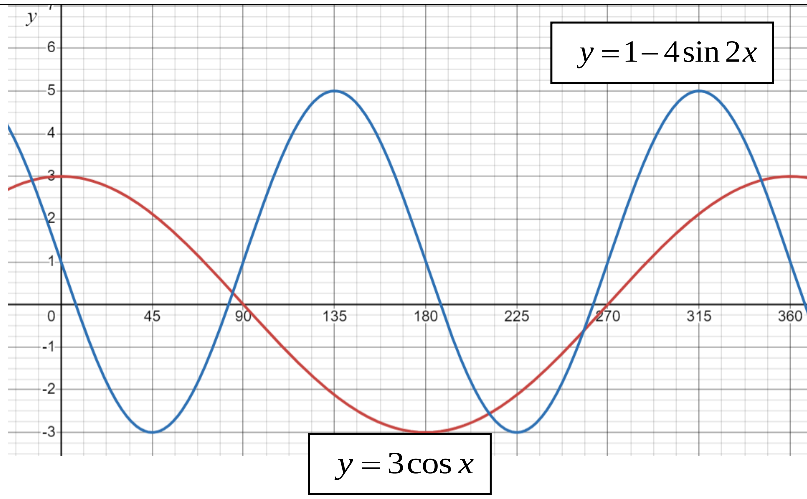
Qns	Suggested Solutions	Remarks
1	$\dfrac{-3x^2+12x-1}{(3x+1)(x-1)^2}=\dfrac{A}{3x+1}+\dfrac{B}{x-1}+\dfrac{C}{(x-1)^2}$ $-3x^2+12x-1=A(x-1)^2+B(3x+1)(x-1)+C(3x+1)$	[M1]
	When $x=1$, $-3+12-1=C(4)$ $C=2$	[M1]
	When $x=-\dfrac{1}{3}$, $-3\left(-\dfrac{1}{3}\right)^2+12\left(-\dfrac{1}{3}\right)-1=A\left(-\dfrac{4}{3}\right)^2$ $A=-3$	[M1]
	When $x=1$, $A=-3$, $C=2$ $-1=(-3)(-1)^2+B(1)(-1)+(2)(0+1)$ $B=0$	[M1]
	$\dfrac{-3x^2+12x-1}{(3x+1)(x-1)^2}=-\dfrac{3}{3x+1}+\dfrac{2}{(x-1)^2}$	[A1]
2(a)(i)	$3\cos x$ Amplitude = 3 Period = 360°	[B1]
2(a)(ii)	$1-4\sin 2x$ Amplitude = 4	[B1]
	Period $=\dfrac{360^\circ}{2}$ $=180^\circ$	[B1]

2(b)		[M1] Correct shape [M1] Smoothness [M1] Intersection
2(c)	4 solutions	[B1]
3(a)(i)	$\left(2-\frac{x}{3}\right)^7=2^7+\binom{7}{1}(2)^6\left(-\frac{x}{3}\right)+\binom{7}{2}(2)^5\left(-\frac{x}{3}\right)^2+\ldots$ $=128-\frac{448}{3}x+\frac{224}{3}x^2+\ldots$	[M1] [A1]
3(a)(ii)	$(p+x)^2\left(2-\frac{x}{3}\right)^7=\left(p^2+2px+x^2\right)\left(128-\frac{448}{3}x+\frac{224}{3}x^2+\ldots\right)$ $=\frac{224}{3}p^2x^2-\frac{896}{3}px^2+128x^2+\ldots$ $=\left(\frac{224}{3}p^2-\frac{896}{3}p+128\right)x^2+\ldots$ $\frac{224}{3}p^2-\frac{896}{3}p+128=-\frac{32}{3}p^2$ $256p^2-896p+384=0$ $2p^2-7p+3=0$ $(p-3)(2p-1)=0$ $p=3$ $p=\frac{1}{2}$ (rejected)	 [M1] [M1] [M1] [A1]
3(b)	$\left(x^2-\frac{1}{2x}\right)^{17}$	

	$T_{r+1}=\binom{17}{r}\left(x^2\right)^{17-r}\left(-\frac{1}{2x}\right)^r$ $=\binom{17}{r}\left(x^{34-2r}\right)(-1)^r\left(\frac{1}{2}\right)^r\left(\frac{1}{x}\right)^r$ $=\binom{17}{r}(-1)^r\left(\frac{1}{2}\right)^r x^{34-3r}$ Let $x^{34-3r}=x^0$ $34-3r=0$ $r=\frac{34}{3}$ $=11\frac{1}{3}$ Since r is <u>not an integer</u>, there is no independent term.	[M1] [M1] [M1] [A1]
4(i)	$y=ax^2+4x+3a-7\cdots(1)$ $y=2ax-5\cdots(2)$ Sub (2) into (1) $ax^2+4x+3a-7=2ax-5$ $ax^2+(4-2a)x+3a-2=0$ $b^2-4ac=0$ $(4-2a)^2-4a(3a-2)=0$ $16-16a+4a^2-12a^2+8a=0$ $-8a^2-8a+16=0$ $a^2+a-2=0$ $(a+2)(a-1)=0$ $a=-2, a=1$	[M1] [M1] [M1] [A1]
4(ii)	Since gradient is positive, a = 1. $y=x^2+4x-4\cdots(1)$ $y=2x-5\cdots(2)$ Sub (2) into (1)	

$x^2+4x-4=2x-5$

$x^2+2x+1=0$

$(x+1)^2=0$

$x=-1$ [M1]

Sub $x=-1$,

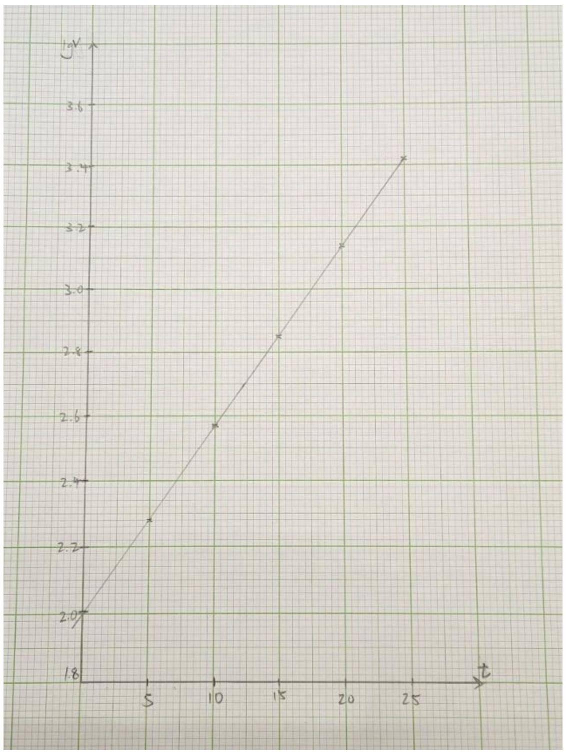
$y=-2-5$

$y=-7$

$Q(-1,-7)$ [A1]

5(i)

t (years)	5	10	15	20	25
V (in thousands $)	192	370	714	1374	2642
lgV	2.28	2.57	2.85	3.14	3.42



[G1] Axes

[G1] Points plotted correctly

[G1] Straight line passing through all plotted points.

5(ii)	From graph $\lg V = 2$ $V = 10^2$ $V = 100$ The initial value of investment is $100 000.	[M1] [A1]
5(iii)	$V = V_o k^t$ $\lg V = t \lg k + \lg V_o$ From graph, Gradient $= \frac{3.08 - 2.4}{19 - 7} = \frac{17}{300}$ $\lg k = \frac{17}{300}$ $k = 1.13937$ $k \approx 1.14$	 [M1] [M1] [A1]
5(iv)	Increase by 50% → investment is at $150 000 $V = 150$ $\lg V \approx 2.18$ From graph, when $\lg V = 2.18$, $t = 3.25$ years	 [M1] [A1]
6(a)	$2x(x-3) \geq 3 - 5x$ $2x^2 - 6x \geq 3 - 5x$ $2x^2 - x - 3 \geq 0$ $(x+1)(2x-3) \geq 0$ $x \leq -1$ or $x \geq 1\frac{1}{2}$	 [M1] Expansion [M1] Factorisation [A1]
6(b)	$2y + k = x \cdots (1)$ $y = 2x + \frac{6}{x} \cdots (2)$ Sub (2) into (1)	

	$2\left(2x+\frac{6}{x}\right)+k=x$ $4x+\frac{12}{x}+k=x$ $4x^2+12+kx=x^2$ $3x^2+kx+12=0$ $b^2-4ac<0$ $k^2-4(3)(12)<0$ $k^2-144<0$ $(k+12)(k-12)<0$ $-12<k<12$ Therefore, the greatest value of k is 11.	[M1] [M1] [M1] [A1]
7(a)	$y=2x^2-6x+9$ $=2\left(x^2-3x\right)+9$ $=2\left[\left(x-\frac{3}{2}\right)^2-\frac{9}{4}\right]+9$ $=2\left(x-\frac{3}{2}\right)^2+\frac{9}{2}$ Since $\left(\frac{3}{2},\frac{9}{2}\right)$ is a minimum point, it is not possible for the curve to have a value smaller than $\frac{9}{2}$.	[M1] [A1]
7(b)	$y=3x+5\cdots(1)$ $y=2x^2-6x+9\cdots(2)$ Sub (1) into (2) $2x^2-6x+9=3x+5$ $2x^2-9x+4=0$ $(x-4)(2x-1)=0$ $x=4, x=\frac{1}{2}$	[M1]

	Sub $x = 4$ $y = 3(4) + 5$ $= 17$ $(4, 17)$ Sub $x = \frac{1}{2}$ $y = 3\left(\frac{1}{2}\right) + 5$ $= \frac{13}{2}$ $\left(\frac{1}{2}, \frac{13}{2}\right)$ Length PQ $= \sqrt{\left(4 - \frac{1}{2}\right)^2 + \left(17 - \frac{13}{2}\right)^2}$ $= \sqrt{\frac{490}{4}}$ $= \frac{\sqrt{490}}{2}$ or $\frac{7\sqrt{10}}{3}$ or $\frac{7}{2}\sqrt{10}$	[M1] [M1] [A1]
8(a)	$y = 27 + \frac{8}{(x-2)^3}$ $\frac{dy}{dx} = -24(x-2)^{-4}$ $= -\frac{24}{(x-2)^4}$ Since $\frac{dy}{dx} \neq 0$ for all $x \neq 2$, the curve does not have a stationary point.	[M1] [M1] [A1]
8(b)	Sub $x = 0$	

	$y=27+\frac{8}{(-2)^3}$ $=26$ $B(0,26)$ Sub y = 0 $0=27+\frac{8}{(x-2)^3}$ $-27=\frac{8}{(x-2)^3}$ $-27(x-2)^3=8$ $(x-2)^3=-\frac{8}{27}$ $x-2=-\frac{2}{3}$ $x=1\frac{1}{3}$ $A\left(1\frac{1}{3},0\right)$	[B1] [B1]
8(c)	Sub x = 0 $\frac{dy}{dx}=-\frac{24}{(-2)^4}$ $=-\frac{3}{2}$ Gradient of normal $=\frac{2}{3}$ $y-26=\frac{2}{3}(x-0)$ $y=\frac{2}{3}x+26$	 [M1] [M1] [A1]
8(d)	Sub y = 0	

	$\frac{2}{3}x=-26$ $x=-39$ Area of triangle *BOC* $=\frac{1}{2}\times 39\times 26$ $=507$ units^2	[M1] [A1]
9(i)	$\angle PAB=\angle BCA$ (alternate segment Thm) $\angle ABC=90^\circ$ (angle in semi-circle) $\angle ABP=90^\circ$ (angle on a straight line) $\therefore \angle ABP=\angle ABC=90^\circ$ $\therefore \Delta BAP$ is similar to ΔBCA (AA)	[M1] [A1]
9(ii)	Let $\angle AKH=x$, $\angle KAP=90^\circ$ (tangent perpendicular to radius) $\angle APH=90^\circ-x$ (sum of angles in triangle) $\angle HPB=90^\circ-x$ (angle bisector) $\angle ABC=90^\circ$ (angle in semi-circle) $\angle ABP=90^\circ$ (angle on a straight line) $\angle BHP=180^\circ-90^\circ-\left(90^\circ-x\right)$ (sum of angles in triangle) $=x$ $\angle AHK=x$ (vertically opposite angles) $\therefore$ Since $\angle AHK=\angle AKH$, $AH=AK$ (isosceles triangle)	[M1] [A1]
9(iii)	Since ΔBHP is similar to ΔAKP $\frac{AK}{BH}=\frac{KP}{HP}$ $\Rightarrow \frac{AH}{BH}=\frac{KP}{HP}$ ($AK=AH$ from (ii)) $\Rightarrow AH\times HP=KP\times BH$ (Proved)	[M1] [A1]

10	$y=\frac{e^{2x}}{1-x}$ $\frac{dy}{dx}=\frac{2e^{2x}(1-x)+e^{2x}}{(1-x)^2}$ $=\frac{3e^{2x}-2xe^{2x}}{(1-x)^2}$ $=\frac{e^{2x}(3-2x)}{(1-x)^2}$ Since $x>\frac{3}{2}$, $(1-x)^2>0$ $\frac{1}{(1-x)^2}>0$ $\frac{(3-2x)}{(1-x)^2}<0$ $\frac{e^{2x}(3-2x)}{(1-x)^2}<0$ $\frac{dy}{dx}<0$ Since $\frac{dy}{dx}<0$, y is always decreasing.	 [M1] [M1] [M1] [M1] [A1]
11	$\frac{d^2y}{dx^2}=36e^{3x}-e^{-x}$ $\frac{dy}{dx}=\int 36e^{3x}-e^{-x}dx$ $=\frac{36}{3}e^{3x}+e^{-x}+c$ $=12e^{3x}+e^{-x}+c$ Sub $\frac{dy}{dx}=14, x=0$ $14=12+1+c$ $c=1$	 [M1] [M1] [M1]

	$\frac{dy}{dx}=12e^{3x}+e^{-x}+1$ $y=\int 12e^{3x}+e^{-x}+1dx$ $y=4e^{3x}-e^{-x}+x+d$ Sub $x=0, y=5$ $5=4-1+0+d$ $d=2$ $y=4e^{3x}-e^{-x}+x+2$	[M1] [M1] [M1] [A1]
12(i)	When t = 0, $v=4\sin^2\left(\frac{0}{2}\right)-1$ = –1 m/s $\therefore$ initial velocity = –1 m/s	[B1]
12(ii)	When acceleration, a = 2 m/s^2 $\frac{dv}{dt}=8\sin\left(\frac{t}{2}\right)\times\cos\left(\frac{t}{2}\right)\times\frac{1}{2}$ $4\sin\left(\frac{t}{2}\right)\cos\left(\frac{t}{2}\right)=2$ $2\sin t=2$ $\sin t = 1$ $t=\frac{\pi}{2}$ $\therefore$ when a = 2 m/s^2, $t=\frac{\pi}{2}$ and $v=4\sin^2\left(\frac{\pi}{4}\right)-1$ $v=4\times\left(\frac{1}{\sqrt{2}}\right)^2-1$ = 1 m/s	[M1] [M1] [M1] [A1]
12(iii)	Instantaneous rest $\Rightarrow$ v = 0 $\Rightarrow$ $4\sin^2\left(\frac{t}{2}\right)-1=0$ $\Rightarrow$ $\sin\left(\frac{t}{2}\right)=\pm\frac{1}{2}$ $\Rightarrow$ $\frac{t}{2}=\frac{\pi}{6},\frac{5\pi}{6},\frac{13\pi}{6},\ldots,$ For first instantaneous rest, $\frac{t}{2}=\frac{\pi}{6}$ $\Rightarrow$ $t=\frac{\pi}{3}$ *or* 1.05 second	[M1] [M1] [A1]

12(iv)	$\cos t = 1 - 2\sin^2\left(\frac{t}{2}\right)$ $2\sin^2\left(\frac{t}{2}\right) = 1 - \cos t$ $4\sin^2\left(\frac{t}{2}\right) = 2 - 2\cos t$ $S = \int 4\sin^2\left(\frac{t}{2}\right) - 1 dt$ $= \int 2 - 2\cos t - 1 dt$ $= \int 1 - 2\cos t dt$ $= t - 2\sin t + c$ When $t = 0$, $S = 0$, $c = 0$ When $t = 2$ $S = 2 - 2\sin(2)$ ≈ 0.18140 When $t = 3$ $S = 3 - 2\sin(3)$ ≈ 2.7177 Distance travelled during the 3rd second $2.7177 - 0.18140 \approx 2.54$ m	[M1] Double angle formula [M1] Finding *S* either $t = 2$ or $t = 3$ [A1]