RAFFLES INSTITUTION
PHYSICS DEPARTMENT

14 CURRENT OF ELECTRICITY

Self-check Questions

S1. Explain what is meant by an electric current.

S2. State the equation relating charge Q and current I.

S3. State the equation relating current and drift velocity. Explain all symbols used.

S4. State the definition for potential difference between two points.

S5. State the definition of electromotive force.

S6. Use energy considerations to distinguish between electromotive force (e.m.f.) and potential difference (p.d.).

S7. State Ohm's Law.

S8. Sketch and explain the I-V characteristics of a metallic conductor at constant temperature, a semiconductor diode and a filament lamp.

S9. Sketch the temperature characteristic of a thermistor.

S10. For an electric circuit element with potential difference V between its terminals and current I passing through it, what is the electric power P delivered to or extracted from the circuit element? If the circuit element is a resistor of resistance R, what is the electric power delivered to the resistor in terms of R?

S11. How does the internal resistance of a battery affect its terminal p.d. and output power?

S12. State the maximum power theorem.

Self-Practice Questions

P1. A torch is rated as '2.5 V, 0.030 A'.

(a) How much charge flows through the bulb in 1.0 hour when it is operating at its rated current?

(b) At what rate is electrical energy dissipated in the bulb when it is operating at its rated voltage?

(c) What is its resistance under these operating conditions?

P2. **(a)** The Earth as a whole carries negative charges. On a day with fine weather, the total charge on the surface of the Earth is 5.5×10^5 C. The Earth revolves around the Sun at a distance of 1.5×10^{11} m once a year. Estimate the average electric current along the orbit.

(b) It is estimated that the average quantity of electric charge transported in a lightening flash is 30 C. If the energy liberated is 2.1×10^{10} J, what is the p.d. involved?

P3. Distinguish between resistance and resistivity.

P4. A cylindrical conductor with a length of 2.00 m and a diameter of 0.500 cm is found to carry a current of 4.60 A when a p.d. of 60.0 V is applied between its ends.

(a) What is the resistance of this conductor?

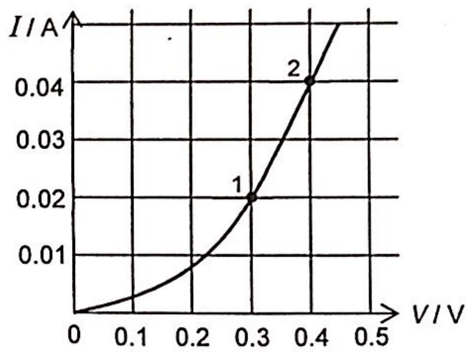
(b) What is the resistivity of the material from which it is made?

P5. **(a)** A constantan wire of length 15 m is to carry a current of 25 A with a potential drop of no more than 5.0 V along its length. What is the minimum acceptable diameter of this cable?

(b) Calculate the resistance per metre of constantan wire of diameter 0.35 mm. What length of this wire would be needed to make a '12 V, 30 W' heater filament?

(resistivity of constantan = 4.8×10^{-7} Ω m)

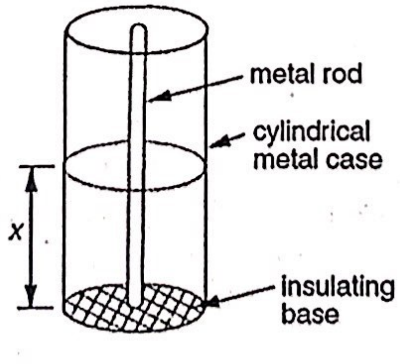
P6. An electrical device has the I-V graph as shown.



(a) What is the resistance at point 1 and point 2?

(b) What is the power dissipated at point 1 and point 2?

P7. A conducting liquid fills a cylindrical metal case to a depth x as shown in the diagram.

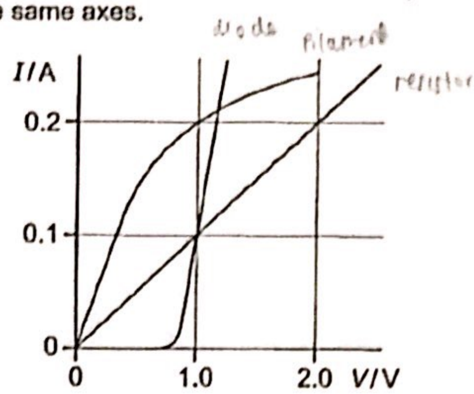


The resistance between the case and the metal rod is

A proportional to x^2.
B proportional to x.
C independent of x.
D inversely proportional to x.
E inversely proportional to x^2.

N88/I/13

P8. The graph shows the I-V characteristics of three electrical components, a diode, a filament lamp and a resistor, plotted on the same axes.



Which statement is correct?

A The resistance of the diode equals that of the filament lamp at about 1.2 V.

B The resistance of the diode is constant above 0.8 V.

C The resistance of the filament lamp is twice that of the resistor at 1.0 V.

D The resistance of the resistor equals that of the filament lamp when $V = 0.8$ V.

P9. For proper balance in some small aircraft, the 12.0 V engine battery must be located in the tail of the plane. The aluminum cables connecting the battery and the motor has a length of 6.10 m, a diameter of 8.25 mm, a resistivity of $2.80 \times 10^{-8}\ \Omega$ m and it carries a current of 125 A.

(a) What is the potential drop across the aluminum cables?

(b) What is the power dissipated in the cable?

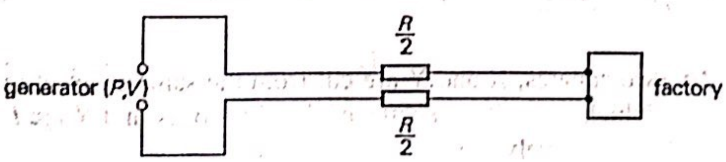
P10. A cell has e.m.f. 1.5 V and internal resistance 0.5 Ω.

(a) Calculate the power delivered to an external 2.5 Ω resistor.

(b) What is the value of the external resistance if the power delivered is to have a maximum value?

(c) Calculate the terminal p.d. of the cell when maximum power is delivered.

P11. A generator, with output power P and output voltage V, is connected to a factory by cables of total resistance R.



What is the power input to the factory?

A P

B $P - \left(\frac{P}{V}\right)\frac{R}{2}$

C $P - \left(\frac{P}{V}\right) R$

D $P - \left(\frac{P}{V}\right)^2 \frac{R}{2}$

E $P - \left(\frac{P}{V}\right)^2 R$

N92/I/14

SELF-PRACTICE ANSWERS

P1. (a) 108 C (b) 0.075 W (c) 83 Ω

P2. (a) 0.017 A (b) 7.0×10^{8} V

P4. (a) 13.0 Ω (b) 1.28×10^{-4} Ω m

P5. (A) 6.8×10^{-3} m, 5.0 Ω m^{-1} (B) 0.96 m

P6. (a) 15 Ω, 10 Ω (b) 6.0×10^{-3} W, 16×10^{-3} W

P7. D

P8. A

P9. (a) 0.400 V (b) 50.0 W

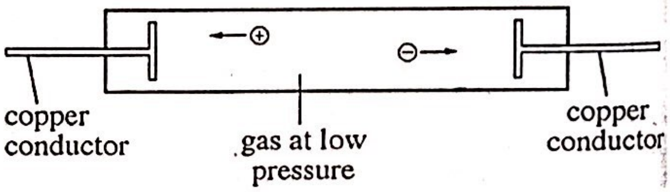
P10. (a) 0.63 W (b) $R = 0.5$ Ω (c) 0.75 V

P11. E

Discussion Questions

D1. [N99/3/5 part]

In a gas, conduction occurs as a result of negative particles flowing one way and positive particles flowing in the opposite direction as shown below.



In this case, the copper conductors to the gas carry a current of 0.28 mA. The number of negative particles passing any point in the gas per unit time is 1.56×10^{15} s^{-1} and the charge on each negative particle is -1.60×10^{-19} C.

Calculate

(a) the negative charge flowing past any point in the gas per second,

(b) the positive charge flowing past any point in the gas per second,

(c) the number of positively charged particles passing any point in the gas per second, given that the charge on each positive particle is $+3.20 \times 10^{-19}$ C.

(d) By considering the significant figures available, explain why your answers to (b) and (c) are unreliable.

[0.250×10^{-3} C s^{-1}, 0.03×10^{-3} C s-1, 9×10^{13} s^{-1}]

1.0 mm 1.0 A

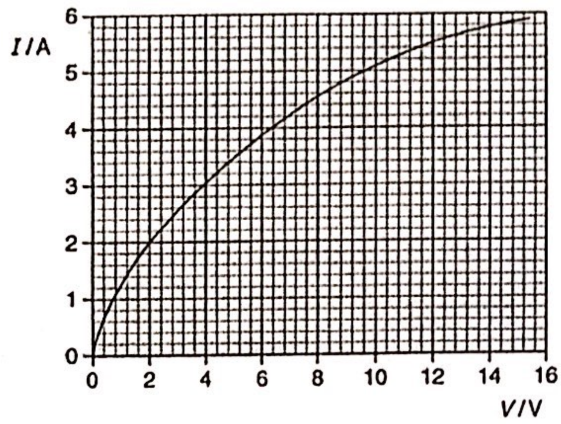
D2. A copper wire of diameter 1 mm carries a current of 1 A. Given that each copper atom contributes one free electron, estimate the drift velocity of the free electrons in the wire.

(~~resistivity~~ of copper = 8960 kg m^{-3}, mass of each copper atom = 1.06×10^{-25} kg)

density

D3. [H1 N2011/2/7 part]

(a) The I-V characteristic of a 12 V car headlamp is drawn in the following figure. Only positive values of V and I are shown.



(i) Explain why the graph stops at $V = 15.4$ V.

(ii) Deduce the resistance of the headlamp at 4.0V and at 12.0V.

(iii) Explain why the resistance at either of these voltages values is not the reciprocal of the gradient of the graph at the same voltage values.

(b) The filament of a lamp cannot be manufactured from a straight piece of tungsten wire of diameter 0.084mm.

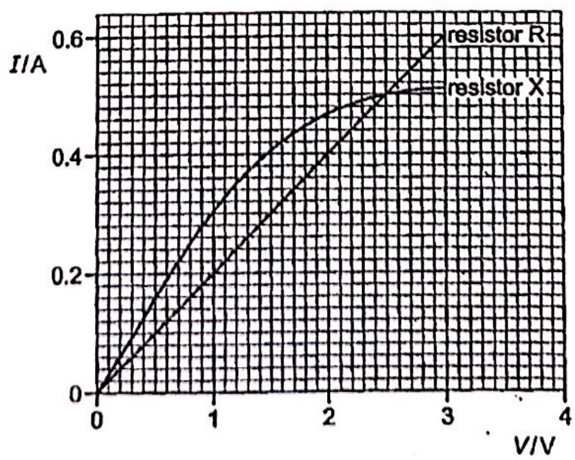
(i) Calculate the length of wire required of a resistance of 0.40Ω when the wire is at room temperature. The resistivity of tungsten at room temperature is $5.5 \times 10^{-8}\,\Omega$ m.

(ii) Explain why this straight length of wire is impractical.

(iii) Suggest two ways of making a filament wire more practical.

[(a) (ii) 1.33Ω , 2.18Ω ,(b) (i) 0.040m]

D4. The graph below shows the I-V characteristics of two resistors R and X.



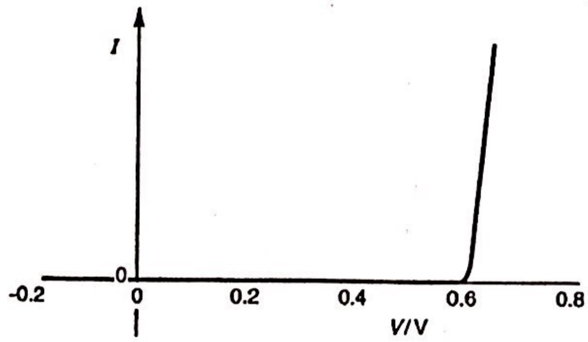
(a) The resistors R and X are connected in series with a cell of negligible internal resistance. The current in the circuit is 0.2 A. Calculate the e.m.f of the cell.

(b) Estimate the new current flowing through the cell if its emf is doubled.

[(a) 1.62-1.63V (b) 0.38A]

D5. [H1 N2007/2/2 part]

(a) The following figure shows the *I-V* characteristic of a particular semiconductor diode.



Describe, with the help of equations, how the resistance of this diode depends on the potential difference V across it.

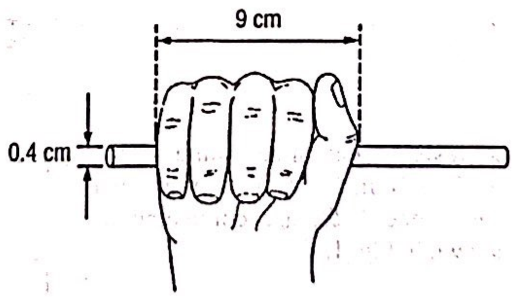
(b) A resistance wire of length L and cross-sectional A has a resistance R. The resistance wire is fixed between two supports. The length of the resistance wire increases when the two supports move apart. The volume of the wire and its resistivity both remain constant.

(i) Show that the resistance of the wire is directly proportional to the square of its length.

(ii) The length of the resistance wire increases by 5.2%. Calculate the ratio of final resistance to initial resistance. [1.11]

D6. [N00/3/4 part]

The resistivity of the human body is low compared with the resistivity of skin, which is about $3.0 \times 10^4\ \Omega\ \text{m}$ for dry skin.



(a) For a layer of dry skin 1.0 mm thick, determine the resistance of a 1.0 cm^2 area of skin.

(b) A person, who is well earthed, accidentally grabs a wire of diameter 0.40 cm at a potential of 50 V. His hand makes contact with the whole circumference of the wire over a distance of 9 cm as shown above. The average thickness of the skin of his hand is 1.0 mm. Estimate the current through the person.

(c) Discuss **two** factors, referred to above, which affect the magnitude of the current and hence affect the possible danger from electric shock.

(d) One obvious safety precaution is to keep live wires well insulated. What other safety precautions do you suggest?

[$3.0 \times 10^5\ \Omega$, 1.88 mA]

D7. A battery is connected in series with a 2.0 Ω resistor and a switch S. A high resistance voltmeter is connected parallel to the resistor. The voltmeter reads 12.5 V when the switch is open, but 8.1 V when the switch is closed. What is the internal resistance of the battery? [1.1 Ω]

D8. An electric utility company supplies a customer's house from the main power lines of 120 V with two copper wires, each of which is 50.0 m long and has a resistance of 0.108 Ω per 300 m.

(a) Find the voltage at the customer's house for a load current of 110 A.

(b) For the load in **(a)**, find

(i) the power that the customer is receiving and

(ii) the power lost in the copper wires.

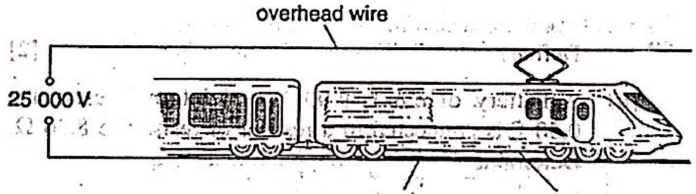
(iii) the efficiency of transmission.

(c) If 1 kWh of electrical energy costs $0.17, how much will the customer need to pay if he uses electricity for 3 hours?

[(a)116 V, (b)(i) 12.8 kW, (ii) 436 W, (iii) 97 %, (c)$6.53]

D9. [N98/3/4 part]

The figure shows the arrangement for supplying power to an engine. A 25 kV supply is used and the return current from the engine returns through the track. The resistance per kilometre of the overhead wire is 0.344 Ω and the resistance of the track can be neglected.



(a) Consider first when the engine is close to the power supply and require 6700 kW of power. Calculate the current which is needed.

When the engine is 30 km from the power supply, it is supplied with a current of 241 A. Calculate

(b) **(i)** the resistance of the overhead wire between the power supply and the engine.

(ii) the potential difference across the engine

(iii) the power supplied to the engine

(iv) the fraction of the power supplied which is used by the engine

(c) Explain the following facts about the supply to the engine.

(i) A railway employee who touches the track through which there is a current of 241 A does not get an electric shock.

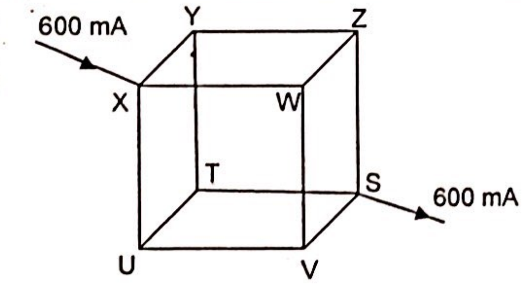
(ii) For constant power transmitted, a high voltage supply is essential for a railway system such as this.

(iii) A different current is needed when the train is climbing a hill from that required when travelling at the same speed on the flat.

[(a)268 A (b)10.3 Ω, (ii) 22500 V, (iii) 5430 kW, (iv) 0.900]

Challenging Questions

C1. A resistance network is formed by connecting twelve identical constantan wires, each of length 20 mm and cross-sectional area 0.66 mm^2, as the edges of a cube as shown in the figure on the right.



(a) Find the resistance of *one* of the constantan wires, given that the resistivity of constantan is $4.9 \times 10^{-7}\ \Omega$ m. [$1.5 \times 10^{-2}\ \Omega$]

(b) A current of 600 mA enters at the corner X and leaves at the diagonally opposite corner S. By considering the symmetry of the cube, find the currents in each of the wires. [200 mA, 100 mA]

(c) The potential difference between X and S is the algebraic sum of the potential differences across each of the wires making up a continuous path between X and S. Hence, find the resistance of the network between X and S. [7.5×10^{-3} V, $1.3 \times 10^{-2}\ \Omega$]

C2. A thin layer of copper is deposited uniformly on the surface of an iron wire of radius 0.50 mm. Calculate the thickness of copper required to reduce the resistance between the ends of the copper-plated wire to 75 % of the resistance of the bare iron wire.
(Resistivity of iron = $8.9 \times 10^{-8}\ \Omega$ m, resistivity of copper = $1.6 \times 10^{-8}\ \Omega$ m) [0.015 mm]

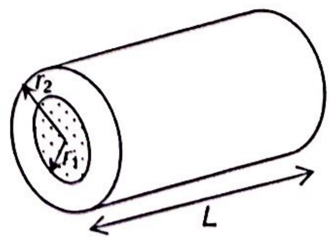
C3. An electrical transmission line connects a generator to a load. The line has resistance X, giving rise to power loss in the line. The load resistance is R.

(a) Show that, for a given power P delivered by the generator, the power loss in the line is inversely proportional to V^2, where V is the output voltage of the generator.

(b) This system is to transmit power of 200 kW from the generator. The line resistance is 0.60 Ω. It is required that the power loss in the line should not exceed 0.015 % of the total power. Find the minimum output voltage to achieve this. [2.83×10^4 V]

C4. A resistor is made from a material of resistivity ρ. It is formed in the shape of a hollow cylinder with inner radius r_1, outer radius r_2 and length L.
Show that the resistor's resistance to current flow perpendicular to the cylinder's axis, in the radial direction, is given by

$$R = \frac{\rho}{2\pi L}\ln\frac{r_2}{r_1}.$$



Current of Electricity Suggested Solutions

SELF-CHECK QUESTIONS

S1. Electric current is defined as the rate of flow of charge.

S2. $Q = It$, where t is time.

S3. $I = nAv_Dq$, where I is the current, n is the number density of charges, A is the cross-sectional area perpendicular to the direction of the current, v_D is the drift velocity and q is the charge carried by each charge carrier.

S4. The **potential difference** between two points in a circuit is defined as the amount of electrical energy per unit charge that is converted *to* other forms of energy when charges pass from one point to the other.

S5. The **electromotive force of a source** is defined as the amount of electrical energy per unit charge that is converted *from* other forms of energy to drive charges around a complete circuit.

S6.

p.d. across an external load, V	e.m.f. of a source, E
$V = \frac{W}{Q}$	$E = \frac{W}{Q}$
W is the energy converted **from electrical energy** to other forms	W is the energy converted **from other forms** to electrical energy

S7. Ohm's Law states that the current through a conductor is proportional to the potential difference across it provided there is no change in the physical conditions (e.g. temperature) of the conductor.

Note: Ohm's Law deals with proportionality. $R = \frac{V}{I}$ defines resistance R for any conductor, whether or not it obeys Ohm's law. Only when R is constant can we say that the conductor obeys Ohm's law.

S8. Refer to lecture notes.

S9. Refer to lecture notes.

S10. $P = IV$, $P = I^2R$ OR $P = V^2/R$.

S11. When a current I flows through the battery, there is a potential drop of Ir across its internal resistance r. Hence, the p.d. across the battery (terminal p.d.) is

$$V = E - Ir$$

Multiplying this equation by I, we find

$$P = IV = IE - I^2r$$

IE is the power supplied by the source (rate of conversion of non-electrical energy to electrical energy within the source). I^2r is the power dissipated in the internal resistance of the source. The difference P is the net electrical power output of the source.

S12. A given source of e.m.f. delivers the maximum amount of power to a load when the **resistance of the load is equal to the internal resistance** of the source.

SELF-PRACTICE QUESTIONS

P1. **(a)** $Q = It = 0.03(60 \times 60) = 108\ \text{C}$

(b) $P = IV = 0.030(2.5) = 0.075\ \text{W}$ OR $W = VQ = 2.5(108) = 270\ \text{J}$

$$P = \frac{270}{60 \times 60} = 0.075\ \text{W}$$

(c) $R = \frac{V}{I} = \frac{2.5}{0.03} = 83\ \Omega$

P2. **(a)** In one year, a total of 5.5×10^5 C of charge passes a point in the orbit.

$$I = \frac{Q}{t} = \frac{5.5 \times 10^5}{365 \times 24 \times 60 \times 60} = 0.017\ \text{A}$$

(b) $V = \frac{W}{Q} = \frac{2.1 \times 10^{10}}{30} = 7.0 \times 10^8\ \text{V}$

P3. The resistance of a conductor is the ratio of the potential difference across it to the current flowing through it. It is not constant for a given material, at a fixed temperature, but depends on its shape and size.

S.I. unit: Ohm, Ω

The resistivity of a material is numerically equal to the resistance between opposite faces of a cube of the material of unit length and unit cross-sectional area. It is constant for a given material, at a fixed temperature, regardless of its shape and size.

S.I. unit: Ohm-metre, Ω m

P4. **(a)** $R = \frac{V}{I} = \frac{60.0}{4.60} = 13.0\ \Omega$

(b)
$$A = \pi r^2 = \pi\left(\frac{0.500 \times 10^{-2}}{2}\right)^2$$

$$\rho = \frac{RA}{l} = \frac{13.0\pi \times \left(\frac{0.500 \times 10^{-2}}{2}\right)^2}{2.00} = 1.28 \times 10^{-4}\ \Omega\ \text{m}$$

P5. **(a)**

$$R_{max} = \frac{V_{max}}{I} = \frac{5.0}{25} = 0.20\ \Omega$$

$$R = \rho\frac{l}{A} = \rho\frac{l}{\pi\left(\frac{d}{2}\right)^2} = \rho\frac{4l}{\pi d^2}$$

$$d_{min} = \sqrt{\rho\frac{4l}{\pi R_{max}}} = \sqrt{\left(4.8\times10^{-7}\right)\left(\frac{4\times15}{\pi\times0.20}\right)} = 6.77\times10^{-3}\text{ m}$$

(b)

$$R = \rho\frac{l}{A}$$

Resistance per metre = $\frac{R}{l} = \frac{\rho}{A} = \frac{4.8\times10^{-7}}{\pi\left(\frac{d}{2}\right)^2} = \frac{4\left(4.8\times10^{-7}\right)}{\pi\left(0.35\times10^{-3}\right)^2} = 5.0\ \Omega\ \text{m}^{-1}$

Resistance of filament, $R = V^2 / P = 12^2 / 30 = 4.8\ \Omega$

length of wire, $l = 4.8 / 5.0 = 0.96$ m

P6. **(a)**

Point 1: $R = \frac{V}{I} = \frac{0.3}{0.02} = 15\ \Omega$

Point 2: $R = \frac{V}{I} = \frac{0.4}{0.04} = 10\ \Omega$

(b) Point 1: $P = IV = 0.02\ (0.3) = 6.0\times10^{-3}$ W

Point 2: $P = IV = 0.04\ (0.4) = 16\times10^{-3}$ W

P7. Current is travelling radially to or from the centre rod.

Length through which current flows in the radial direction = r

Area A normal to current flow = cylindrical surface area = $2\pi rx$

Since $R=\rho l/A=\rho r/2\pi rx=\text{constant}/x \Rightarrow$ resistance is inversely proportional to x

Ans: D

P8. curved graph (as V increases I increases at a decreasing rate) – filament lamp

straight line – resistor

curved then straight – diode

option A: true, the 2 graphs intersect at that point

option B: not true, the ratio of V/I is not constant

option C: not true, resistance of filament lamp is half that of the resistor

option D: not true, the 2 graphs DO NOT intersect at that point.

Ans: A

P9. **(a)**

$$R = \rho\frac{l}{A} = \left(2.80\times10^{-8}\right)\left(6.10\Big/\pi\left(\frac{8.25\times10^{-3}}{2}\right)^2\right) = 3.20\times10^{-3}\ \Omega$$

$V = RI = (3.20 \times 10^{-3})\,(125) = 0.400$ V

(b) $P = IV = (125)\,(0.40) = 50.0$ W

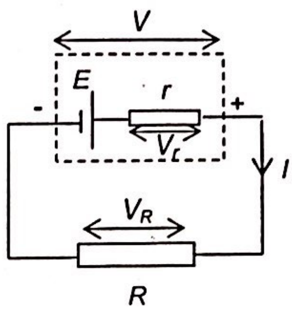
OR $P = I^2R = (125)^2\,(3.20 \times 10^{-3}) = 50.0$ W

OR $P = V^2 / R = (0.40)^2 / (3.20 \times 10^{-3}) = 50.0$ W

P10. $E = 1.5$ V, $r = 0.5\ \Omega$, $R = 2.5\ \Omega$

$E = V_r + V_R = Ir + IR = I\,(r + R)$

$$I = \frac{E}{r+R}$$



(a)

$$I = \frac{1.5}{0.5+2.5} = 0.5\text{ A}$$

Power delivered to $R = I^2R = (0.5)^2\,(2.5) = 0.63$ W

(b) For maximum power, $R = r = 0.5\ \Omega$

(c)

$$I = \frac{1.5}{2\times0.5} = 1.5\ A$$

Terminal p.d., $V = E - Ir = 1.5 - 1.5\,(0.5) = 0.75$ V

P11. Current through circuit $I = P/V$

$P_{gen} = P_{cable} + P_{factory}$

$P_{factory} = P_{gen} - P_{cable} = P - I^2R = P - (P/V)^2R$

Ans: E