Measurement

CONTENT

1. Physical Quantities and SI Units
2. Errors and Uncertainties
3. Scalars and vectors

LEARNING OUTCOMES

Candidates should be able to:

a.	recall the following base quantities and their units: mass (kg), length (m), (s), current (A), temperature (K), amount of substance (mol).
b.	express derived units as products or quotients of the base units and use the named listed in ‘Summary of Key Quantities, Symbols and Units’ as appropriate.
c.	Use of SI base units to check homogeneity of physical equations.
d.	show an understanding and use the conventions for labeling graph axes and table columns as set out in the ASE publication Signs, Symbols and Systematics (The ASE companion to 16-19 Science, 2000).
e.	use the following prefixes and their symbols to indicate decimal sub-multiples or multiples of both base and derived units: pico (p), nano (n), micro (μ), milli (m), centi (c), deci (d), kilo (k), mega (M), giga (G), tera (T).
f.	make reasonable estimates of physical quantities included within the syllabus.
g.	distinguish between scalar and vector quantities, and give examples of each.
h.	add and subtract coplanar vectors.
i.	represent a vector as two perpendicular components.
j.	Show an understanding of the distinction between systemic errors (including zero error) and random errors.
k.	show an understanding of the distinction between precision and accuracy.
l.	assess the uncertainty in a derived quantity by simple addition of actual, fractional or percentage uncertainties or by numerical substitution (a rigorous statistical treatment is not required).

Physics Books for further reading :
Physics for Scientists and Engineers. Serway.
College Physics. Sears and Zemansky.
Physics. Robert Hutchings.
Physics. Tom Duncan.

1.0 Quantities and Units

The science of physics is based upon taking measurements. All scientific theories and laws must be tested experimentally, and all experiments necessitate making measurements.

1.1 Physical Quantities: Base Quantities and Derived Quantities

In Physics, quantities that can be measured are known as **physical quantities**. Each quantity consists of a **numerical magnitude** and a **unit**. (For vectors, there are **directions** as well.) **Base quantities** are physical quantities that are the most fundamental and they are independent of each other. The corresponding units for the base quantities are called the **base units**. It is important at all times to think of and write the value and the unit of any quantity together.

The metric system of units was introduced at the time of the French Revolution to rationalise the chaos of units that existed at that time. It has been modified since then and now most countries use the metric system of units called the Systeme International (SI). The advantage of the SI system of units is that any quantity has only one unit in which it can be measured.

- Quantities can be classified as **base quantities** or **derived quantities**.
- Base quantities are the 7 physical quantities of the *S.I. system* by which all other physical quantities are defined. They arbitrarily chosen by scientists so that they:
  a) form the smallest set of physical quantities that will lead to a complete description of physics in the simplest terms.
  b) are based on international agreement by scientists.
- Derived quantities are obtained from one or more of the base quantities through a **defining equation.**

Base Units

- There are 7 base units, one for each of the base quantities.
- **Base units** are the 7 base units of the *S.I. system*, related to a base quantity, whose magnitude is defined without referring to any other units. The units used in measurement are the International System of Units (SI).

Base Quantity	Base Unit	Name
Length	**m**	**metre**
Mass	**kg**	**kilogram**
Time	**s**	**second**
Electric current	**A**	**ampere**
Temperature	**K**	**kelvin**
Amount of substance	**mol**	**mole**
Luminous Intensity	**cd**	**candela**

Derived Units

- A **derived unit** can be expressed in terms of base units by using the defining equation of the quantity.
- It is obtained from the base units by multiplication and/or division; without including any numerical factors.

DERIVED QUANTITIES	Defining Equation	Base SI Units	Derived Unit
Volume	$V = l^3$	m^3	-
Velocity	$v = d/t$	ms^{-1}	-
Frequency	$f = 1/T$	s^{-1}	Hz
Force	$F = ma$	$Kgms^{-2}$	N
Work or Energy	$W = Fd$	kgm^2s^{-2}	J
Pressure	$P = F/A$	$kgm^{-1}s^{-2}$	Pa
Charge	$Q = It$	As	C

Example 1:
The energy of a photon of light frequency *f* is given by *hf*, where h is the Planck constant. What are the base units of *h*?

Solution:
The defining equation is $E=hf$

$$h = \frac{E}{f}$$

Base units of $h = \frac{kgm^2s^{-2}}{s^{-1}} = kgm^2s^{-1}$

Example 2:
The drag coefficient C_D of a car moving with speed *v* through air of density ρ is given by $C_D = \frac{F}{2pv^2A}$ where *F* is the drag force exerted on the car and *A* is the maximum cross-sectional area of the car perpendicular to the direction of travel. Show that C_D is dimensionless.

Solution:

$$C_D = \frac{F}{2\rho v^2 A}$$

Units of $C_D = \frac{(kgms^{-2})}{(kgm^{-3})(ms^{-1})^2(m^2)} = \frac{kgms^{-2}}{kgms^{-2}} = 1$ C_D is dimensionless.

(Quantities that have no units are known as **dimensionless**.)

Prefixes

In physics, it is common to encounter quantities that are very large or minute in magnitude. For example, Earth's mean radius is estimated to be 6 400 000 m. Radius of a hydrogen nucleus is approximately 0.000 000 000 000 001 3 m. To include all the zeros in all computation or steps will be undesirable and hence scientists adopted any of the 2 methods: use of scientific notation (standard form) or **prefixes**.

Prefix	Symbol	Sub-multiple
Pico	p	10^{-12}
Nano	n	10^{-9}
Micro	μ	10^{-6}
Milli	m	10^{-3}
Centi	c	10^{-2}
Deci	d	10^{-1}

Prefix	Symbol	Sub-multiple
Kilo	k	10^{3}
Mega	M	10^{6}
Giga	G	10^{9}
Tera	T	10^{12}

Example 3:
Convert the following to suitable units with prefix: 800, 000 W; 0.0000325 m; 2.65 x 10^{-10} s

Solution

800, 000 W = 800 kW = 0.8 MW
0.0000325 m = 0.0325 mm = 32.5 μm
2.65 x 10^{-10} s = 2.65 x 10^{-1} x (10^{-9}) = 2.65 x 10^{-1} ns

Estimates Of Physical Quantities

You are expected to make reasonable **estimates of the order of magnitude** of common physical quantities.

Orders of Magnitude of Some Common Data

Distances

Radius of Earth	6400 km	Moon-Earth Distance	3.8×10^{8} m
Earth-Sun Distance	1.5×10^{11} m	Size of atom	10^{-10} m (1 Å)
Size of nucleus	10^{-15} m	Wavelength of visible light	$4\text{-}7 \times 10^{-7}$ m

Density

Density of air	about 1 kgm^{-3}	Density of water	1000 kgm^{-3}
Density of metals	about 10^{3} kgm^{-3}		

Mass

Mass of atom	10^{-27} kg	Mass of person	60 kg
Mass of earth	6.0×10^{24} kg		

Speed

Speed of walking person	~ 3 kmh^{-1} or 0.8 ms^{-1}
Speed of running person	~ 10 kmh^{-1} or 2.7 ms^{-1}
Speed of car	~ 60 kmh^{-1} or 17 ms^{-1}
Speed of molecular movement in air (Room temp)	300 ms^{-1} to 400 ms^{-1}
Speed of light	3×10^{8} ms^{-1}
Speed of sound	~ 300 ms^{-1}

Others

Atmospheric pressure	1.01×10^{5} Pa	Room temperature	303 K (30 ^{o}C)

Example 4:

What is a reasonable estimate for the volume of a wooden metre rule found in a school laboratory?

A 1.5 cm^3 **B** 15 cm^3 **C** 150 cm^3 **D** 1500 cm^3

Volume = 100 cm x 3 cm x 0.5 cm = 150 cm^3

Answer C
2007 P1Q1

Example 5:

Which of the following gives the estimated number of atoms in your body?

A 10^{24} **B** 10^{27} **C** 10^{30} **D** 10^{33}

VJC 2010 Prelim

Solution

The mass of an average person is about 60 kg and it consists of mainly water. Molar mass of water is 18 g = 0.018 kg.
60 kg consist of 60/0.018 = 3.3×10^{3} moles.
Hence no. of molecules = $3.3 \times 10^{3} \times 6.02 \times 10^{23}$ = **2×10^{27} molecules**.

1.2 Homogeneity Of Physical Equations

When we try to form equations that describe a physical system or phenomenon, it is important to understand how the concept of homogeneity helps us:

1. Equations that are not homogeneous are definitely wrong.
2. Equations that are homogeneous may or may not be correct. Possible causes of equations that are homogeneous but are incorrect.
   (i) Presence/absence of dimensionless constant
   (ii) Incorrect coefficient.
   (iii) Presence of extra term(s)/ Missing term(s).

- When an equation is *homogeneous (or dimensionally consistent)*, the following rules must be applicable:
  - **Only terms with the same units can be added or subtracted** e.g. $C = A + B$ implies that A and B must have the same units.
  - **Units on both sides of the equation must be the same** e.g. $A = B$ implies that A and B must have the same units.
  - **The exponent of a term must not have any unit** e.g. $e^{\frac{-t}{RC}}$ implies that units of RC combined must have the same unit as time, t.

1.3 Plausibility of Physical Equations

Checking the homogeneity of an equation using base units (or dimensional analysis) is a powerful way of establishing if the physical equation is reasonable. It narrows the numerous combinations that may exist.

For example, consider the period T of a simple pendulum. The possible factors which may affect it are its length l, its mass m and the acceleration due to gravity g.

We can therefore form an equation that describes the factors that affect the period of a simple pendulum:

$T \propto l^x m^y g^z$ where x, y and z are pure numbers without units

$\Rightarrow$ $T = k\, l^x m^y g^z$ where k is a “unit-less” constant of proportionality.

In some situation, k may have a unit.

Example 6: Check that the equations (i) $T = 2\pi\sqrt{\frac{l}{g}}$ and (ii) $x = ut + \frac{1}{2}at^2$ are homogenous.

Solution

(a) Unit of T = s; units of l = m; units of g = m s^{-2}

Units of $2\pi\sqrt{\frac{l}{g}} = \sqrt{\frac{\text{m}}{\text{m s}^{-2}}} = \sqrt{\frac{1}{\text{s}^{-2}}} = \sqrt{s^2}$ = s

Since units on LHS = units on RHS, equation is homogeneous.

(b) Units of u = m s^{-1}; unit of t = s; units of a = m s^{-2}; unit of s = m.

Units of ut = (m s^{-1})(s) = m; Units of $\frac{1}{2}at^2$ = (m s^{-2})(s^2)= m.

Since units of x = units of ut = units of $\frac{1}{2}at^2$, equation is homogeneous.

Further Readings

The Metre Convention. http://www.bipm.org/en/worldwide-metrology/metre-convention/

This kilogram has weight-loss problem. http://www.npr.org/templates/story/story.php?storyid=112003322

2. Errors and Uncertainties

2 ERRORS

Error refers to the difference between a measured quantity and its true value. There are various factors that give rise to errors. However, in experiments, outright errors such as misreading a measurement (including parallax error), wrong calculations or poor execution of the experiment should not be considered as a legitimate set of errors when discussing the reliability of the results of the experiment. An experiment is only meaningful when such errors are avoided. Errors in measurement can result in costly or catastrophic outcome. An example of an error in measurement is the Hubble telescope. An error in calibration tool during the polishing process of the mirror led to the imperfect image sent back to earth when the Hubble telescope was first used in space in 1990. After a few costly repairs in outer space by the space shuttle missions, it was finally corrected and put to good use in 2009.

While the uncertainty in a reading is an inherent limitation of our measuring instruments, errors could also arise from different sources. They could be due to the following:

(a) **The Instrument**
Mass-produced instruments may not be correctly calibrated. Old instruments like ammeter and voltmeter may suffer drift in accuracy due to the weakening of magnet or spring inside the meters. External conditions, particularly at extreme temperatures, may affect the accuracy of many instruments. The drift of the zero reading contributes to **zero errors**.

(b) **The Experimenter**
Improper use of instruments is a common source of error. Examples are parallax error, misalignment of the zero scale or over-tightening of the micrometer screw gauge.

(c) **Nature of Quantity to be Measured**
Some quantities may inherently change with time during the measurement (e.g. change in the resistance of a wire due to heating) or may give different values of measurements when taken at different points (e.g. diameter of a long wire).

Errors can be classified under 2 categories, namely **Systematic Errors** and **Random Errors**.

2.1 Systematic Errors

A systematic error is one that leads to readings that are **consistently more or consistently less*** than the actual reading.

*Note that the phrase is *consistently more or consistently less* **NOT** *consistently more or less*. A systematic error is either too high or too low, **NOT** both.

(i) A systematic error is a reproducible error caused by imperfect equipment, calibration or technique. (e.g. a shrunken, bent or damaged meter rule)

(ii) It **can be <u>eliminated</u>** if the source is known and removed. (i.e. use more accurate equipment, calibrate the equipment properly or use a better/correct technique)

(iii) To detect systematic errors, you would need to either make measurements under different experimental conditions

OR

use another technique to perform the experiment.

A consistently different set of results reveals the presence of systematic errors.

2.1.1 Some common systematic errors

a. **Zero error**
The pointer of an instrument does not exactly coincide with the zero mark when it is supposed to. Always check for zero errors before using an instrument such as a micrometer screw gauge or a vernier calipers.

b. **End error**
For example, in some rulers, the 0-cm mark starts right at the edge of the ruler. End error occurs when there is wear and tear at the ends of such a ruler after being used for many years such that the 0-cm mark is no longer present.

When conducting experiments, systematic errors that can be accounted for or rectified should be corrected immediately.

2.2 Random Errors

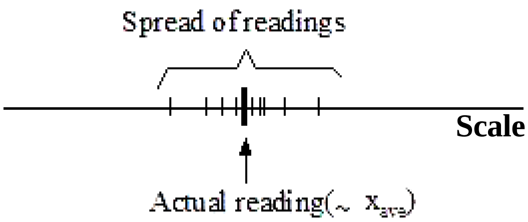
Even if every step of the experiment was done properly and all systematic errors accounted for, a quantity measured a number of times would not give identical results. There will be a fluctuation in the results and this is known as random error.

A random error is one that gives rise to a **scatter of readings about a mean value**.

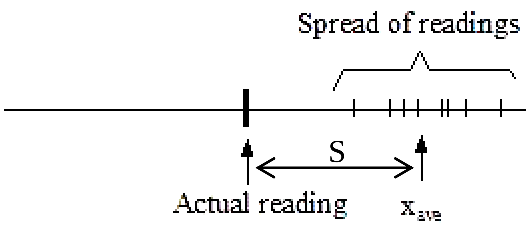
Random errors **can be <u>reduced</u>** by taking the average of repeated readings.

Random errors **cannot be eliminated** even if the source is known because there is no way to reproduce exactly the same conditions in each measurement.

Pure Random Error



Random with Systematic Error



S : Systematic Error

2.2.1 Some common random errors

a. **Fluctuating conditions** of environment - such as temperature, pressure, vibrations etc.

b. **Errors of judgment** - e.g. the observer's estimate of a fraction of the smallest division may vary from time to time.

a) Parallax errors when reading from an inconsistent direction

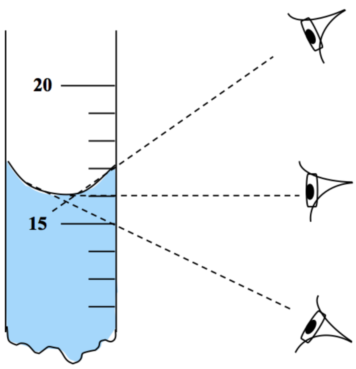


Fig. 1. Reading from a burette from an inconsistent direction.

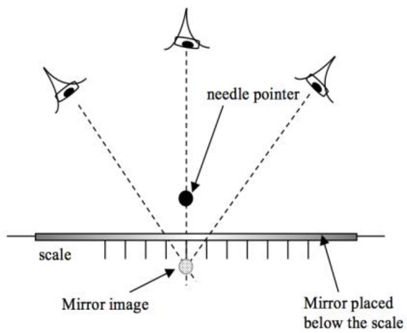


Fig. 2. Reading from a needle deflection pointer meter from an inconsistent

Random errors can arise from parallax when an observer reads a scale from an *inconsistent* direction. In Fig. 2, a plane mirror is placed alongside the scale so that the viewer may make the correct reading when the needle and its mirror image coincide; the plane of view is then perpendicular to the scale.

b) Variations in environmental conditions
Try as we may to preserve or maintain the experimental conditions, certain things are beyond our control e.g. variations in air pressure, temperature, reaction time etc. resulting in slight fluctuations in the readings.

c) Irregularity of the quantity being measured
Certain quantities by nature do not follow a regular pattern e.g. the spontaneous decay of radioactive nuclei, thickness of a wire, loudness of sound, reaction time etc.

d) Limitation of the equipment.
Certain equipment may be so sensitive that it can detect even the slightest variation on the signals which can be due to a number of reasons e.g. different applications of pressure on the jockey to the potentiometer or on the micrometer screw gauge etc. This may not be a good thing if a general reading is what you want.

2.2.2 Ways to Reduce Random Errors

Random errors **cannot be eliminated** but can only be reduced by:
a) Taking repeated measurements to obtain an average value.
b) Plotting a graph to establish a pattern and obtaining the line or curve of best fit. In this way, the discrepancies or errors are reduced.
c) Maintaining good experimental technique (e.g. reading from a correct position)

DISTINCTION BETWEEN SYSTEMATIC ERRORS AND RANDOM ERRORS

	Systematic Errors	Random Errors
How does the error affect the readings?	It causes a random set of readings to be distributed around an average value that is consistently more or consistently less than the actual reading. The error is predictable.	It gives a scatter of readings about a mean value. The error is unpredictable.
How to detect the error?	It can be detected by making measurements under a different experimental conditions or by using another technique to perform the experiment. A consistently different set of results may reveal the presence of systematic errors.	It can be detected by plotting a graph and drawing a best-fit line to the points; the presence of random errors is reflected by the scattering of points about the best-fit line.
How to minimize or eliminate the error?	These errors may be eliminated if the cause is known and rectified, but sometimes can only be minimized through improving experimental techniques. Cannot be minimized or eliminated by averaging repeat readings.	Cannot be eliminated, but can be minimized by averaging repeat readings or by observing the scatter of the graph.

2.3 Precision And Accuracy

Accuracy is a measure of how close a measurement or result is to the true value. It depends on how well systematic errors can be controlled or compensated for.

Precision is a measure of how exact the result is or how close repeated results are to each other. It depends on how well random errors can be overcome or analyzed.

> If an experiment has **small systematic errors**, it is said to have **high accuracy**.
> If an experiment has **small random errors**, it is said to have **high precision**.

Illustration of precision and accuracy using shooting target as an example

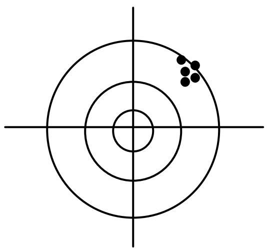
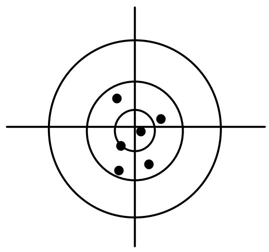
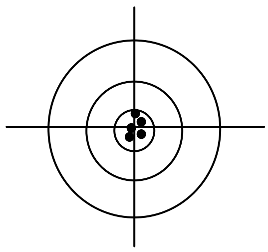
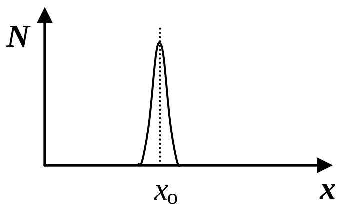
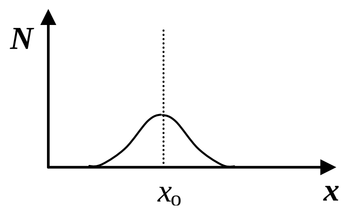
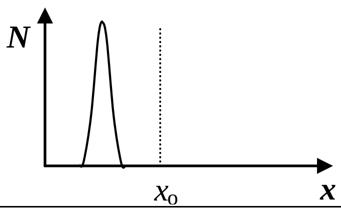
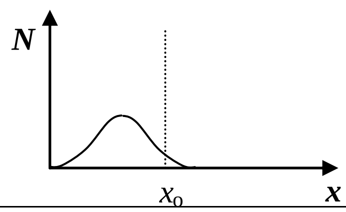
Diagram			
Precision	Good	Poor	Good
Random Error	Small	Large	Small
Accuracy	Poor	Good	Good
Systematic Error	Large	Small	Small

Illustration of precision and accuracy in graphical form

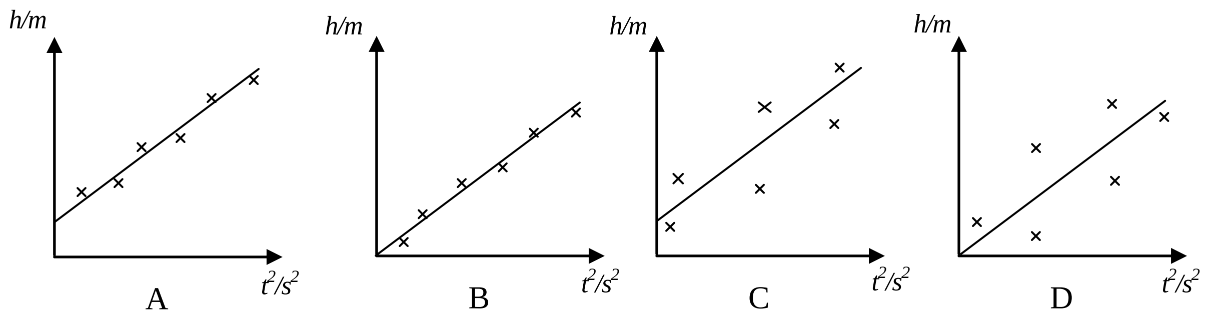
Example 1
Complete the following table.
*N represents the number of data collected, x represents the possible values of the results and x_0 represents the actual value.

	
Accurate and Precise	***Accurate and Imprecise***
	
Inaccurate and Precise	***Inaccurate and imprecise***

Example 2
Four students each made a series of measurements of the acceleration of free fall *g*, where $g = 9.81\ ms^{-2}$. The table shows the results obtained.
Which student obtained a set of results that could be described as precise but not accurate?

Student	Results, g / m s^{-2}				*Average*	*Precise*	*Accurate*
A	9.81	9.79	9.84	9.83	*9.82*	✓	✓
B	9.81	10.12	9.89	8.94	*9.69*	x	✓
C	9.45	9.21	8.99	8.76	*9.10*	x	x
D	8.45	8.46	8.50	8.41	*8.46*	✓	x

Worked Example 3
A student measures the time *t* for a ball to fall from rest through a vertical height, *h*. He repeats the measurement for various different values of *h*. Knowing that the equation $h = \frac{1}{2}gt^2$ applies, he plots a graph of *h* against t^2. Comment on the accuracy and precision of the given data in the boxes below.



A	B	C	D
Not accurate but more precise than C	***Accurate and more precise than C***	***Not accurate and not precise***	***Accurate but not precise***

Questions:

1. How do you know these graph are accurate or precise?

2. What type of errors resulted in these graphs?

2.4 Significant Figures And Precision

When a number such as 0.78 is quoted, it has 2 significant figures (s.f), whereas 0.8, it has only 1 significant figure (s.f). The number of s.f is directly related to the precision of the measuring instrument. The greater the number of subdivisions, the more the precise is the instrument (and the greater the number of s.f in the measurements).

When quoting a final value in a computation of an equation in a physics experiments, the investigator is expected to be able to quote the answer to an appropriate number of s.f.

Example 4
The potential difference (p.d) across a resistor is 2.5 V and the current passing through it is 0.22 A. Calculate the resistance.

R = 2.5 V/0.22 A = 11.363636…Ω
Is this answer correct?

Calculation cannot improve the precision of the measuring instruments. A p.d quoted as 2.5 V means that the measurement lies between 2.45 V and 2.55 V. Similarly, the current is between 0.215 A and 0.225 A. This enables us to calculate a range of possible values for the resistance.

R_{max} = 2.55 V/0.215 A = 11.860465… Ω
R_{min} = 2.45V/0.225 A = 10.888888… Ω

Hence the best we can say is that the resistance is near enough 11 Ω. The figures after the decimal pont are meaningless – they are not significant figures. Rather than doing this maximum/minimum calculation all the time, we use the rule that the calculated value can be no precise than the values used to obtain it. Hence we should only quote the answer to the same number of significant figures as the least precise measurement. Since the values 2.5 V and 0.22 A are quoted to 2 s.f, our answer must be quoted to 2 s.f, i.e R = 11 Ω.

2.4 Uncertainty In A Measurement

Whenever a measurement of a physical quantity is made, some measuring instrument has to be used to make that measurement. Built in to the instrument is a limit of accuracy within which the experimenter is working. The result of this is that the readings that the experimenter takes have a degree of **uncertainty**.

Physical quantities cannot be measured exactly with any instrument. An accurate clock might measure a time interval to a millionth of a second, but even this is not quite exact.

If a person uses a ruler, the measurement will probably be taken to the nearest millimetre. The measurement might be stated as (208 ± 1) mm. This implies that the person taking the measurement thinks that the best value is 208 mm, and that the value will not fall outside the range from 207 mm to 209 mm. The ±1 mm is the **uncertainty** in the measurement. (Note: The symbol " ± " is read as *plus-minus*.)

Uncertainty refers to the range of values on both sides of a measurement in which the actual value of the measurement is expected to lie. The **uncertainty** in a reading can be based on the **finite precision** in the instrument used.

The uncertainty in a reading **A** is represented by the symbols Δ**A** (pronounced as "delta **A**") .

The experimental reading of A together with its associated uncertainty is properly expressed in the form: **A** ± Δ**A**. For example: the value of *g* is 9.81 ± 0.01 ms^2.

In general, express
- the uncertainty (ΔA) to **1 significant figure**,
- the calculated value (A) to the **same decimal place** as the uncertainty
- both with the same units.

1 sig. fig.

For example: 20.0**0** ± 0.0**1** g; 19.**1** ± 0.**1** cm

2.4.1 Absolute Uncertainty

When a measurement is made using an instrument, the value obtained will always carry an uncertainty. The uncertainty of the measurement obtained is then determined by the division of the scale of an instrument.

Scale reading:
Uncertainty is read to **nearest half of the smallest division**.
Example: Using a thermometer to take temperature by reading the height of the mercury thread from the fixed scale.

Measurement:
Uncertainty is read to **nearest smallest division**.
Example: Using a ruler to measure the length of a strip of paper.

As these uncertainties are only estimated values, they are always quoted to **ONE SIGNIFICANT FIGURE** .

The numerical value of the uncertainty is known as the **absolute uncertainty**.

The uncertainty of an instrument determines the order of magnitude (or the number of decimal places) that should be quoted for the measurements made with it. The order of magnitude (or number of decimal places) in a reading is the same as that in the uncertainty. For example, an exact measurement of 20 cm using a ruler must be recorded as (20.0 ± 0.1) cm and not as (20 ± 0.1) cm.

2.4.2 Fractional Uncertainty: How Suitable Is Your Instrument?

Generally, all readings can be recorded in the form $R \pm \Delta R$ in which ΔR is the **absolute uncertainty**. However, the suitability of an instrument in relation to a certain measurement is not reflected by the absolute uncertainty but by the fractional uncertainty (or percentage uncertainty) which is defined as follows:

Fractional Uncertainty = $\dfrac{\Delta R}{R}$

Percentage Uncertainty = $\dfrac{\Delta R}{R} \times 100\%$

Note: *A **small reading (R)** may give rise to a **large fractional uncertainty (**$\frac{\Delta R}{R}$**)**.*

The fractional and percentage uncertainty give a better indication of the <u>significance of the error</u> and the <u>reliability of the measurement</u> and the <u>suitability of the instrument</u> used for it.

For example, when measuring small values such as the diameter of a wire, a precise instrument such as a micrometer screw gauge should be used in order to reduce the fractional and percentage uncertainty.

Example 5

Using a meter rule, a length is measured to be (20.0 ± 0.1) cm.

The percentage uncertainty is equal to (0.1/20.0)x100% = 0.5%

However, if the value is (2.0 ± 0.1) cm, the percentage uncertainty is (0.1/2.0)x100% = 5%

Comment on your answers obtained from the 2 calculations.

Using a ruler to measure a small object, the percentage uncertainty of the measurement will be large. A more precise instrument has to be used to reduce the percentage uncertainty of the measurement.

2.5 Consequential Uncertainty

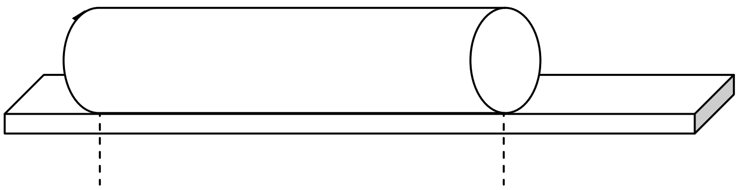
The final result of an experiment is seldom obtained directly from one measurement. It is often calculated from a few measurements with the use of an appropriate equation. The calculated value would thus carry a consequential uncertainty. Here are some rules for estimation of consequential uncertainty:

2.5.1 Addition and Subtraction

If $Y = a + b$ or $Y = a - b$, then the consequential uncertainty of Y is given by

$$\Delta Y = \Delta a + \Delta b$$

The length of a rod is measured with a meter rule.



$x_1 = (1.40 \pm 0.05)$ cm $x_2 = (12.80 \pm 0.05)$ cm

x_2 has values ranging from 12.75 cm to 12.85 cm
x_1 has values ranging from 1.35 cm to 1.45 cm

Maximum length L of rod = 12.85 cm - 1.35 cm = 11.50 cm
Minimum length L of rod = 12.75 cm - 1.45 cm = 11.30 cm

Thus, the length of rod L has values ranging from 11.30 cm to 11.50 cm.

Base on the measurements, L = x_2- x_1 = 12.80 cm -1.40 cm = 11.40 cm
The uncertainty of L = $\Delta x_2 + \Delta x_1$ = 0.05 cm + 0.05 cm = 0.1 cm
As the uncertainty of the measurements is 0.1 cm, the value of L cannot be read more precise than 0.1 cm.
Hence, L = (11.4 $\pm$ 0.1) cm

Example 6

Given that a = (41.2 ± 0.1) cm, b = (20.1 ± 0.5) cm:
a) What is the value of X and its consequential uncertainty, if $X = a + b$?

X= 41.2 +20.1 = 61.3 cm
ΔX =ΔA+ΔB = 0.6 cm

(61.3 $\pm$ 0.6) cm

b) What is the value of Y and its associated uncertainty, if $Y = a – b$?

(21.1$\pm$ 0.6) cm

Important note:

In general, if $X = nA \pm mB$, where n and m are constants then

$$\Delta X = n\Delta A + m\Delta B$$

Example 7

Given that A = (41.2 ± 0.1) cm, B = (20.1 ± 0.5) cm:
Find X and its uncertainty, if $X = 5A – 2B$.
X = 5(41.2) -2 (20.1) = 165.8
ΔX = 5ΔA+2ΔB = 1.5 cm = 2 cm (1s.f)

X = (166 $\pm$ 2) cm

2.5.2 Product and Quotient

If $Y = a \times b$ or $Y = a \div b$, then the consequential uncertainty of Y is given by

$$\frac{\Delta Y}{Y} = \frac{\Delta a}{a} + \frac{\Delta b}{b}$$

Derivation (not required in syllabus)

If $R(x) = A(x)\,B(x)$, where A and B are functions of x:

By differentiation (Product Rule) $\Rightarrow \frac{dR}{dx} = \frac{dA}{dx}B + A\frac{dB}{dx}$,

"Multiplying" dx throughout $\Rightarrow dR = dA \,.\, B + A \,.\, dB$

Dividing by R throughout $\Rightarrow \frac{dR}{R} = \frac{dA}{AB}B + A\frac{dB}{AB}$

$$\Rightarrow \frac{dR}{R} = \frac{dA}{A} + \frac{dB}{B}$$

Replacing d with Δ $\Rightarrow \frac{\Delta R}{R} = \frac{\Delta A}{A} + \frac{\Delta B}{B}$.

Example 8

Given that a = (10.3 $\pm$ 0.1) cm, b = (5.6 $\pm$ 0.1) cm:
Find X and its consequential uncertainty, where $X = a \times b$.

X = 57.68

$\frac{\Delta X}{X}$ = (0.1/10.3) + (0.1/5.6)

= 0.00971 + 0.01786
= 0.0276

Thus ΔX = 1.59 = 2 (1 sf)
Hence X should be written as
X = (58 $\pm$ 2) cm

Generally, if a quantity is a function of the product of a few variables, the percentage uncertainty of the quantity is the sum of the percentage uncertainties of the variables.

If $\mathbf{Y = a^n = a \times a \times a \ldots}$, then

$$\frac{\Delta Y}{Y} = \mathbf{n}\frac{\Delta a}{a}$$

It can be seen that if a variable is raised to a power n, it contributes an uncertainty n times that of the variable towards the consequential uncertainty. Thus all high powered variables warrant careful measurement with instruments with small absolute uncertainties.

Example 9

Given that a = (3.04 $\pm$ 0.02) g and $W = 3a^5$, what is the actual uncertainty in W?

W =778.9

$$\frac{\Delta W}{W} = 5\frac{\Delta a}{a}$$

= 5 (0.02/3.04)
= 0.0329

ΔW = 0.0329x778.9
= 25.6
= 30 (1 s.f)

W = (780 $\pm$ 30) g^5

Example 10

The mass and diameter of a steel ball were used to determine its density. The mass was measured within 1% and the diameter within 3%. What is the percentage uncertainty in the calculated density of the steel?

$$\rho = \frac{M}{V}$$

$= M / (4/3 \,.\, \pi r^3)$

$= M / (4/3 \,.\, \pi (D/2)^3)$

$= (3/4).\,(8/\pi).\, M/D^3$

$= 6M/\pi D^3$

$$\frac{\Delta\rho}{\rho} x 100\% = \left(\frac{\Delta M}{M} + 3\frac{\Delta D}{D}\right) x\ 100\ \%$$

$= [0.01 + 3\ (0.03)]$ x100 %

$= 0.1$ x100 %

$= 10$ %

Important note:

In general, if $X = nA^{\alpha}.mB^{\beta}$ or $X = \frac{nA^{\alpha}}{mB^{\beta}}$, where α, β, n and m are constants, then

$$\frac{\Delta X}{X} = \alpha\frac{\Delta A}{A} + \beta\frac{\Delta B}{B}$$

2.5.3 Quantity with Mathematical Functions

For mathematical function such as trigonometric function of a reading, we can estimate the error as being the difference of the average value with either the smallest or largest possible value. You may need to consider the shape of the curve in order to make a meaningful judgement of the absolute uncertainty.

General Method:

$\Delta z_1 = z - z_{min}$ or $\Delta z_2 = z_{max} - z$

Consider z = sin θ where $\theta = (48 \pm 3)°$,
Calculated value of z = sin 48° = 0.743

Using the smallest and largest possible value of θ,
z_{min} = sin 45° = 0.707
z_{max} = sin 51° = 0.777

$z - z_{min} = 0.743 - 0.707 = 0.036$
$z_{max} - z = 0.777 - 0.743 = 0.034$

Choosing the larger of the two:
Therefore, $\Delta z = 0.036 = 0.04$ (1 s.f.)

$z \pm \Delta z = 0.74 \pm 0.04$

½ (Max – Min) Method:

Consider z = sin θ where $\theta = (48 \pm 3)°$,
Calculated value of z = sin 48° = 0.743

Using the smallest and largest possible value of θ,
z_{min} = sin 45° = 0.707
z_{max} = sin 51° = 0.777

$\Delta z = ½\ (z_{max} - z_{min})$
$= ½\ (0.777-0.707)$
$= 0.035$

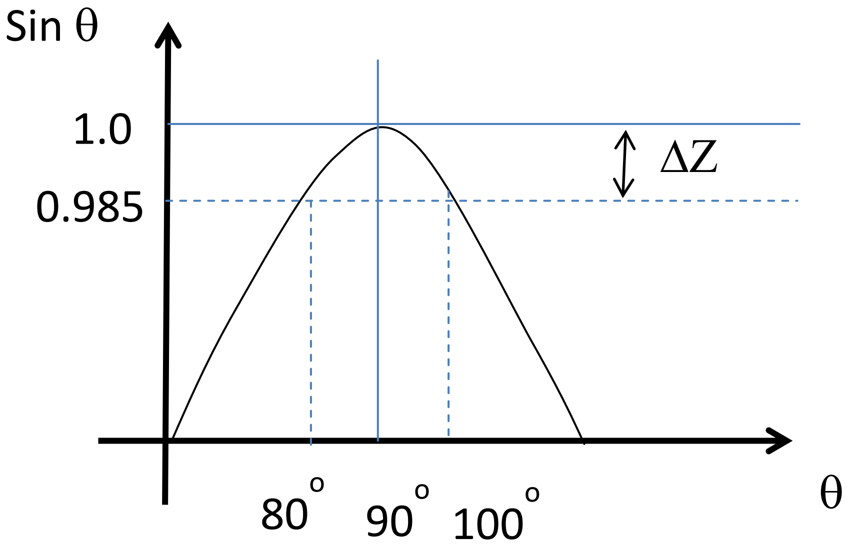
Therefore, $\Delta z = 0.035 = 0.04$ (1 s.f.)

$z \pm \Delta z = 0.74 \pm 0.04$

Usually the ½ (Max – Min) method will give a reasonable absolute uncertainty value. The general method may lead to a larger absolute uncertainty value.

The general method is usually used when the ½ (Max – Min) fails to give a sensible value. For example, consider the above problem z = sin θ where $\theta = (90 \pm 10)°$.

$$z = \sin\theta \text{ where } \theta = (90 \pm 10)°.$$



$Z_{max} = 1.0$

$Z_{min} = \sin 80°$ or $\sin 100°$

$= 0.985$

$\Delta Z = 1.0 - 0.985 = 0.015 = 0.02$

$Z = 1.00 \pm 0.02$ (Wrong)

$Z = 1.00 - 0.02$ (Not meaningful)

Hence $0.98 \le Z \le 1.00$

Calculating Uncertainties In A Derived Quantity

The table below shows the formulae used in calculating uncertainties in a derived quantity. The symbols k, m, n and p represent constants (e.g. 4, 2.7 and 6π) while A, B and C represents variables.

Addition	$R = m\,A + n\,B$	$\Delta R = \lvert m\rvert\,\Delta A + \lvert n\rvert\,\Delta B$	Adding Absolute Uncertainties
Subtraction	$R = m\,A - n\,B$	$\Delta R = \lvert m\rvert\,\Delta A + \lvert n\rvert\,\Delta B$	
Multiplication by Constant	$R = m\,A$	$\Delta R = \lvert m\rvert\,\Delta A$	
Power	$R = k\,A^m$	$\frac{\Delta R}{R} = \lvert m\rvert \frac{\Delta A}{A}$	Adding Relative Uncertainties
Product	$R = k\,A^m \times B^n$	$\frac{\Delta R}{R} = \lvert m\rvert \frac{\Delta A}{A} + \lvert n\rvert \frac{\Delta B}{B}$	
Quotient	$R = k\,A^m \div B^n$	$\frac{\Delta R}{R} = \lvert m\rvert \frac{\Delta A}{A} + \lvert n\rvert \frac{\Delta B}{B}$	
Other Functions	For example, $R = \sin x$ $R = \ln x$	$\Delta R = ½\,(R_{max} - R_{min})$	General Method Or ½ (Max – Min) method

Suggested problem-solving strategy for uncertainty problems

To determine the uncertainty of a physical quantity (A):

1. Write down the relevant equation relating all physical quantities.
2. <u>Make (A) the subject of the equation.</u>
3. Based on the equation, choose the uncertainty formulae that apply.
4. Convert the equation into an equation for uncertainty. (i.e. remove all constants)
5. Put in the uncertainty values and calculate the final uncertainty of (A).

In terms of presentation:

6. Calculate and express the uncertainty (ΔA) to **1 significant figure.**
7. Write the calculated value (A) to the **same place value** as the uncertainty.

Worked Example 11

Given that $T = 2\pi\sqrt{\frac{l}{g}}$, find an expression for the fractional uncertainty of

(a) *T* using measured values of *l* and *g*.
(b) *g* using measured values of *T* and *l*.

(a) $T = 2\pi(l)^{0.5}(g)^{-0.5}$

$$\frac{\Delta T}{T} = (0.5)\frac{\Delta l}{l} + (0.5)\frac{\Delta g}{g}$$

(b) Rearranging the equation and making g the subject:

$$g = \frac{4\pi^2 l}{T^2} = 4\pi^2 l T^{-2}$$

$$\frac{\Delta g}{g} = \frac{\Delta l}{l} + (2)\frac{\Delta T}{T}$$

Worked Example 12

Given the following quantities and their corresponding uncertainties:

r_1 = 2.2 ± 0.1 cm
r_2 = 3.5 ± 0.1 cm
h = 7.2 ± 0.1 cm
m = 0.800 ± 0.001 kg

Determine the value of *S* and its associated uncertainty for each of the cases:

(a) $S = \frac{1}{2}(r_1 + r_2)h$ (b) $S = \frac{3m}{4\pi(r_2^{\ 2} - r_1^{\ 2})}$

Solution:

(a) Let $R = r_1 + r_2$, then $S = \frac{1}{2}Rh$ = 20.52

and $\frac{\Delta S}{S} = \frac{\Delta R}{R} + \frac{\Delta h}{h}$ ------- (1)

Consider $R = r_1 + r_2$ = 5.7

$\Delta R = \Delta r_1 + \Delta r_2 = 0.1 + 0.1 = 0.2$

From (1): $\frac{\Delta S}{S} = \frac{0.2}{5.7} + \frac{0.1}{7.2}$ = 0.049

ΔS = 1.005 = 1 (to 1 sf)

Therefore, **$S \pm \Delta S$ = (21 ± 1) cm^2**

Using ½ (Max – Min) Method:

S = ½ (2.2+3.5)7.2 = 20.52

S_{max} = ½ (2.3 +3.6)7.3 = 21.535

S_{min} = ½ (2.1+3.4)7.1 = 19.525

ΔS = ½ (S_{max}- S_{min}) =1/2 (21.535-19.525) = 1.005 = 1 (1 s.f)

Therefore, **$S \pm \Delta S$ = (21 ± 1) cm^2**

(b) Let $R = (r_2^2 - r_1^2)$ then $S = \frac{3}{4\pi}\frac{m}{R}$ = 0.0258

Consider R = ($r_2^2 - r_1^2$) and let P = r_2^2 = 12.25 and Q= r_1^2= 4.84
R = P – Q = 7.41

$$\frac{\Delta P}{P} = 2\frac{\Delta r_2}{r_2}$$

and $\frac{\Delta Q}{Q} = 2\frac{\Delta r_1}{r_1}$

$$\Delta P = 2\frac{\Delta r_2}{r_2}P$$

= 0.7

and $\Delta Q = 2\frac{\Delta r_1}{r_1}Q = 0.44$

$\Delta R = \Delta P + \Delta Q$ = 0.7 +0.44 = 1.14

then $S = \frac{3}{4\pi}\frac{m}{R} = 0.0258$ (Use diff method)

and $\frac{\Delta S}{S} = \frac{\Delta R}{R} + \frac{\Delta m}{m} = 1.14/7.41 + 0.001/0.800 = 0.154 + 0.00125 = 0.155$

ΔS = 0.155 x S = 0.004
S = (0.026 ± 0.004) kg cm^{-2}

Using ½ (Max - Min) Method:

$$S = \frac{3}{4\pi}\frac{m}{R} = 0.0258$$

$$S_{max} = \frac{3(0.800)}{4\pi(3.4^2-2.3^2)} = 0.03046$$

$$S_{min} = \frac{3(0.799)}{4\pi(3.6^2-2.1^2)} = 0.02231$$

ΔS = ½ (0.03046 -0.02331) = 0.00408 = 0.004 (1 s.f)

S = (0.026 ± 0.004) kg cm^{-2}

Example 13 (2012 P1Q2)

The equation connecting object distance u, image distance v and focal length f for a lens is

$$\frac{1}{u}+\frac{1}{v}=\frac{1}{z}$$

A student measures values of u and v, with their associated incertainties. These are
Given: u = (50 $\pm$ 3) mm and v = (200 $\pm$ 5) mm.
He calculates the value of f as 40 mm. What is the uncertainty in this value?

<u>Using ½ (Max – Min) Method:</u>

$$\frac{1}{f}=\frac{1}{u}+\frac{1}{v}=\frac{1}{50}+\frac{1}{200}$$

$$f = 40.0 \text{ mm}$$

$$\frac{1}{f_{\text{min}}}=\frac{1}{u_{\text{min}}}+\frac{1}{v_{\text{min}}}$$

$$\frac{1}{f_{\text{min}}}=\frac{1}{47}+\frac{1}{195}$$

$$f_{\text{min}} = 37.9 \text{ mm}$$

$$\frac{1}{f_{\text{max}}}=\frac{1}{u_{\text{max}}}+\frac{1}{v_{\text{max}}}$$

$$\frac{1}{f_{\text{max}}}=\frac{1}{53}+\frac{1}{205}$$

$$f_{\text{max}} = 42.1 \text{ mm}$$

$$\tfrac{1}{2}(f_{\text{max}}-f_{\text{min}})=\tfrac{1}{2}(42.1-37.9)=2.1 \text{ mm}$$

$$\therefore \Delta f = 2.1 \text{ mm} = 2 \text{ mm (1 s.f.)}$$

f = <u>(40 ± 2) mm</u>

3. Scalars and Vectors

1. DEFINITIONS OF SCALAR AND VECTOR QUANTITIES

Scalar quantities are physical quantities that can be represented by a **magnitude** only. They do not have a direction associated with them.

Vector quantities are physical quantities that can be represented by a **magnitude** and a **direction** in space**.**

Examples of scalar and vector quantities

Scalar Quantity	Vector Quantity
distance	displacement
speed	velocity
temperature	acceleration
energy	force
power	momentum
mass	weight
density	moment

Vector notation

A vector A will be represented as $\vec{A}$.

The magnitude of vector A will be represented as $\left|\vec{A}\right|$ or *A*.

Some mathematical operations involving scalars and vectors

(a) scalar $\times$ (or $\div$) scalar gives a scalar quantity

(b) vector $\times$ (or $\div$) scalar gives a vector quantity
For *e.g., weight = mass $\times$ g, momentum = m $\times$ v*

(c) vector $\times$ vector can give:

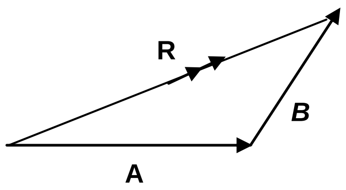
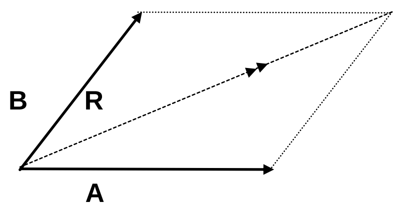
- a scalar quantity
This type of multiplication is called the **scalar (dot) product of two vectors** and is in your H2 Mathematics syllabus.
For e.g., work done (scalar) = force (vector) $\bullet$ displacement (vector).

- a vector quantity
This type of multiplication is called the **vector (cross) product** and might be something you would learn in university.

2. ADDITION OF VECTORS

There are in general two methods of vector addition – Triangle Law and Parallelogram Law.

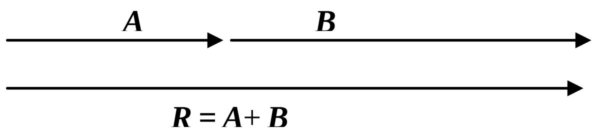
We will focus on using the Triangle Law for the 'A' Levels.

TRIANGLE LAW	PARALLELOGRAM LAW
• **RULE**: ***Head*** of one to ***Tail*** of the other and the **Resultant** is from the tail of the first to the head of the second.	• **RULE:** ***Tail* to *Tail*.** Then draw a parallelogram in the following way.
	
• Useful for <u>subtraction</u> of vectors. • Useful for addition of <u>more than two vectors</u> (polygon addition).	• Useful for giving an approximate indication of the direction of resultant. • Useful for <u>perpendicular</u> vectors.
$\vec{R} = \vec{A} + \vec{B}$ Magnitude and direction of $\vec{R}$ can be found by using ***cosine rule*** or ***sine rule***.	

Note: Addition is commutative, i.e. $\vec{A} + \vec{B} = \vec{B} + \vec{A}$.

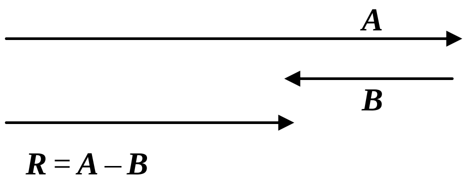
Addition of vectors in the same direction

If the two vectors are in the same direction, the resultant force is in the same direction.
Its magnitude is the sum of the magnitudes of the two, refer to the figure below, i.e. ***R*** = ***A*** + ***B***.



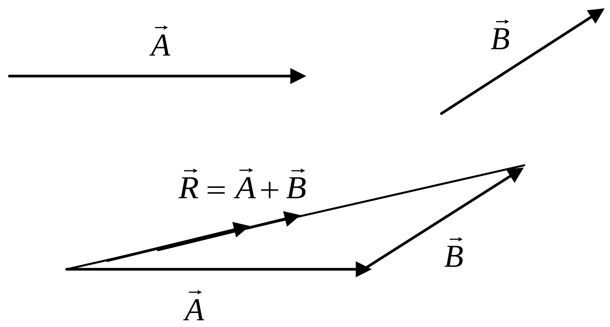
Addition of vectors in opposite or anti-parallel directions

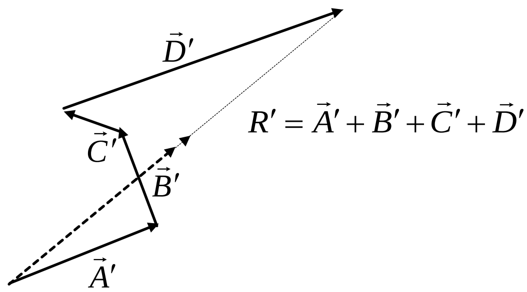
If two vectors are acting in the opposite direction, the magnitude of their sum is the difference between the magnitudes of the two vectors, refer to the figure below, i.e. ***R*** = ***A*** – ***B***. Note that the direction of the resultant force is the same as the direction of the vector with a larger magnitude.



Addition of vectors in two dimensions

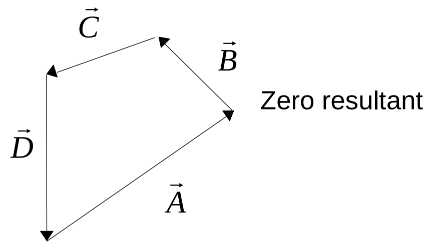
Two or more coplanar vectors ($\vec{A}$, $\vec{B}$, $\vec{C}$, $\vec{D}$, etc.) can be added by using the parallelogram method or "head to tail" method (preferred). Please refer to the figures below.





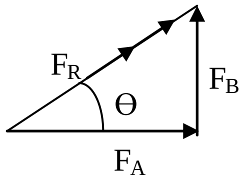
Polygon Addition of Vectors

- Polygon addition is used when there are more than two vectors to be added. This is a mere extension of the triangular addition.
- To add several vectors, just place the tail of one vector to the head of another vector consecutively. The resultant is the vector which joins from the tail of the first vector to the head of the last vector.
- If the polygon is closed, the resultant is zero.



Example 1:
Find the resultant of the following forces:
F_A = 5.00 N along the east and F_B = 3.00 N along the north.

Solution:



$F_R = \sqrt{(F_A^2+F_B^2)}$

$= \sqrt{(5.00^2+3.00^2)}$

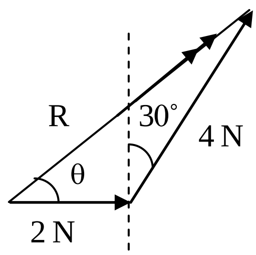
$= 5.83$ N

$\tan\theta = F_B/F_A = 3.00/5.00 = 0.600$

$\theta = 30.96^o = 31^o$ measured anti-clockwise from the eastward direction.

Example 2:
Two forces act on a body. One force has magnitude of 2.00 N acting in the easterly direction, the other 4.00 N acting in the direction 30^o East of North. Find the resultant of these two forces.

Solution:

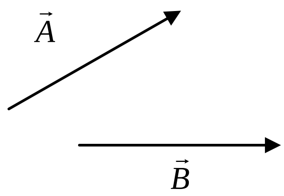


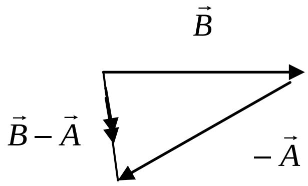
Using cosine rule:
$R^2 = 4.00^2+2.00^2-2x4.00x2.00\cos 120^o$
$R = 5.29$ N

Using sine rule:
$4/\sin\theta = R/\sin 120^o$
$\theta = 40.9^o$ measured anti-clockwise from the easterly direction.

3. SUBTRACTION OF VECTORS

- Subtraction is a special form of addition.
- To subtract vector $\vec{A}$ from $\vec{B}$ is actually to add vector $\vec{B}$ to vector $-\vec{A}$,
  i.e. $\vec{B}-\vec{A}=\vec{B}+\left(-\vec{A}\right)$





Hence subtracting a vector is equivalent to adding the negative of the vector. The negative vector has the same magnitude as the original (positive) vector but is rotated 180°.

Applications of Vector Subtraction

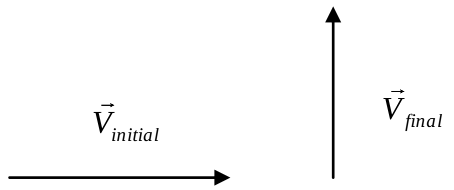
<u>Change in vector</u>

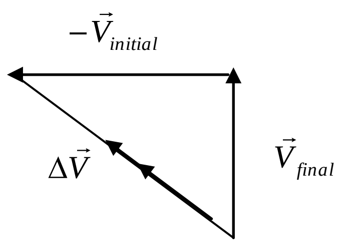
If a vector $\vec{V}$ changes either its magnitude or direction (or both), the change in $\vec{V}$, $\Delta\vec{V}$ is defined as $\Delta\vec{V}=\vec{V}_{final}-\vec{V}_{initial}$ where $\vec{V}_{initial}$ is the initial vector and $\vec{V}_{final}$ is the final vector.

The above vector equation can be re-written as:

$\Delta\vec{V}=\vec{V}_{final}+(-\vec{V}_{initial})$

Now the change in vector $\Delta\vec{V}$ can be found by the vector addition of the vector $\vec{V}_{final}$ and $(-\vec{V}_{initial})$ as shown in the figure below.





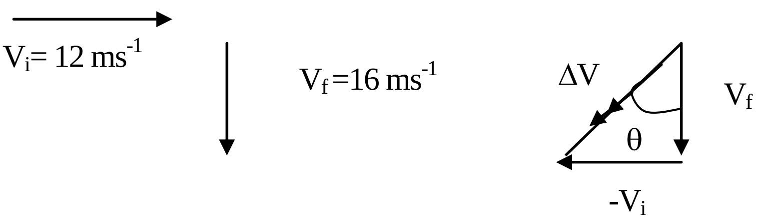
Example 3:
A car is initially travelling 12 ms^{-1} due east. What is the change in velocity if its final velocity is

(a) 16 ms^{-1} due east? [4 ms^{-1} due east]

(b) 9 ms^{-1} due east ?
Δv = 9 - (12) = - 3ms^{-1} = 3ms^{-1} due west

(c) 10 ms^{-1} due west?
Δv = -10 - (12) = - 22ms^{-1} = 22 ms^{-1} due west

(d) 16 ms^{-1} due south?



$\Delta V = \sqrt{(16^2+12^2)}$
$= 20$ ms^{-1}

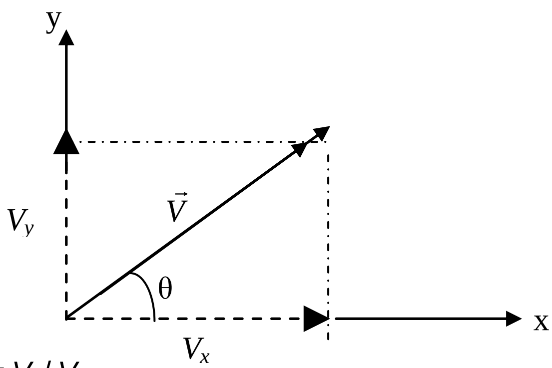
Tan θ = 16/12
θ = 36.9^{o} west of south

4. RESOLVING VECTORS INTO TWO PERPENDICULAR COMPONENTS

The vector $\vec{V}$ in the figure below is the resultant of two perpendicular vectors, $\vec{V}_x$ and $\vec{V}_y$ along the *x* and *y*-axis respectively.

We can represent any vector lying in the *xy*-plane as the sum of a vector parallel to the *x*-axis and a a vector parallel to the *y*-axis. In the figure below, these two vectors are labelled $\vec{V}_x$ and $\vec{V}_y$; they are called the **component vectors** of vector $\vec{V}$, and their vector sum is equal to $\vec{V}$.

V_x (equal to the magnitude of $\vec{V}_x$) is known as the x-component of $\vec{V}$ and V_y (equal to the magnitude of $\vec{V}_y$) is known as the y-component of $\vec{V}$.



From the figure, sin θ = V_y / V.

Thus, we have $V_y = V \sin\theta$.

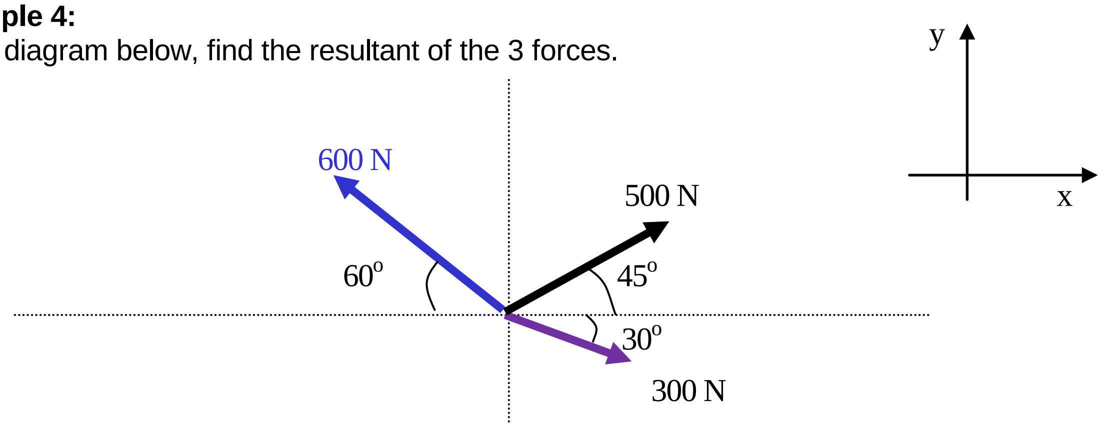
Similarly, cos θ = V_x / V and we have $V_x = V\cos\theta$.

The magnitude of $\vec{V}$ is given by $\left|\vec{V}\right| = \sqrt{{V_x}^2 + {V_y}^2}$.

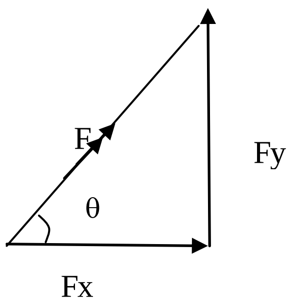
The direction of V is given by $\tan\theta = \dfrac{|V_y|}{|V_x|}$.

Note: When we resolve the vector $\vec{V}$ into its perpendicular component vectors $\vec{V}_x$ and $\vec{V}_y$, **DOTTED ARROW LINES** are used to represent $\vec{V}_x$ and $\vec{V}_y$.

Example 4:
In the diagram below, find the resultant of the 3 forces.



Solution:



$\Sigma Fy = 500 \sin 45^o + 600 \sin 60^o - 300 \sin 30^o$
$= 723.2N$

$\Sigma Fx = -600 \cos 60^o + 500 \cos 45^o + 300 \cos 30^o$
$= 313.4$ N
Resultant $F = \sqrt{[(723.2)^2 + (313.4)^2]} = 788$ N

Tan θ =Fy/Fx
= 723.2/313.4

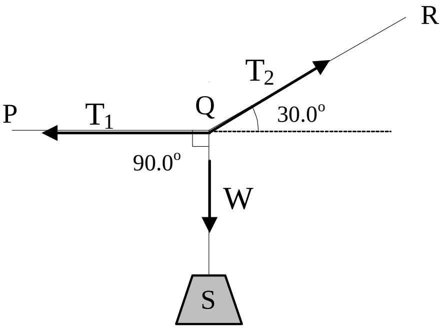
$\Theta = 66.6^o$
Measured anti-clockwise from the x-axis

Example 5:
In the diagram below, a body S of weight *W* hangs vertically by a thread tied at Q to the string PQR. If the system is in equilibrium, what is the tension in terms of W in the section PQ?

Solution:

$T_2 \sin 30^o = W$
$T_2 = W/\sin 30^o = 2W$

$T_1 = T_2 \cos 30^o$
$= 2W \sqrt{3}/2$
$= \sqrt{3}W$ or $1.73\ W$

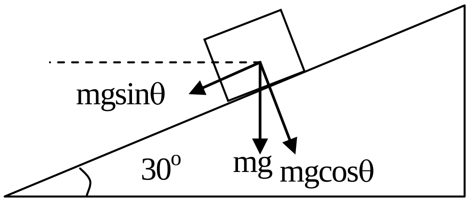


Example 6:
An object of mass 10 kg lies on a slope that is inclined at 30° to the horizontal. Find the components of the weight of the object along and perpendicular to the slope.

Solution: (draw a diagram)
Step 1 - Draw the components of the vector parallel and perpendicular to the plane.
Step 2 - Indicate the angle that one of the components makes with the original vector.
(You should be able to identify the angle immediately without much calculation after some practice).
Step 3 - Calculate the magnitude of each component.

Weight along the slope = $mg\sin\theta$
$= 10 \times 9.81 \sin 30^o = 49.1$ N

Weight perpend. to the slope = $mg\cos\theta$
$= 10 \times 9.81 \cos 30^o$
$= 85.0$ N



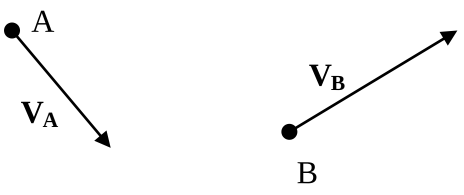
Relative Velocity

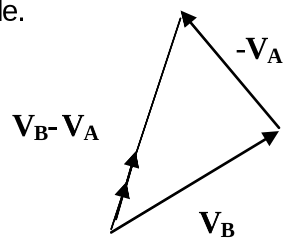
If A and B are two moving objects, then the apparent (relative) velocity of B when observed from A is called the velocity of B relative to A, i.e.

Velocity of B relative to A= Velocity of B – velocity of A

$$\mathbf{V_{BA} = V_B - V_A}$$

The velocity of B relative to A may be found by using a vector triangle.

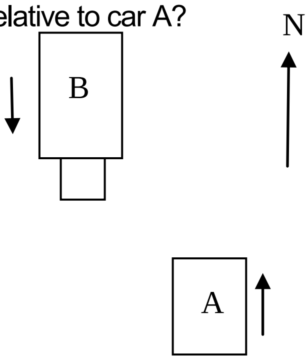




Vector diagram

Example 7

Suppose car A is travelling north at a constant speed of 80 kmh^{-1} and a truck B is approaching the car in opposite direction at a constant speed of 100 kmh^{-1}. What is the truck's velocity relative to car A?



Let velocity of truck B = V_B
Let the velocity of car A = V_A

Taking northward as positive direction
Velocity of truck relative to car = $V_B - V_A$
= -100 – 80
= -180 kmh^{-1}

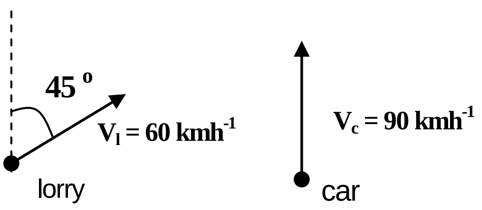
The truck is moving at 180 kmh^{-1} south relative to car.

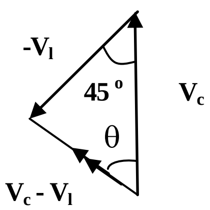
Example 8

A car is travelling due north on a straight road at 90 kmh^{-1}. The car is observed by the driver pf a lorry travelling north-east on another road at 60 kmh^{-1}. Find the velocity of car relative to the lorry.

Let the speed of car relative to the lorry = $V_c = V_c - V_l$

Using cosine rule $V_{lc}^2 = 90^2 + 60^2 - 2 \times 90 \times 60 . \cos 45^o$
V_{cl} = 63.7 kmh^{-1}





Vector diagram

<u>Calculation of direction:</u>
Sin θ /V_l = sin 45^o/ V_{cl}
Sin θ = (sin45^o/63.7) x 60 = 0.666
Θ=41.8^o anti- clockwise from North