{2\sqrt{10}}$ | | | |
| | | | | | |
| | (c) | From (b), $\cos(60^\circ + A)$ is a negative ratio.<br>So $60^\circ + A$ lies in the 2nd or 4th quadrant.<br>Both $60^\circ$ and A are both acute, $60^\circ + A$ cannot exceed $180^\circ$. Thus $60^\circ + A$ lies in the 2nd quadrant and is obtuse. | | | |
| | | | | | |
| 7 | (a) | $y=(3x-1)\left(\sqrt{6x+1}\right)$ | | | |
| | | | | | |
| | | $\dfrac{\mathrm{d}y}{\mathrm{d}x}=3\left(\sqrt{6x+1}\right)+\dfrac{1}{2}(3x-1)(6x+1)^{-\frac{1}{2}}(6)$ | | | |
| | | $=\dfrac{18x+3+9x-3}{\sqrt{6x+1}}$ | | | |
| | | $=\dfrac{27x}{\sqrt{6x+1}}$ | | | |
| | | | | | |
| | (b) | $\int_0^4 \dfrac{9x-3}{\sqrt{6x+1}}\,\mathrm{d}x$ | | | |
| | | $=\dfrac{1}{3}\int_0^4 \dfrac{27x}{\sqrt{6x+1}}\,\mathrm{d}x-\int_0^4 \dfrac{3}{\sqrt{6x+1}}\,\mathrm{d}x$ | | | |
| | | $=\dfrac{1}{3}\left[(3x-1)\sqrt{6x+1}\right]_0^4-3\left[\dfrac{(6x+1)^{\frac{1}{2}}}{6\times\frac{1}{2}}\right]_0^4$ | | | |
| | | $=\dfrac{1}{3}\left(11\sqrt{25}+\sqrt{1}\right)-\left(\sqrt{25}-\sqrt{1}\right)$ | | | |
| | | $=\dfrac{44}{3}$ or 16.7 (3s.f) | | | |
| | | | | | |
| 8 | (a) | $\mathrm{T}_3=\binom{n}{2}(2x)^{n-2}\left(-\dfrac{3}{x}\right)^2$ | | | |
| | | $=\binom{n}{2}(2)^{n-2}(x)^{n-2}(-3)^2\left(x^{-1}\right)^2$ | | | |
| | | $\dfrac{n(n-1)}{2}(2)^{n-2}(-3)^2=\dfrac{270}{8}\left(2^n\right)$ | | | |
| | | $\dfrac{n(n-1)}{2}(2)^{-2}(-3)^2=\dfrac{270}{8}$ | | | |

| | | | | | |
|---|---|---|---|---|---|
| | | $\frac{n(n-1)}{2}=15$ | | | |
| | | $n^2-n-30=0$ | | | |
| | | $(n-6)(n+5)=0$ | | | |
| | | $n=6$ or $n=-5$ (reject) | | | |
| | | | | | |
| | | | | | |
| 8 | (b) | $\left(1+x^2\right)\left(2x-\frac{3}{x}\right)^6=\left(1+x^2\right)\left(-576x^4+2160x^2+...\right)$ | | | |
| | | Coefficient of $x^4$ = $-576+2160=1584$ | | | |
| | | | | | |
| 9 | (a) | $a=-3$ | | | |
| | | $b=4$ | | | |
| | | | | | |
| | (b) |  | | | |
| | | | | | |
| | (c) | $1-\frac{1}{2}\cos 2x=0$ | | | |
| | | $6-\frac{6}{2}\cos 2x=0$ | | | |
| | | $4-3\cos 2x=-2$ | | | |
| | | $y=-2$ | | | |
| | | No solution | | | |
| | | | | | |
| 10 | (a) | $49^x-7^{x+1}=18$ | | | |
| | | $7^{2x}-7^x\times 7=18$ | | | |
| | | Let $y=$ $7^x$ | | | |
| | | $y^2-7y-18=0$ | | | |

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|---|---|---|---|---|---|
| | | $(y-9)(y+2)=0$ | | | |
| | | $y=-2$ or $y=9$ | | | |
| | | (reject) | | | |
| | | $7^x=9$ | | | |
| | | $x=\log_7 9$ | | | |
| | | $a=7,\ b=9$ | | | |
| | | | | | |
| | (b) | $-3x^2+qx-8<0$ | | | |
| | | $b^2-4ac<0$ | | | |
| | | $q^2-4(-3)(-8)<0$ | | | |

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|---|---|---|---|---|---|
| | | $q^2-96<0$ | | | |
| | | $(q-\sqrt{96})(q+\sqrt{96})<0$ | M1 | | |
| | | $-\sqrt{96}<q<\sqrt{96}$ | | | |
| | | $-9.798<q<9.798$ | | | |
| | | Largest value of the integer $q$ = 9 | A1 | | |
| | | | | | [8] |
| 11 | (a) | $DA = GC + x$ | | | |
| | | $x=26\cos\theta$ | | | |
| | | $GC=13\sin\theta$ | | | |
| | | $AD = 13\sin\theta+26\cos\theta$ | | | |
| | | $T$ = 11+13 +26 + $26\cos\theta+13\sin\theta$ | | | |
| | | $T$ = 50 + $26\cos\theta+13\sin\theta$ | | | |
| | | | | | |
| | (b) | $\sqrt{26^2+13^2}$ | | | |
| | | $=\sqrt{845}$ or $13\sqrt{5}$ | | | |
| | | $\tan^{-1}\dfrac{13}{26}$ | | | |
| | | $T = 50+13\sqrt{5}\cos(\theta-26.6°)$ | | | |
| | | | | | |
| | (c) | Max $T$ = 50 + $13\sqrt{5}$ | | | |
| | | =79.1 (3s.f) | | | |
| | | This is not possible as the maximum distance of $T$ is 79.1 m. | | | |
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| 12 | (a) | $a + 4$ | | | |
| | | | | | |
| | (b) | centre = $(a, a+4)$ | | | |
| | | $\sqrt{(a-8)^2+(a+4-13)^2}=a$ | | | |
| | | $(a-8)^2+(a-9)^2=a$ | | | |
| | | $a^2-16a+64+a^2-18a+81=a^2$ | | | |
| | | $a^2-34a+145=0$ | | | |
| | | $a=29$ (rejected) or $a=5$ | | | |
| | | centre = $(5,9)$ | | | |
| | | $(x-5)^2+(y-9)^2=25$ | | | |
| | | | | | |
| | (c) | Gradient of radius = $\frac{13-9}{8-5}=\frac{4}{3}$ | | | |
| | | Gradient of line $L$ = $-\frac{3}{4}$ | | | |
| | | Equation of line $L$: $y=-\frac{3}{4}x+c$ | | | |
| | | $13=-\frac{3}{4}(8)+c$ | | | |
| | | $c=19$ | | | |
| | | $y=-\frac{3}{4}x+19$ | | | |
| | | | | | |
| 13 | (a) | $\frac{4x^3+2x^2-5}{x^2(2x-1)}=2+\frac{4x^2-5}{x^2(2x-1)}$ | | | |
| | | | | | |
| | | $\frac{4x^2-5}{x^2(2x-1)}=\frac{A}{x}+\frac{B}{x^2}+\frac{C}{2x-1}$ | | | |
| | | $4x^2-5=A(x)(2x-1)+B(2x-1)+Cx^2$ | | | |
| | | Let $x=0$ | | | |
| | | $-5=B(-1)$ | | | |
| | | $B=5$ | | | |
| | | Let $x=\frac{1}{2}$ | | | |

| | | | | | |
|---|---|---|---|---|---|
| | | $-4=\frac{1}{4}C$ | | | |
| | | $C=-16$ | | | |
| | | Let $x=1$ | | | |
| | | $-1=A+5-16$ | | | |
| | | $-1=A+5-16$ | | | |
| | | $A=10$ | | | |
| | | $\frac{4x^3+2x^2-5}{x^2(2x-1)}=2+\frac{10}{x}+\frac{5}{x^2}-\frac{16}{2x-1}$ | | | |
| | | | | | |
| | (b) | $\int\frac{4x^3+2x^2-5}{x^2\left(2x-1\right)}\,\mathrm{d}x$ | | | |
| | | $=\int 2+\frac{10}{x}+\frac{5}{x^2}-\frac{16}{2x-1}\,\mathrm{d}x$ | | | |
| | | $=2x+10\ln x-\frac{5}{x}-8\ln\left(2x-1\right)+c$ | | | |
| | | | | | |