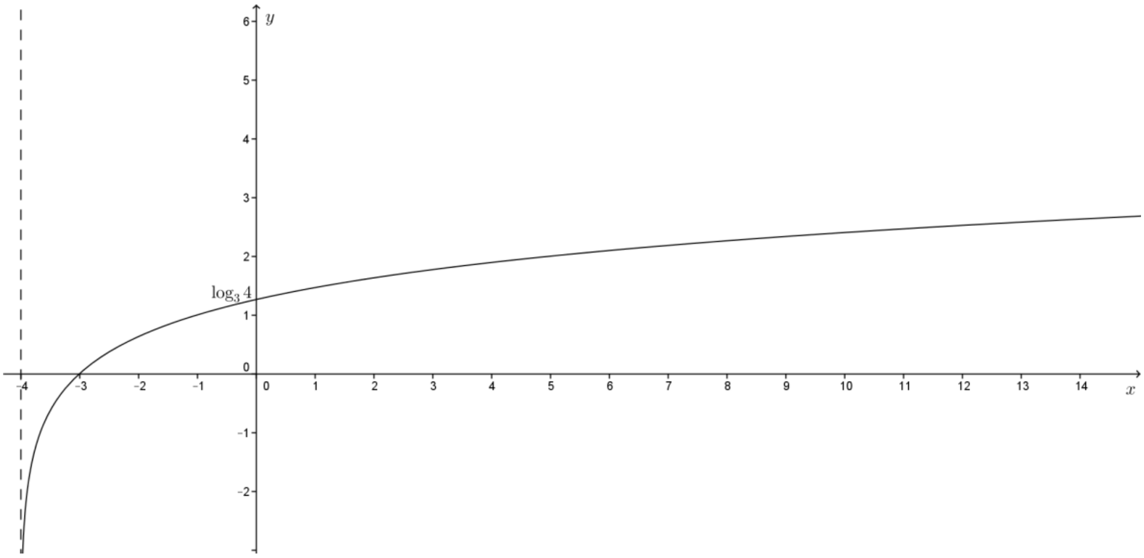
National Junior College
2016 – 2017 H2 Mathematics
Revision: Exponential, Logarithmic and Modulus Functions and their Graphs

Solutions to Practice Questions

1.



2a $|2x-3| = x$

$x = 2x-3$ or $x = -(2x-3)$

$3 = x$ $\quad$ $x = -2x+3$

$3x = 3$

$x = 1$

2b $|x+4| = |2-x|$

$x+4 = 2-x$ or $x+4 = -(2-x)$

$2x = -2$ $\quad$ $x+4 = -2+x$

$x = -1$ $\quad$ $4 = -2$ (rej.)

2c $|x^2+6| = 5x$ $\quad$ notice here that x must be positive

$x^2+6 = 5x$

$x^2-5x+6 = 0$

$(x-3)(x-2) = 0$

$x = 2$ or 3

2d $\left|x+\sqrt{6}\right|\left|x-\sqrt{6}\right|=-5x$ notice here that x must be negative

$\left|x^2-6\right|=-5x$

$x^2-6=-5x$ if $x^2-6\geq 0$ or $-\left(x^2-6\right)=-5x$ if $x^2-6\geq 0$

$x^2+5x-6=0$

$(x+6)(x-1)=0$

$x=1$ (rej.) or -6

$-x^2+6=-5x$

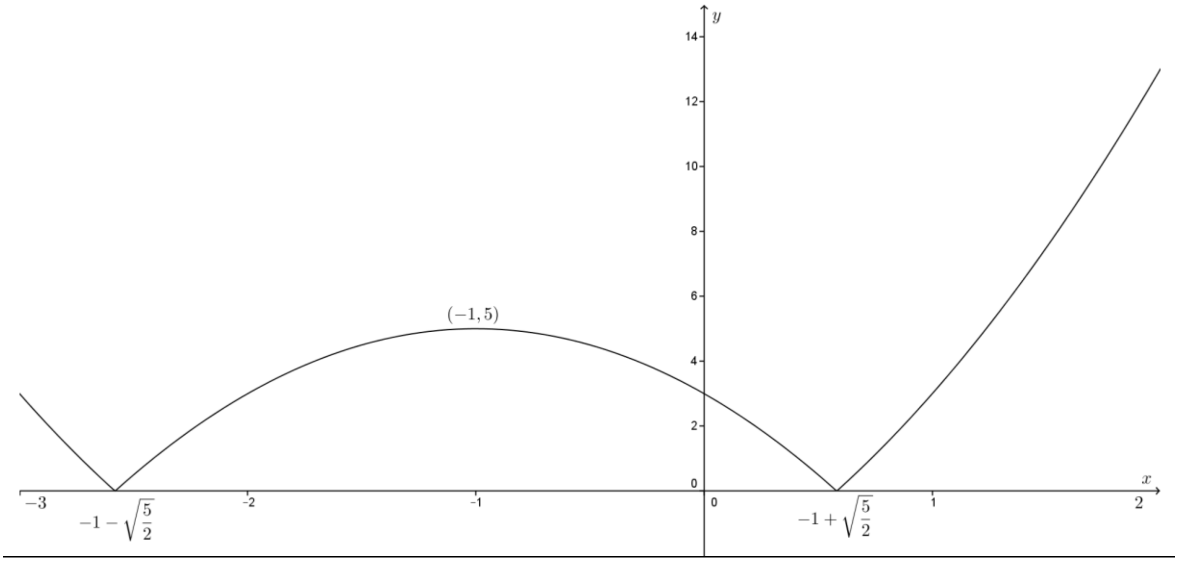
$x^2-5x-6=0$

$(x-6)(x+1)=0$

$x=6$ (rej.) or -1

3 $2x^2+4x-3=2\left(x^2+2x\right)-3$

$=2\left(x+1\right)^2-5$



4 $\frac{r^2}{4}(3x)^r\left(\frac{2}{9x^2}\right)^{6-r}=\frac{r^2}{4}3^r x^r 2^{6-r}\left(9x^2\right)^{r-6}$

$=\frac{r^2 3^r 2^{6-r} 9^{r-6}}{2^2}x^r x^{2r-12}$

$=r^2 3^r 3^{2r-12} 2^{6-r-2} x^{r+2r-12}$

$=r^2 3^{r+2r-12} 2^{4-r} x^{3r-12}$

$=r^2 3^{3r-12} 2^{4-r} x^{3r-12}$

$\therefore x^{3r-12}=x^{-3}$

$3r-12=-3$

$3r=9$

$r=3$

$\Rightarrow k=(3)^2 r^{3\times 3-12} 2^{4-3}=18r^{-3}=18(3)^{-3}=\frac{18}{27}=\frac{2}{3}$

5a

$$3\left(9^x\right)-3^{x+1}+1=3^x$$

$$3y^2-3y+1=y$$

$$3y^2-4y+1=0$$

$$(3y-1)(y-1)=0$$

$y=1$ or $y=\frac{1}{3}$

$3^x=1$ $\quad$ $3^x=3^{-1}$

$x=0$ $\quad$ $x=-1$

5b

$$2^{2x}-3\left(2^x\right)-10=0$$

$$y^2-3y-10=0$$

$$(y-5)(y+2)=0$$

$y=5$ or $y=-2$

$2^x=5$ $\quad$ $2^x=-2$ (rej.)

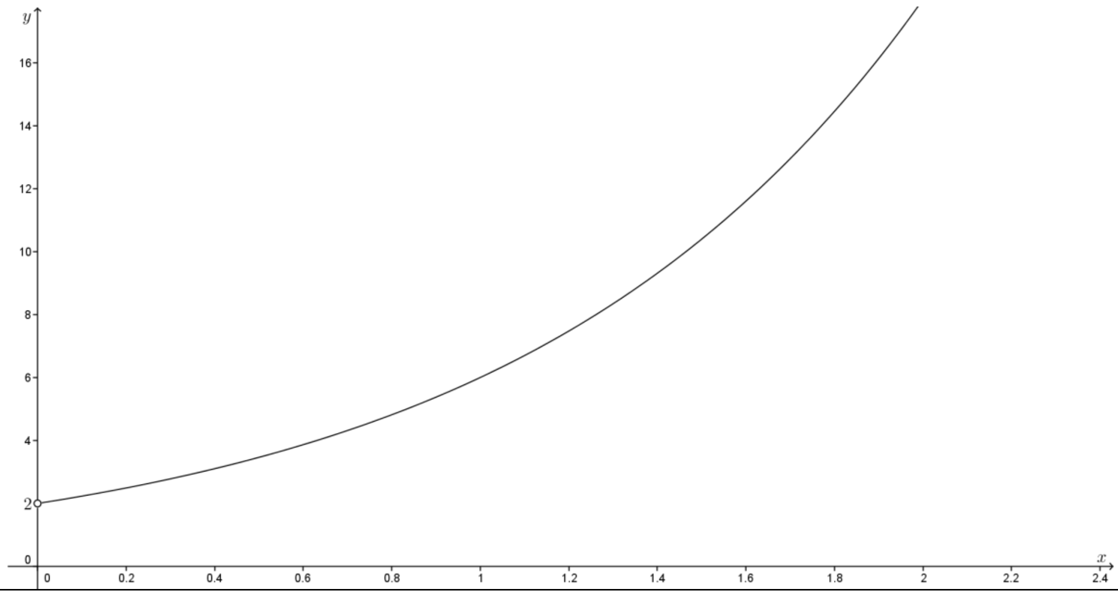
$$x=\frac{\ln 5}{\ln 2}$$

6

$$y=k3^x$$

$$x=0,\ y=2$$

$$\therefore 2=k\left(3^0\right)=k$$



7 $e^{\ln x} = y$

$\ln e^{\ln x} = \ln y$

$\ln x \ln e = \ln y$

$\ln x = \ln y$

$\therefore x = y = e^{\ln x}$

$x = e^{\ln x}$

8 $\log_b a \bullet \log_c b \bullet \log_a c = \dfrac{\log_a a}{\log_a b} \cdot \dfrac{\log_a b}{\log_a c} \cdot \log_a c = \log_a a = 1$

9a $\lg(2x+5) = 1 + \lg x$

$\lg(2x+5) = \lg 10 + \lg x = \lg 10x$

$2x + 5 = 10x$

$8x = 5$

$x = \dfrac{5}{8}$

9b $\log_4 y + \log_2 y = 12$

$\dfrac{\log_2 y}{\log_2 4} + \log_2 y = 12$

$\dfrac{x}{2} + x = 12$

$1.5x = 12$

$x = 8 \Rightarrow \log_2 y = 8 \therefore y = 2^8 = 256$

9c $\lg(x+3) - \lg x = \lg 7$

$\log\left(\dfrac{x+3}{x}\right) = \lg 7$

$\dfrac{x+3}{x} = 7$

$x + 3 = 7x$

$6x = 3 \Rightarrow x = 0.5$

9d $\dfrac{8^{2y}}{4^{y+1}} = 2^{2y+1} \Rightarrow \dfrac{2^{6y}}{2^{2y+2}} = 2^{2y+1}$

$2^{6y-2y-2} = 2^{2y+1}$

$4y - 2 = 2y + 1$

$2y = 3 \Rightarrow y = 1.5$

10a

$y+2x=3 \Rightarrow y=3-2x$

$y=|2x-1|$

$y=2x-1$ or $y=-(2x-1)$

$y=-2x+1$

$3-2x=2x-1$ | $3-2x=-2x+1$

$4=4x$ | $3=1$ (rej.)

$x=1 \therefore y=1$

10b

$2x+3y=19 \Rightarrow x=\frac{19-3y}{2}$

$|x-y|=3$

$x-y=3$ or $-(x-y)=3$

$-x+y=3$

$\frac{19-3y}{2}-y=3$	$-\frac{19-3y}{2}+y=3$
$19-3y-2y=6$	$-19+3y+2y=6$
$19-5y=6$	$-19+5y=6$
$5y=13$	$5y=25$
$y=\frac{13}{5}$	$y=5$
$x=\frac{28}{5}$	$x=2$

11a

$$\ln(3x-y)=2\ln 6-\ln 9$$
$$=\ln\frac{36}{9}=\ln 4$$
$$3x-y=4$$
$$\frac{\left(e^x\right)^2}{e^y}=e\Rightarrow e^{2x-y}=e^1$$
$$2x-y=1$$

Using GC, $x=3, y=5$

11b

$$3^p=9(27)^q\Rightarrow 3^p=3^{2+3q}$$
$$p=2+3q$$

$$\log_2 7-\log_2(11q-2p)=1$$
$$\log_2\frac{7}{11q-2p}=1$$
$$2^1=\frac{7}{11q-2p}$$
$$22q-4p=7$$

$$22q-4(2+3q)=7$$
$$22q-8-12q=7$$
$$10q=15$$
$$q=1.5\therefore p=6.5$$

12

$$\left|e^x-2\right|=e^x+1$$

$e^x-2=e^x+1$ or $-(e^x-2)=e^x+1$

$-3=0$ (rej.) or $-e^x+2=e^x+1$.

$$2e^x=1$$
$$e^x=0.5$$
$$x=-\ln 2$$

13i Amount = $100(1.05)^8=\$1477.46$

13ii

$$1000(1.05)^t>4000$$
$$(1.05)^t>4$$
$$t\ln 1.05>\ln 4$$
$$t>\frac{\ln 4}{\ln 1.05}$$
$$t>28.41$$

So, the year the amount first exceed \$4000 = 1900 + 29 – 1 = 2018

13iii $2100(1.05^t - 1) > 1000(1.05^t)$

$2.1(1.05^t) - 2.1 > (1.05^t)$

$1.1(1.05^t) > 2.1$

$1.05^t > \frac{2.1}{1.1}$

$t\ln 1.05 > \ln\left(\frac{2.1}{1.1}\right)$

$t > 13.25$

So ,the year is 1900 + 14 – 1 = 2003

14a $(\ln x)^2 + 2\ln x = 3$

Let $y = \ln x$,

$y^2 + 2y - 3 = 0$

$(y+3)(y-1) = 0$

$y = -3$ or $y = 1$

$\ln x = -3$ or $\ln x = 1$

$x = e^{-3}$ or $x = e^1 = e$

14b $\log_2(a) = \log_4(3b+13)$

Change of base of logarithms,

$\Rightarrow \log_2(a) = \frac{\log_2(3b+13)}{\log_2(4)} = \frac{\log_2(3b+13)}{\log_2(2^2)} \Rightarrow \log_2(a) = \frac{\log_2(3b+13)}{2}$

$\Rightarrow 2\log_2(a) = \log_2(3b+13)$

$\Rightarrow a^2 = 3b + 13 \ldots(1)$

$3^a = \frac{9^b}{27} \Rightarrow 3^a = 3^{2b-3}$

$\Rightarrow a = 2b - 3 \ldots(2)$

Subt. (2) into (1),

$\Rightarrow (2b - 3)^2 = 3b + 13$

$4b^2 - 12b + 9 = 3b + 13$

$4b^2 - 15b - 4 = 0$

$(4b + 1)(b - 4) = 0$

$b = -\frac{1}{4}$ or $b = 4$

When $b = -\frac{1}{4}$, $a = -3.5$

(reject because $\log_2(-3.5)$ is undefined)

When $b = 4$, $a = 5$