Instructions: After completing your topical revision, complete Practice Papers A to E under time constraint of 2.5 hours each.

Practice Paper A (2.5 hours, 80 marks)

1 A musical was held in a conference hall that has a maximum capacity of 800 seats. Three categories of tickets Cat A, Cat B and Cat C were sold at \$60, \$45 and \$15 respectively. It was known that 720 tickets were sold and a total of \$19200 was collected from the ticket sales. Furthermore, the combined number of Cat A and Cat B tickets sold was half of the number of Cat C tickets sold. For each category of tickets, find the amount of money collected from their sales. [4]

2 Jason is considering to invest \$50,000 in a fund at the end of 2018. The fund will credit an amount of \$300 at the end of 2019. At the end of each subsequent year, the amount credited increases by \$150 as compared to the amount credited in the previous year.

(i) Find the amount of money that will be credited in the fund at the end of 2022. [2]

(ii) Given that the money in Jason's account first exceeds \$70,000 at the end of year N, find the value of N. [4]

3 The curve C has equation $y=\dfrac{\mathrm{e}x^2-12\mathrm{e}x-30}{x-5}$, where e is the exponential constant.

(i) Sketch C, indicating clearly the equations of the asymptotes and the coordinates of any points of intersection with the axes. [4]

(ii) State the exact coordinates of the point of intersection of the asymptotes. [1]

(iii) Deduce the exact range of values of m such that the equation

$$\frac{\mathrm{e}x^2-12\mathrm{e}x-30}{(x-5)^2}=m-\frac{2\mathrm{e}}{x-5}$$

has no real root. [2]

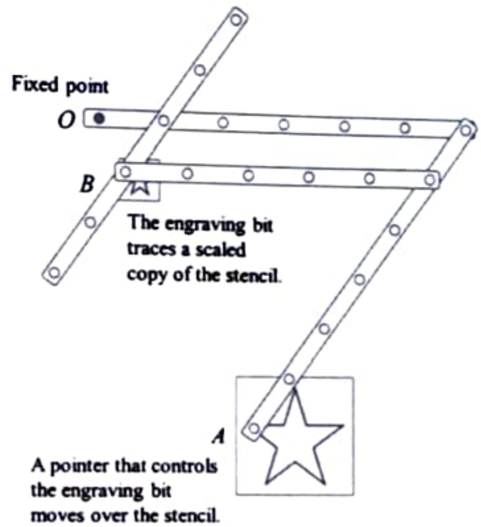
4 The sum of the first n terms of a series is given by $20-5\left(\dfrac{3^n}{4^{n-1}}\right)$.

(i) Show that the series is a geometric series. [3]

(ii) Explain why this series converges and find its sum to infinity. [2]

A new geometric series is formed by using the even-numbered terms (i.e. $u_2, u_4, u_6, \ldots$) of the above series. Find the sum of the first 20 terms of this new series. [3]

9 **(a)** Engraving is the process of incising a design or text onto a surface. For instance, serial numbers are commonly engraved onto items such as machine parts, in order to identify them individually. Today, computers are used for engraving, but in the past the job required a mechanical linkage known as a pantograph.



The diagram shows a simple pantograph, in the *x*-*y* plane, consisting of two pairs of parallel rigid rods, joined together at four joints. The rods are free to rotate at these joints. One end of the pantograph is fixed at the origin *O* represented by the point (0, 0). Points (x, y) are defined relative to the fixed point *O*. A pointer is attached to *A*, while the engraving bit is attached to *B*. The engraver controls the pantograph by moving the pointer over a stencil, which then causes the engraving bit to trace out a copy of the shape.

(i) A pantograph is set up such that the engraving bit, point *B*, always lies on the same line as *OA* as the pointer moves. The ratio of the distances $OA:OB$ is set at $5:1$. Describe a sequence of geometric transformations which maps the shape of the stencil (at *A*) to the shape of the engraving (at *B*). [2]

(ii) Suppose that the shape on the stencil is a circle with centre at (h,k) and radius *r*. State the equation of the circle. Find the equation of the engraved shape, showing your working clearly. [3]

(iii) State the area enclosed by the engraved shape. [1]

(b) The curve *S* undergoes the following transformations in order:

A: Scaling parallel to the *x*-axis with scale factor 3

B: Translation of 1 unit in the negative *y*-direction

C: Reflection about the *y*-axis

(i) The point $P(-\pi, 3)$ lies on the curve *S*. Find the co-ordinates of the corresponding point after the transformations have been applied. [2]

(ii) The equation of the resulting curve, after the transformations, is $y=-2\cos\left(\frac{x}{3}\right)$.

Find the equation of curve *S*, showing your working clearly. [4]

8 The function f is defined as

$$f : x \mapsto x^2 - 2x + 3, \quad x \in \mathbb{R}, \quad x \le 1.$$

(i) Define f^{-1} in a similar form. [3]

(ii) Sketch the graphs of f and f^{-1} in a single diagram and state the relationship between the graphs of f and f^{-1}. [3]

Another function g is defined as

$$g : x \mapsto \frac{2-x}{x-3}, \quad x \in \mathbb{R}, \quad x \ne 3.$$

(iii) Determine whether the composite function fg exists. [1]

(iv) Given that $h : x \mapsto \dfrac{2-x}{x-3}$, $x \in \mathbb{R}$, $x > 3$ and that fh exists, find the range of fh. [2]

(v) Find the exact solution of $\mathrm{fh}(x) = 7$. [3]

Practice Paper E **(2.5 hours, 80 marks) EJC 2020 June Common Test JC1 H2 Maths[Modified]**

1 A curve C has equation $y = ax^2 + b\sqrt{x} + c$, where a, b and c are constants. It is given that C crosses the x-axis at $x = 3$ and has a turning point at $(1, -2.5)$. Find the values of a, b and c. [4]

2 The function f, with domain the set of non-negative integers, is given by

$$f(n) = \begin{cases} 1 & \text{for } n = 0, \\ 2f\left(\frac{1}{2}n\right) & \text{for } n > 0,\ n \text{ even}, \\ 2 - f(n-1) & \text{for } n > 0,\ n \text{ odd}. \end{cases}$$

(i) Find $f(3)$, $f(4)$, and $f(6)$. [3]

(ii) Does f have an inverse? Justify your answer. [2]

3 On the same axes, sketch the graphs of

(i) $y = \dfrac{x+1}{kx+1}$, where $k > 1$, indicating clearly the asymptotes and the axial intercepts; [3]

(ii) $4x^2 + 16x + 15 + (y-1)^2 = 0$, indicating clearly the centre and any other relevant features. [3]

Hence, deduce the number of real roots of the equation $4x^2 + 16x + 15 + \left(\dfrac{x+1}{kx+1} - 1\right)^2 = 0$. [1]

4 **(a)** Without using a calculator, solve the inequality $\dfrac{6x^2 + 3x + 2}{2 - x} \leq 1 + x^2$. [4]

(b) On the same diagram, sketch the graphs of $y = \left|-x^2 + 3x - 1\right|$ and $y = \sin x$, where $0 \leq x \leq \dfrac{3\pi}{2}$.

Hence solve $\sin x < \left|-x^2 + 3x - 1\right|$, where $0 \leq x \leq \dfrac{3\pi}{2}$. [5]

5 The points A, B and C have position vectors $\mathbf{a}$, $\mathbf{b}$ and $\frac{3}{2}\mathbf{a}+\frac{1}{2}\mathbf{b}$ respectively. The point P on AB is such that $AP : PB = \lambda : 1-\lambda$ and the point P on OC is such that $OP : PC = \mu : 1-\mu$.

(i) Express $\overrightarrow{OP}$ in terms of λ, $\mathbf{a}$ and $\mathbf{b}$. [1]

(ii) By expressing $\overrightarrow{OP}$ in terms of μ, $\mathbf{a}$ and $\mathbf{b}$, find the values of λ and μ. Hence show that P is the midpoint of OC. [3]

It is given that the position vectors of the points A and B are $2\mathbf{j}+\mathbf{k}$ and $12\mathbf{i}-2\mathbf{j}-3\mathbf{k}$ respectively. The point Q lies on OA such that PQ is perpendicular to OA.

(iii) Find the position vector of the point Q. [4]

6 The curve C_1 has parametric equations

$$x=3\sin\theta+4, \qquad y=3\cos\theta-3, \quad \text{where } 0\le\theta\le 2\pi,$$

and the curve C_2 has equation $x^2-y^2=4$.

(i) Sketch C_1 and C_2 on the same diagram, stating clearly the co-ordinates of any points of intersection with the axes and the equations of any asymptotes. [5]

(ii) Show algebraically that the points of intersection of C_1 and C_2 satisfy the equation

$$3(\sin^2\theta-\cos^2\theta)+8\sin\theta+6\cos\theta+1=0.$$ [2]

(iii) Hence, find the co-ordinates of the points of intersection of C_1 and C_2. [3]

7 The sum of the first n terms of a series G is given by $S_n = \mathrm{e}-\frac{\mathrm{e}^{n+1}}{k^n}$, where k is a positive constant.

(i) Find the n^{th} term of the series G. [2]

(ii) Hence, show that the series G is a geometric series. [2]

(iii) Find the exact range of values of k for the sum to infinity to exist. [1]

(iv) The second term of the series G is $\frac{\mathrm{e}}{4}$. Show that $k=2\mathrm{e}$, and find the exact value of S, the sum to infinity of the series G. [3]

The first term of an arithmetic series A is equal to the first term of G. The common difference of A is $\sqrt{2}$.

(v) The sum of the first n terms of series A is denoted by L_n. Find the least value of n such that $\frac{1}{100}L_n$ differs from S by more than 10. [4]

10 A painter is tasked to paint the interior of a HDB flat. First, the painter fills up paint cans each having a volume of 2 litres. To prevent spillage, the tap used to fill the paint cans has a special mechanism that can control the volume of paint poured into the paint can every second. In the 1st second, 100 ml of paint is poured into the paint can and for each subsequent second, the volume of paint poured into the paint can is 5% less than the volume in the previous second.

(i) Find the volume of paint poured into the paint can in the 24th second. [1]

(ii) Find, to the nearest ml, the total volume of paint in the paint can after 1 minute. [2]

(iii) Explain why the paint will never overflow the paint can. [2]

At the start of each day of painting, the painter paints an area of 70 m^2 in the 1st hour. In the subsequent hours, the fatigue of painting results in the painter painting 3 m^2 less compared to the previous hour. For each day, the painter continuously paints for only 10 hours.

(iv) In which hour of each day does the painter paint an area of exactly 49 m^2? [2]

(v) Find the total area painted in a day. [1]

(vi) The painter takes up a project to paint a house with an interior area of 2045 m^2. Find the number of complete hours the painter takes to finish painting. [4]

5 The line l_1 passes through the point P with coordinates $(3,-3,-1)$ and is parallel to the vector $-\mathbf{i}+\mathbf{j}+\mathbf{k}$. The line l_2 has vector equation $\mathbf{r}=6\mathbf{i}-\mathbf{k}+\alpha(-3\mathbf{i}+\mathbf{j}+2\mathbf{k})$ where α is a real parameter.

(i) Write down a vector equation of the line l_1. [1]

(ii) If the lines l_1 and l_2 intersect at the point R, find the position vector of R. [3]

(iii) Point Q has coordinates $(6,0,-1)$ and the foot of the perpendicular from Q to l_1 is P. Find a vector equation of the line which is the reflection of l_2 in l_1. [3]

6 The curve C has parametric equations

$$x=-\frac{1}{2}\cos 2\theta, \quad y=\sqrt{2}\sin\theta-1, \text{ where } -\pi<\theta<\pi.$$

(i) Sketch the curve C, giving the coordinates of its vertex, endpoints and any points where C crosses the x- and y- axes. [5]

(ii) The line $y=2x-1$ intersects C. Find the point(s) of intersection. [3]

7 The curve C has equation $y=\dfrac{(x-2)^2}{x+4}$.

(i) Find, using an algebraic method, the set of values that y can take. [3]

(ii) Sketch the curve C, stating the coordinates of any points of intersection with the axes, any turning points, and the equations of any asymptotes. [4]

(iii) By drawing $x^2+y^2=1$ on the same diagram, deduce the number of real roots for the equation

$$x^2+\left[\frac{(|x|-2)^2}{|x|+4}\right]^2=1.$$

[2]

8 Functions f and g are defined by

$$f(x)=\begin{cases} x(2-x) & \text{for } 0\le x<1, \\ 2-x & \text{for } 1\le x<2, \end{cases}$$

$$g(x)=e^x, \quad x\in\mathbb{R},\ x\ge 0.$$

(i) Sketch the graph of $y=f(x)$ for $0\le x<2$. [3]

(ii) Give a definition (including the domain) of the composite function gf and its range. [3]

(iii) Given further that $f(x-2)=f(x)$ for all real values of x, find the exact value of $f\left(\frac{101}{4}\right)$. [2]

9 The function f is defined as

$$f: x\mapsto \sqrt{x+1}-\frac{1}{2}, \quad x\in\mathbb{R},\ x>-1.$$

(i) Sketch the graph of $y=f(x)$. Your sketch should state the coordinates of any points of intersection with the axes and endpoints. [2]

(ii) Find $f^{-1}(x)$, stating the domain of f^{-1}. [3]

(iii) On the same diagram as in part (i), sketch the graph of $y=f^{-1}(x)$. [1]

(iv) Write down the equation of the line in which the graph of $y=f(x)$ must be reflected in order to obtain the graph of $y=f^{-1}(x)$, and hence find the exact solution of the equation $f(x)=f^{-1}(x)$. [4]

(v) Using (iii) and (iv) to deduce the solution set of $f(x)\ \ge\ f^{-1}(x)$. [2]

4 In mathematics, the inequality of arithmetic and geometric means, or more briefly the AM–GM inequality, states that the AM (Arithmetic mean) of a series of real numbers is always greater or equal to the GM (Geometric mean).

The simplest non-trivial case of the AM-GM inequality (i.e. when $n = 2$) states that:

For real numbers a and b such that $a \geq 0$ and $b \geq 0$,

$$\frac{a+b}{2} \geq \sqrt{ab}$$

and that equality holds if and only if $a = b$.

(i) By considering $\left(\sqrt{a}-\sqrt{b}\right)^2 \geq 0$, prove that $\frac{a+b}{2} \geq \sqrt{ab}$. [2]

(ii)

Functions f and g are defined by

$$\mathrm{f}(x) = 9x\sin x + \frac{4}{x\sin x} \qquad 0 < x < \pi,$$

$$\mathrm{g}(x) = \ln x \qquad x > 5.$$

Use the AM-GM inequality to show that $\mathrm{f}(x) \geq 12$. [2]

(iii) Determine whether the composite function gf exists. [2]

Practice Paper D (**2.5 hours, 80 marks) EJC 2019 MYE JC1 H2 Maths[Modified]**

1 A vegetable store sells three types of vegetables, namely, water cress, long beans and cabbages. The store sells each type of vegetable at a different price per kilogram. Wayne, Andy and Olivia each buys various amounts of each vegetable. However, they cannot remember the individual prices per kilogram, but can remember the total amount that they each paid. The masses of each type of vegetable bought and the total amount paid are shown in the following table.

	Wayne	Andy	Olivia
Water Cress (kg)	2.50	0.75	1.60
Long Beans (kg)	0.50	1.25	0.95
Cabbages (kg)	1.50	0.50	1.05
Total amount paid ($)	33.15	21.46	28.47

Keith visits the same vegetable store and buys 3 kg of water cress, 0.25 kg of long beans and 1.75 kg of cabbages. Assuming that, for each type of vegetable, the price per kilogram paid by Wayne, Andy, Olivia and Keith is the same, determine the amount Keith has to pay. [5]

2 Solve algebraically

$$\frac{x^3+x^2-8x-12}{x^3+4x^2+5x} \leq 0.$$ [5]

3 **(a)** The curve $y = \mathrm{f}(x)$ has asymptotes $x=-1$ and $y=4$. State the equations of asymptotes of the curve $y = 2\mathrm{f}(-x)-3$. [3]

(b) The graph of $y = \mathrm{f}(x)$ undergoes the following sequence of transformations

A: Stretch with scale factor $\frac{2}{3}$ parallel to the *x*-axis

B: Reflect about the *x*-axis

C: Translate 4 units in the negative *x*-direction

Given that the equation of the resulting curve is $y=-\frac{1}{3x+13}$, find the equation of the curve before the 3 transformations were effected. [4]

10 John intends to purchase a 35-year savings plan from Company X. The details of the plan are given below:

A. Starting from the beginning of 2019, he deposits \$12,000 into his account at the start of each year for 20 years.

B. After the first 20 years, he makes no payment for the next 15 years.

C. The company pays compound interest at 4% per annum on the last day of each of the 35 years.

(i) Taking 2019 to be the first year, show that the total in John's account at the end of the second year is \$25459.20. [2]

(ii) By considering the sum of an appropriate geometric sequence, show that the total in his account at the end of n years, where $1 \leq n \leq 20$ is $\$312000(1.04^n - 1)$. [3]

(iii) Hence, find the total in his account at the end of 35 years. [3]

Instead of withdrawing all the money in his account at the end of the 35th year, John has the option to receive monthly pay-outs starting from the first month in the 36th year. The first monthly pay out is \$2500 and on each subsequent month, the pay-out is \$25 more than that of the preceding month.

(iv) After how much time will the total received by John first exceed \$715,000? Give your answer in years and months. [4]

11 The function f is defined by

$$\mathrm{f}: x \mapsto (1-x)(x-7), \qquad x \in \mathbb{R},\ x \leq 3.$$

(i) Show that the inverse function of f exists. [2]

(ii) Define the inverse function of f in a similar manner. [4]

(iii) State the relationship between the graphs of $y = \mathrm{f}(x)$ and $y = \mathrm{f}^{-1}(x)$, and sketch both graphs on the same diagram, indicating clearly the coordinates of any intersections with the axes and the coordinates of any end-point(s). [4]

(iv) Find the exact value of the y-coordinate of the point of intersection of the two graphs in part **(iii)**. [3]

(v) On a separate diagram, sketch the graph of $y = \mathrm{f}\mathrm{f}^{-1}(x)$. [2]

8 The n^{th} term of a sequence is given by

$$u_n = \frac{2}{(n+1)!}, \text{ for } n \geq 1.$$

(i) Show that $u_n - u_{n+2} = \dfrac{2(n^2+5n+5)}{(n+3)!}$. [1]

(ii) Hence, find $\displaystyle\sum_{n=1}^{N} \frac{n^2+5n+5}{(n+3)!}$, leaving your answer in the form $\dfrac{2}{3} + \dfrac{a}{(N+2)!} + \dfrac{b}{(N+3)!}$, where a and b are integers to be determined. [4]

(iii) Deduce the sum of $\dfrac{29}{6!} + \dfrac{41}{7!} + \ \ldots \ + \dfrac{209}{15!}$. [3]

(iv) Explain why $\displaystyle\sum_{n=1}^{\infty} \frac{n^2+5n+5}{(n+3)!}$ converges, and state the sum to infinity. [2]

(v) Use the result in part (ii) to find $\displaystyle\sum_{n=2}^{N} \frac{n^2+3n+1}{(n+2)!}$. [3]

9 Relative to an origin O, the points A and B have position vectors $4\mathbf{i}+2\mathbf{j}$ and $3\mathbf{i}+\mathbf{k}$ respectively. The point C is such that $OACB$ is a parallelogram and the point E is such that $\overrightarrow{BE} = \dfrac{1}{4}\overrightarrow{BC}$. The point M is the midpoint of AB.

(i) Find the vector equations of the lines AB and OE. [4]

(ii) Find the position vector of point F, the point of intersection of AB and OE.

Hence deduce $\dfrac{OF}{OE}$. [4]

(iii) Find angle OMA, giving your answer correct to the nearest 0.1°. [3]

(iv) Find the exact length of the projection of OM onto AB. [2]

5 A curve C is described by the equation

$$\frac{(x-2)^2}{\left(\frac{5}{2}\right)^2}-\frac{(y-3)^2}{b^2}=1,$$

where b is a positive constant.

It is given that the line $y=\frac{8}{5}x-\frac{1}{5}$ is one of the asymptotes of C.

(i) Show that the value of b is 4 [2]

(ii) Sketch the curve of C, showing clearly the equation of the asymptotes and the coordinates of the vertices. [3]

(iii) Hence, state the set of values of m for which the line with equation $y=m(x-2)+3$ does not intersect C. [1]

6 The sequence u_1, u_2, u_3, ... is a geometric progression with first term 3 and common ratio r.

(i) Write down u_k in terms of k and r. Hence, show that $\ln u_1$, $\ln u_2$, $\ln u_3$, ... is an arithmetic progression. [3]

It is further given that $\sum_{k=1}^{30} \ln u_k = 45$.

(ii) Find the value of r. [2]

(iii) Show that $\frac{1}{u_1},\frac{1}{u_2},\frac{1}{u_3},\ldots$ is a geometric progression. Explain why this progression is convergent and find the sum to infinity. [5]

7 Express x^3-8 in the form $(x-2)(ax^2+bx+c)$, where a, b and c are constants to be determined. [1]

Without using a calculator, solve the inequality $\frac{x^2}{x+\pi}\geq\frac{8}{x(x+\pi)}$. [4]

Hence, solve $\frac{(\ln p)^2}{\ln p+\pi}\geq\frac{8}{\ln p(\ln p+\pi)}$. [2]

Practice Paper C (2.5 hours, 80 marks)

1 Ryan purchased storybooks, Lego sets and sticker sets at a bookshop. If he had bought another 6 more sticker sets, the number of storybooks he bought would have been twice the number of sticker sets. The price of each storybook, Lego set and sticker set was \$14.50, \$20.30, and \$6.10 respectively. Given that he bought a total of 30 items and paid \$513.60 for his purchase, how many of each item did he buy? [3]

2 On the same axes, sketch the graphs of $y = |x^2 + 12x|$ and $y = 6 - x$. [2]

Hence, solve the inequality $|x^2 + 12x| + x - 6 > 0$. [3]

3 Functions f and g are defined by

$$f : x \mapsto 1 + \lambda \cos x, \quad x \in \mathbb{R}, \ -\frac{\pi}{2} < x < \frac{\pi}{2},$$

$$g : x \mapsto x^2 - 6x + 5, \quad x \in \mathbb{R}^+,$$

where λ is a constant and $\lambda > 4$.

(i) Show that the composite function gf exists. [2]

(ii) State the domain of gf and find, in terms of λ, the range of gf. [3]

4 It is given that

$$f(x) = \begin{cases} x^2, & 0 < x \le 1, \\ 2 - x, & 1 < x \le 2, \end{cases}$$

and that $f(x) = f(x + 2)$ for all real values of x.

(i) Sketch the graph of $y = f(x)$ for $-0.5 \le x \le 3$. [3]

(ii) Solve $f(x) = -x + \frac{3}{4}$, giving your answer(s) in exact form. [2]

8 **(a)** The function f is defined by

$$f : x \mapsto \frac{1}{(x-3)^2}, \quad \text{where } x \in \mathbb{R},\ 2 \le x < 3.$$

(i) Explain why f^{-1} exists. [1]

(ii) Find $f^{-1}(x)$ and state the domain of f^{-1}. [3]

(iii) Sketch the graphs of $y = f(x)$, $y = f^{-1}(x)$ and $y = f^{-1}f(x)$ on the same diagram. [4]

(iv) If $f(x) = f^{-1}(x)$, show that x satisfies the equation $x^3 + px^2 + qx - 1 = 0$, where p and q are constants to be determined. [3]

(b) Let g and h be functions defined by

$$g(x) = x^3 - 3x^2 + 4, \quad x \in \mathbb{R}$$

$$h(x) = \begin{cases} 0, & x < 2 \\ 1, & x \ge 2 \end{cases}$$

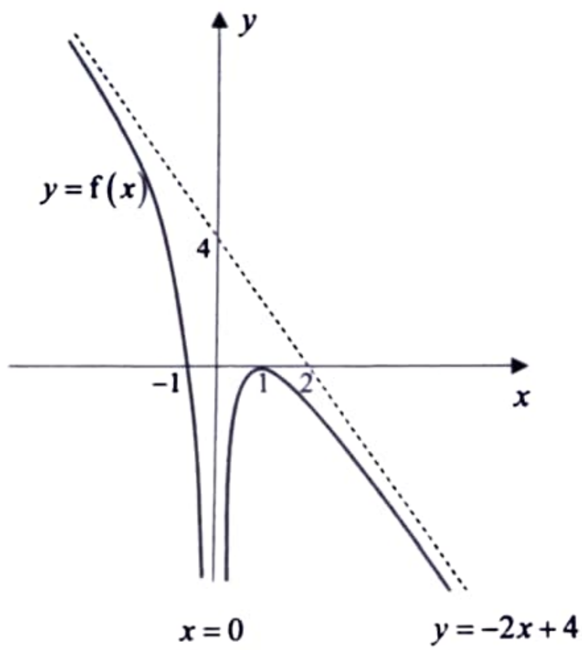
Show that the composite function gh exists. Define the function gh. [3]

7 **(a)** A curve C has equation $(x-1)^2 - y^2 = 4$.

(i) Sketch the graph of curve C, indicating clearly the relevant features. [2]

(ii) Describe a sequence of transformations which would transform the graph of C to the graph of $4x^2 - y^2 + 4y - 20 = 0$. [4]

(b) The graph of $y = \mathrm{f}(x)$ has a maximum point at $(1,0)$ and asymptotes with equation $x = 0$ and $y = -2x + 4$.



Sketch the following curves on separate diagrams. Indicating clearly the equations of asymptotes, stationary points and intersection with axes if any.

(i) $y = \mathrm{f}(|x|) - 1$, [4]

(ii) $y = \dfrac{1}{\mathrm{f}(x)}$. [3]

Practice Paper B (2.5 hours, 80 marks)

1 The sum of the first n terms of a series is given by $S_n = \frac{5}{\pi - 1}\left[1 - \left(\frac{1}{\pi}\right)^n\right]$.

(i) Find the n^{th} term of the series in terms of π [2]

(ii) Show that the series is a geometric progression. Hence, determine whether the sum to infinity of the geometric progression exists. [3]

2 Kesamet Jewellery sells 3 designs of necklaces which are adorned with 3 different kinds of gems. A total of 500 diamonds, 680 sapphires and 530 emeralds are prepared for the making of the necklaces daily. The number of gems required for each design is shown in the table below.

Gems	Design		
	Necklace A	Necklace B	Necklace C
Diamond	25	0	10
Sapphire	10	20	10
Emerald	0	20	10

(i) How many necklaces of each design can be made daily? [4]

(ii) How many diamonds will be left daily? [2]

3 By sketching the curves $y = \frac{6}{x+1}$ and $y = |x|$ on the same diagram, solve the inequality $\frac{6}{x+1} \le |x|$. [4]

Hence solve the following inequalities, leaving your answers in exact form.

(i) $\frac{6}{x^2+1} \le x^2$, [2]

(ii) $\frac{6}{\ln x + 1} \le |\ln x|$. [2]

4 The line l_1 passes through the points M and N with coordinates (4,8,3) and (20,40,19) respectively. The line l_2 has equation $\frac{x-7}{a} = \frac{y-6}{4} = \frac{z-5}{5}$, where a is a constant. The lines l_1 and l_2 intersect at the point P.

(i) Show that $a = 6$ and find the coordinates of P. [4]

(ii) Find the length of projection of $\overline{MP}$ onto the line l_2. Hence find the shortest distance from the point M to the line l_2. [4]

(iii) The point W is on l_2 such that the point N is the foot of the perpendicular from W to line l_1. Find the position vector of W. [3]

5 A farmer needs to plough 100 000 m^2 of land to grow his crops. He ploughs 300 m^2 on Day 1 and increases the area by 50 m^2 on each subsequent day.

(i) Find an expression for the area of land the farmer has ploughed from Day 1 to Day n. [2]

(ii) Deduce the minimum number of days required for him to plough 100 000 m^2 of land. [3]

The farmer's wife will then plant the seedlings when the land is ploughed. On Day 3, she starts to plant the seedlings on 60% of the total land ploughed on Day 1 and Day 2. On subsequent days, she will increase the area by 5% of the previous day.

(iii) Find an expression for the area of land covered by seedlings by the end of Day n. [3]

(iv) Find the number of days the wife will take to catch up with the farmer given that this happens before the land is completely ploughed. [3]

6 A curve with equation $y = ax + b + \frac{c}{x+d}$, where $a > 0$, b, c and d are constants, has two asymptotes which intersect at the point $(0, -1)$.

(i) State the value of d and show that $b = -1$ [2]

(ii) By using differentiation, find the range of values of c if the curve has no stationary points. [3]

For the rest of the question, take $c = 4$.

(iii) Find the coordinates of the turning points of the curve in terms of a. [3]

(iv) Sketch the curve, stating clearly the turning points and equations of asymptotes. [2]

Hence find the range of values of k in terms of a such that the curve $y = ax - 1 + k + \frac{4}{x+d}$ intersects the x-axis. [2]

(i) $y = f(|x|)$, [2]

(ii) $y = \dfrac{1}{f(x)}$, [3]

including the coordinates of the stationary points and the points where the graphs cross the axes, and the equations of the asymptotes.

Deduce the range of values of a such that $f(x) = \dfrac{1}{f(x)}$ have five distinct roots. [2]

7 The function f is defined by

$$f : x \mapsto \begin{cases} x+2, & x < -1, \\ e^{1-x^2}, & x \geq -1. \end{cases}$$

(i) Sketch the graph of f and state its exact range. [3]

(ii) Explain why f^{-1} does not exist. [1]

(iii) State the largest integer value of k such that the restriction function

$$g : x \mapsto \begin{cases} x+2, & x < -1, \\ e^{1-x^2}, & -1 \leq x \leq k, \end{cases}$$

has an inverse. [1]

(iv) With the value of k found in part **(iii)**, find g^{-1} in a similar form. [4]

(v) On the same diagram, sketch the graph of g and g^{-1}, indicating clearly the line of symmetry and its equation, [2]

8 **(a)** (i) State the equations of the asymptotes of the curve $\dfrac{y^2}{a^2} - \dfrac{x^2}{4} = 1$, where a is a positive constant. [1]

(ii) A curve C_1 has equation

$$\frac{(y-1)^2}{a^2} - \frac{x^2}{4} = 1, \text{ where } a > 1.$$

Sketch the graph of C_1, indicating clearly the equations of the asymptotes and the coordinates of the points of intersection with the y-axis. [4]

(b) A curve C_2 is given by the parametric equations $x = t^3 + t$, $y = -e^t + 2e^{-t}$, $-1 \le t \le 1$.

(i) Sketch C_2, indicating clearly where the curve crosses the x- and y-axes. [4]

(ii) Find the coordinates of the point of intersection of C_2 and the line $y = x - 1$. [3]

9 **(a)** Without using a calculator, solve the inequality $\dfrac{x^2 + 3x + 1}{2x + 3} \ge x$. [4]

Hence, find the set of exact values of x for which satisfies

$$\frac{49x^2 - 21x + 1}{3 - 14x} \ge -7x.$$ [2]

(b) It is given that $f(x) = x\sin\left(\dfrac{x}{2}\right)$ for $0 \le x < 2\pi$ and that $f(x) = f(x + 2\pi)$ for all real values of x.

(i) Write down the exact value of $f(5\pi)$. [1]

(ii) Sketch the graph of $y = f(x)$ for $-4\pi \le x \le 5\pi$. (It is not necessary to label the stationary points). [3]

(iii) Show that $x^2 + x + 1$ is positive for all real values of x. [1]

By sketching another graph on the diagram in part **(ii)**, solve the inequality $\dfrac{f(x)}{x^2 + x + 1} > 1$. [2]

5 Relative to the origin O, the points A, B and C have position vectors given by $\overrightarrow{OA} = \mathbf{a}$, $\overrightarrow{OB} = \mathbf{b}$, $\overrightarrow{OC} = \mathbf{c}$ and $4\mathbf{a} = 5\mathbf{b} - \mathbf{c}$.

Given that OAB is an equilateral triangle.

(i) Show that $\mathbf{a} \cdot \mathbf{b} = \frac{1}{2}|\mathbf{b}|^2$. [2]

(ii) The point D lies on OB produced such that $OD = \lambda OB$ and OD is perpendicular to CD. Find the value of λ. [4]

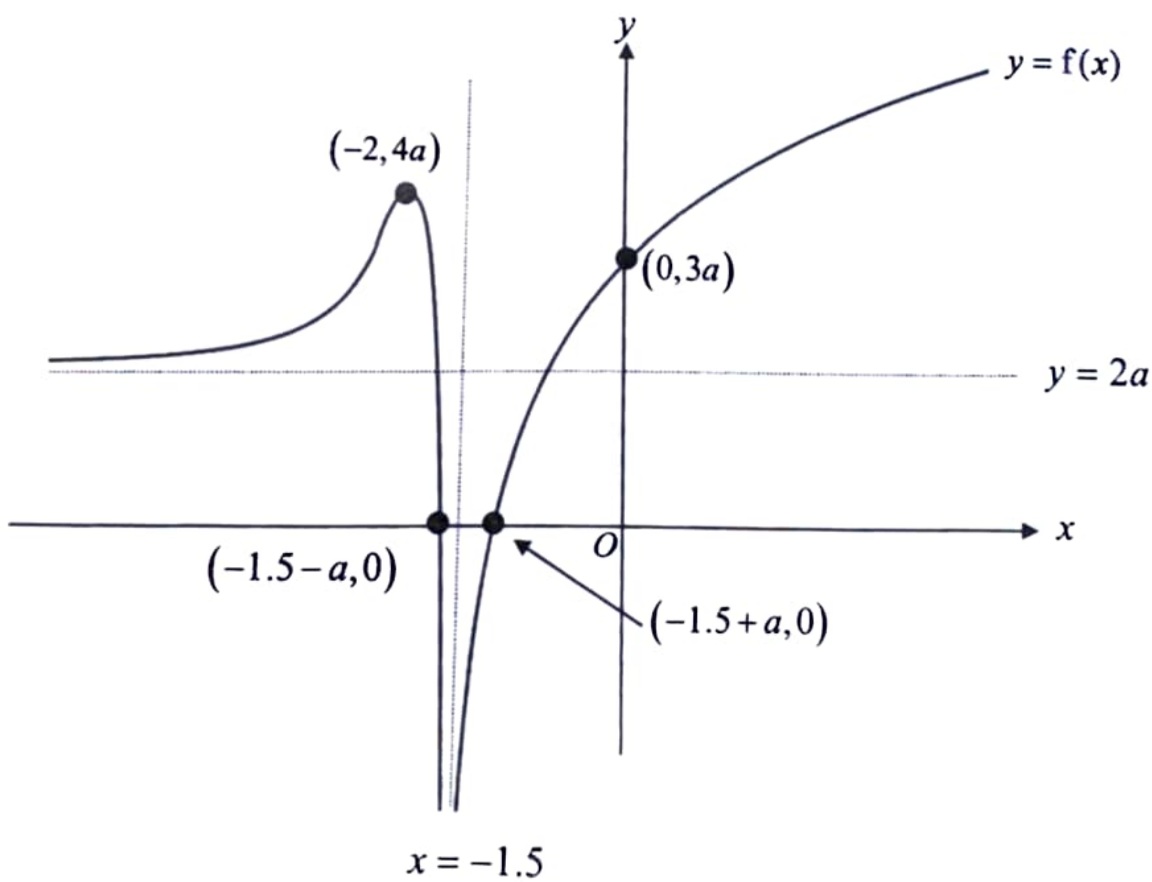
(iii) Hence by using the vector product, find the area of ΔOCD, leaving your answer in the form of $k|\mathbf{b}|^2$, where the exact value of k is to be determined. [3]

6 (a) Describe fully a sequence of transformations which would transform the curve $4x^2 + y^2 = 1$ onto the curve $4x^2 + (2y-4)^2 = 1$.

Describe the locus of the points which satisfies the equation $4x^2 + (2y-4)^2 = 1$. [3]

(b) It is given that $0 < a < 0.5$ is a constant.

The diagram shows the graph of $y = f(x)$, which cuts the y-axis at $(0, 3a)$ and the x-axis at $(-1.5-a, 0)$ and $(-1.5+a, 0)$. The curve has a stationary point at $(-2, 4a)$ and asymptotes with equations $y = 2a$ and $x = -1.5$.



On separate diagrams, sketch the graphs of