Anglo - Chinese School (Independent)

FINAL EXAMINATIONS 2014
YEAR 3 INTEGRATED PROGRAMME
CORE MATHEMATICS
PAPER 1

Friday   **3rd OCTOBER 2014**   **1 h 30 min**

Additional Material
Graph Paper (1 sheet)

INSTRUCTIONS TO CANDIDATES

- Write your index number in the boxes above.
- Do not open this examination paper until instructed to do so.
- You are not permitted access to any calculator for this paper.
- Answer all questions in the spaces provided.
- Unless otherwise stated in the question, all numerical answers must be given exactly or correct to three significant figures.
- The maximum mark for this paper is 80.

For Examiner's Use

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This paper consists of 13 printed pages.

[Turn over

Full marks are not necessarily awarded for a correct answer with no working. Answers must be supported by working and/or explanations. Where an answer is incorrect, some marks may be given for a correct method, provided this is shown by written working. You are therefore advised to show all working.

*Answer **all** the questions in the spaces provided.*

1   *[Maximum mark: 5]*

(a) Evaluate $2 - \dfrac{3}{5 - \dfrac{3}{5 - \dfrac{4}{5}}}$.

[3 marks]

(b) Factorise $25 - \dfrac{x^2}{9}$ completely.

[2 marks]

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2   *[Maximum mark: 4]*

Which of the following is the graph of

(a) $y = -3 + 4x$

[1 mark]

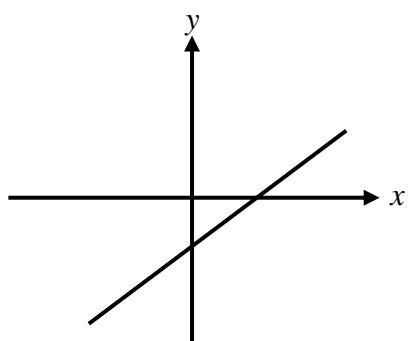(b) $y = \dfrac{2}{x} - \dfrac{2}{x^2}$
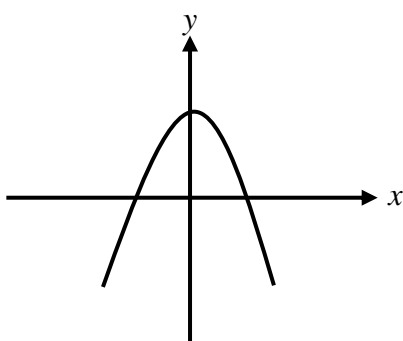
[2 marks]

(c) $y = -x^2 + 1$

[1 mark]

Figure 1
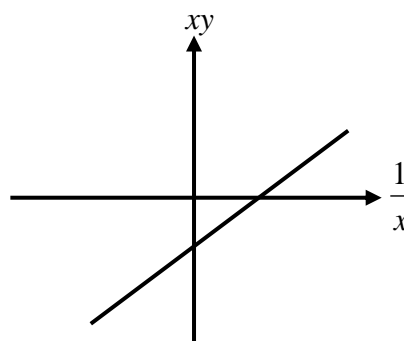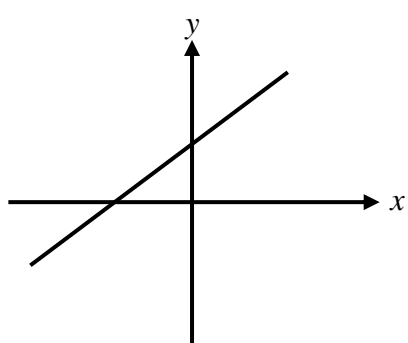

Figure 2

Figure 3

Figure 4
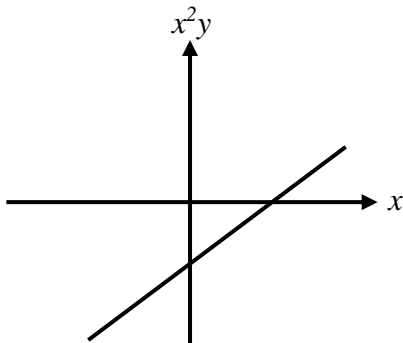
Figure 5
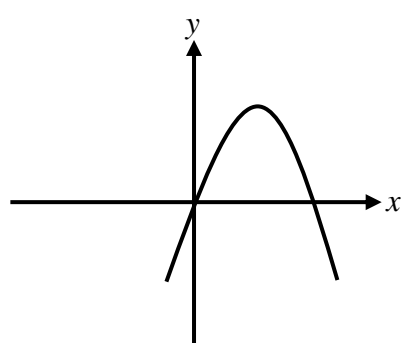
Figure 6

Answer:

(a) Figure _____

(b) Figure _____

(c) Figure _____

3 *[Maximum mark: 8]*

(a) Simplify $\dfrac{20\left(p^{-2}q^3\right)^6}{2\left(pq^{-2}\right)^2} \div \sqrt{\dfrac{p^3q^4}{pq^2}}$.

[4 marks]

(b) If $\log_2\left(\log_x 256\right) = \ln\left(e^7\right) - \left(\dfrac{1}{\log_9 3}\right)^2$, find the value of x.

[4 marks]

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4 *[Maximum mark: 7]*

(a) Given that $\sqrt{\dfrac{ax+by}{3x-4y}} = \dfrac{a}{b}$, express y in terms of a, b and x.

[4 marks]

(b) Simplify $\left(\dfrac{2}{3+\sqrt{7}} - 3 \right)^2$.

[3 marks]

5   *[Maximum mark: 10]*

(a)   Solve $2\log_5 y - \log_{125} y = \log_2 32$.

[4 marks]

(b)   Solve the following simultaneous equations

$$4^{x-4} \div 32^y = 16^{\frac{1}{x}}$$

$$7^x \times \sqrt{7^y} = 5^{\log_3 1}$$

[6 marks]

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6   *[Maximum mark: 7]*

Given that $y = x^2 + 2x + 3$.

(a)   Express y in the form $y = (x+h)^2 + k$, where h and k are constants.

[2 marks]

(b)   Hence, sketch the graph of $y = x^2 + 2x + 3$, clearly labelling the coordinates of the x and y intercepts and turning points, if any.

[3 marks]

(c)   Given that $w = \dfrac{12}{y}$, calculate the largest possible value of w.

[2 marks]

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7   *[Maximum mark: 6]*

Given that $\sin\theta = \sin 148^o$ and $\tan\beta = -\tan 52^o$.

(a)   State an acute angle θ and an obtuse angle β.

[2 marks]

(b)   Hence, calculate $\dfrac{3\cos\left(\dfrac{\beta}{\theta}\right)}{2\sin\left(\beta + 22^o\right)}$, using as much information given in the table below as it is necessary.

	sin	cos	tan
176°	0.0698	-0.9976	-0.0699
30°	0.5000	0.8660	0.5774

[4 marks]

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8 *[Maximum mark: 10]*

The roots of the equation $kx^2 + 4x - 5 = 0$, where k is an integer, are α and β.

(a) Write down the value of $\alpha + \beta$ and $\alpha\beta$ in terms of k.

[2 marks]

(b) Given that $\alpha^2 + \beta^2 = 9$, calculate the value of k.

[3 marks]

(c) Find the quadratic equation in x whose roots are $\dfrac{2}{\alpha+3}$ and $\dfrac{2}{\beta+3}$.
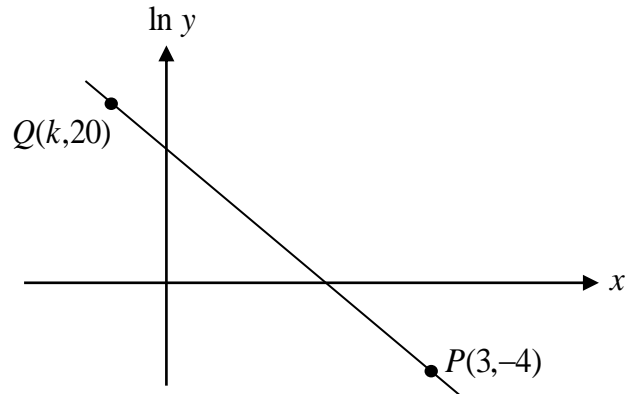
[5 marks]

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9   *[Maximum mark: 5]*

Find the value(s) of the constant k, given that the quadratic equation $4kx^2 + 10kx + 15 = 4x^2 - 5k$ has 2 real and equal roots.

[5 marks]

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10 *[Maximum mark: 4]*

The diagram below shows part of a straight line that passes through the points $P(3,-4)$ and $Q(k,20)$. If x and y are related by the equation $y = e^{h-4x}$, find the values of h and of k, where h and k are constants.

[4 marks]

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11 *[Maximum mark: 7]*

Solutions to this question by accurate drawing will not be accepted.

The diagram shows a trapezium OPQR in which O is the origin, Q is $(-4,-6)$ and R is $(-2,-7)$, $\angle OPQ = 90^o$ and OR is parallel to PQ.

Diagram is not drawn to scale
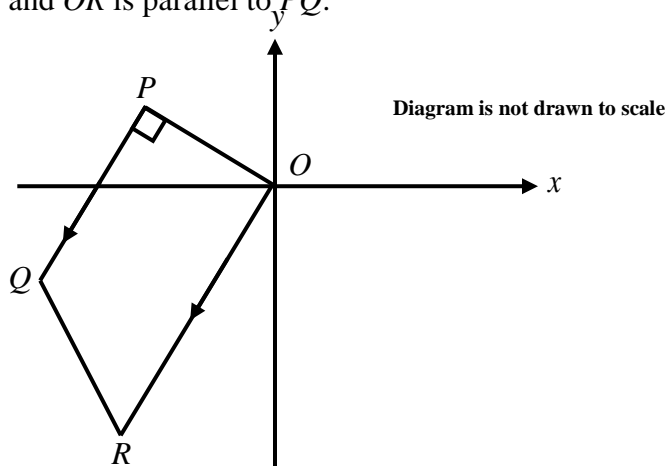
Find

(a) the area of triangle *OQR*.

[3 marks]

(b) the equation of *OP*,

[2 marks]

(c) the equation of *PQ*,

[2 marks]

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12   *[Maximum mark: 7]*

Answer the whole of this question on a sheet of graph paper.

The average number of chickens infected with a disease is connected by the equation $n = 215(2^{-t})$ where t is the time in weeks and n is the average number of chickens.

The table below shows some corresponding values of t and n correct to one decimal place.

t	0	1	2	3	4	5
n	215.0	107.5	53.8	26.9	13.4	6.7

(a)   Using a scale of 4 cm to represent 1 week on the horizontal axis and 1 cm to 20 chickens on the vertical axis, draw the graph of $n = 215(2^{-t})$ for values $0 \le t \le 5$ and $0 \le n \le 220$.

[3 marks]

(b)   Use your graph to find the number of chickens infected when $t = 2.5$.

[1 mark]

(c)   Using your graph, solve the equation $215(2^{-t+2}) - 32t - 80 = 0$.

[3 marks]

**********   END OF PAPER 1   **********

1a) $1\frac{3}{10}$

1b) $\left(5 - \frac{x}{3}\right)\left(5 + \frac{x}{3}\right)$

2a) Figure 1

2b) Figure 5

2c) Figure 2

3a) $\frac{10q^{21}}{p^{15}}$

3b) 2

4a) $y = \frac{3a^2x - ab^2x}{b^3 + 4a^2}$

4b) 7

5a) 125

5b) $y = \frac{2}{3}$ or $y = -2$ and $x = -\frac{1}{3}$ or $\quad x = 1$

6a) $y = (x + 1)^2 + 2$

6b)
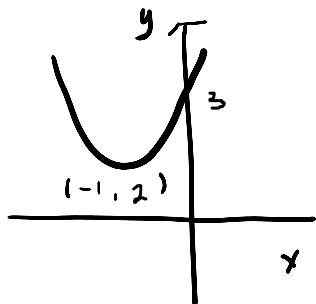
6c) 6

7a) 32, 128

7b) 2.9928

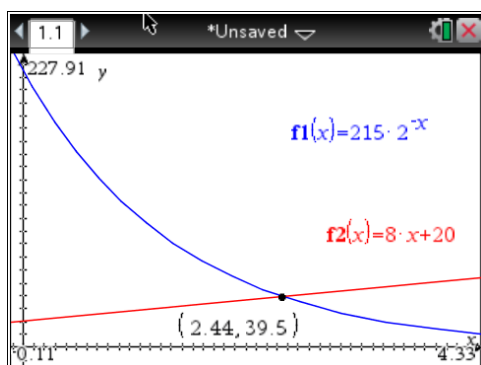8a) 2

8b) $x^2 - 16x + 8 = 0$

9) 6 or 2

10) $h = 8$ and $k = -3$

11a) 8

11b) $y = -\frac{2}{7}x$

11c) $y = \frac{7}{2}x + 8$

12a)

12c) 38