2024 JC1 Promotional Exam

1. Sub $(7,10)$ => $25a-121b-c=0$

Line $y=x-1$ cuts the graph at $x=1$

Sub $x=1$ and $y=0$ into equation of hyperbola => $a-b-c=0$

Using GC:

$a=\frac{5}{4}c$, $b=\frac{1}{4}c$

Sub into equation of hyperbola

$$\frac{5}{4}c(x-2)^2-\frac{1}{4}c(y+1)^2=c$$

Since a,b,c are positive integers

$5(x-2)^2-1(y+1)^2=4$ is a possible equation of the hyperbola

Where $a=5, \quad b=1, \quad c=4$

2(a)

$$\frac{dy}{dx} = 3\sec^2 3x$$
$$= 3\left(1+\tan^2 3x\right)$$
$$= 3\left(1+y^2\right)$$
$$\frac{d^2y}{dx^2} = 0 + 6y\frac{dy}{dx}$$
$$= 6y\frac{dy}{dx}\text{, where } A = 6$$

1st Alternative Method	2nd Alternative Method
$\frac{dy}{dx} = 3\sec^2 3x$ $\frac{d^2y}{dx^2} = 6\sec 3x \cdot 3\sec 3x \cdot \tan 3x$ $= 6(3\sec^2 3x)\tan 3x$ $= 6y\frac{dy}{dx}$, where A = 6	$\tan^{-1} y = 3x$ $\frac{1}{1+y^2}\frac{dy}{dx} = 3$ $\frac{dy}{dx} = 3 + 3y^2$ $\frac{d^2y}{dx^2} = 6y\frac{dy}{dx}$, where A = 6

(b)

$$\frac{d^3y}{dx^3} = 6\frac{dy}{dx}.\frac{dy}{dx} + 6y.\frac{d^2y}{dx^2}$$
$$= 6\left(\frac{dy}{dx}\right)^2 + 6y\left(\frac{d^2y}{dx^2}\right)$$
$$\frac{d^4y}{dx^4} = 6\left(2\frac{dy}{dx}\right).\frac{d^2y}{dx^2} + 6.\frac{dy}{dx}.\frac{d^2y}{dx^2} + 6y\frac{d^3y}{dx^3}$$
$$= 18.\frac{dy}{dx}.\frac{d^2y}{dx^2} + 6y\frac{d^3y}{dx^3}\text{, where } B = 18 \text{ and } C = 6$$

3(a)

Area $= \int_0^1 x^2 \, dx - \frac{1}{2}\left(\frac{1}{2}\right)(1)$ **<u>OR</u>** $= \int_0^1 x^2 \, dx - \int_{\frac{1}{2}}^1 2x - 1 \, dx$

$$= \left[\frac{x^3}{3}\right]_0^1 - \frac{1}{4}$$
$$= \frac{1}{3} - \frac{1}{4}$$
$$= \frac{1}{12}\text{ units}^2$$

<u>Alternative method</u>

$$\text{Area } = \int_0^1 \frac{1}{2}(y+1) - \sqrt{y}\ \mathrm{d}y$$

$$= \left[\frac{1}{4}y^2 + \frac{1}{2}y - \frac{2}{3}y^{\frac{3}{2}}\right]_0^1$$

$$= \frac{1}{4} + \frac{1}{2} - \frac{2}{3}$$

$$= \frac{1}{12}\text{ units}^2$$

(b)

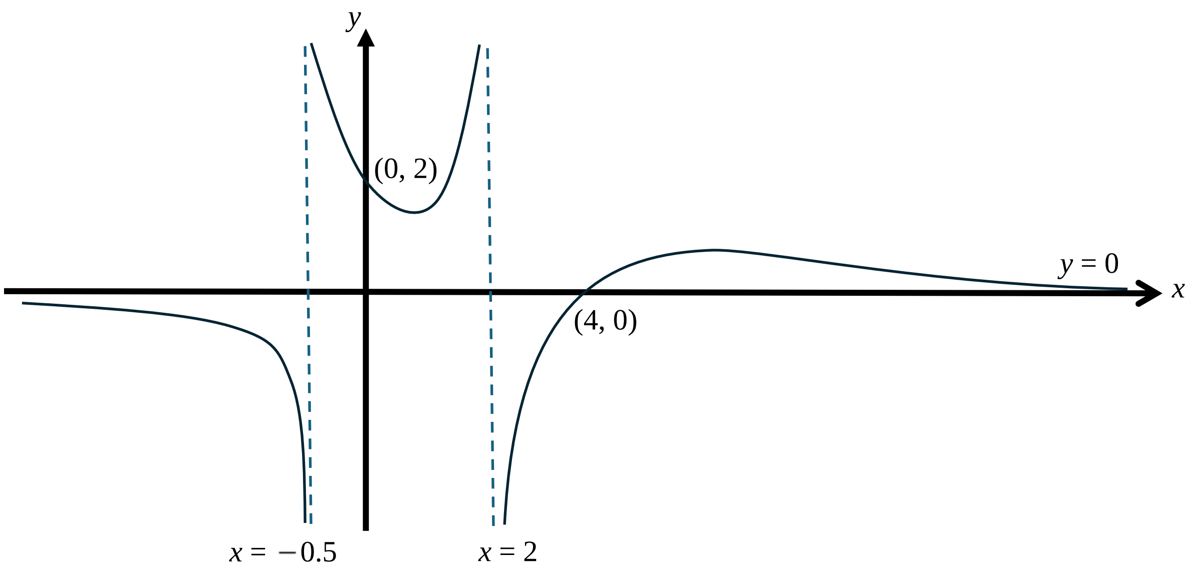
$$\text{Volume } = \pi\int_0^1 \left(x^2\right)^2 \mathrm{d}x - \frac{1}{3}\pi(1)^2\left(\frac{1}{2}\right) \quad \textbf{OR} \quad = \pi\int_0^1 \left(x^2\right)^2 \mathrm{d}x - \pi\int_{\frac{1}{2}}^1 (2x-1)^2\ \mathrm{d}x$$

$$= \pi\left[\frac{x^5}{5}\right]_0^1 - \frac{1}{6}\pi$$

$$= \frac{\pi}{5} - \frac{\pi}{6}$$

$$= \frac{\pi}{30}\text{ units}^3$$

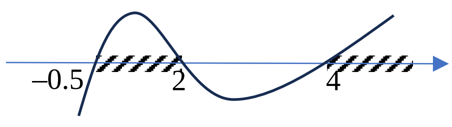
4(a) $y = \dfrac{4-x}{2+3x-2x^2} = \dfrac{4-x}{-(2-x)(1+2x)} \Rightarrow$ Asymptotes are $y = 0$, $x = 2$, $x = -\dfrac{1}{2}$



(b) From the graph, $-0.5 < x < 2$ or $x > 4$

OR: $\dfrac{4-x}{2+3x-2x^2} > 0 \Rightarrow -(4-x)(x-2)(2x+1) > 0$

$\Rightarrow -0.5 < x < 2$ or $x > 4$



(c) Replace x with $|x|$,

$-0.5 < |x| < 2$ or $|x| > 4$

Since $|x| \geq 0 > -0.5$ for all real values of x,

$|x| < 2$ or $|x| > 4$

$-2 < x < 2$ or $x < -4$ or $x > 4$

5(a) $|\mathbf{c} \times \hat{\mathbf{a}}|$ represents the perpendicular distance from the point R to the line PQ.

Area of $\Delta PQR = \frac{1}{2} \times$ base $\times$ height

$= \frac{1}{2} \times |\mathbf{a}| \times |\mathbf{c} \times \hat{\mathbf{a}}|$

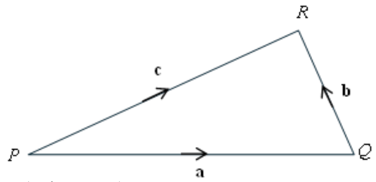
$= \frac{1}{2} \times |\mathbf{a}| \times \left|\mathbf{c} \times \frac{\mathbf{a}}{|\mathbf{a}|}\right|$

$= \frac{1}{2}|\mathbf{c} \times \mathbf{a}|$ units^2 (Shown)

(b) By replacing $\mathbf{a}$ with $-\mathbf{a}$ and $\mathbf{c}$ with $\mathbf{b}$ in part (a), area of $\Delta PQR = \frac{1}{2}|\mathbf{b} \times (-\mathbf{a})| = \frac{1}{2}|\mathbf{b} \times \mathbf{a}|$

$\Rightarrow \frac{1}{2}|\mathbf{c} \times \mathbf{a}| = \frac{1}{2}|\mathbf{b} \times \mathbf{a}|$

$\Rightarrow |\mathbf{c} \times \mathbf{a}| = |\mathbf{b} \times \mathbf{a}|$ (Shown)



OR: From the diagram, $\mathbf{c} = \mathbf{a} + \mathbf{b}$ so $|\mathbf{c} \times \mathbf{a}| = |(\mathbf{a} + \mathbf{b}) \times \mathbf{a}|$

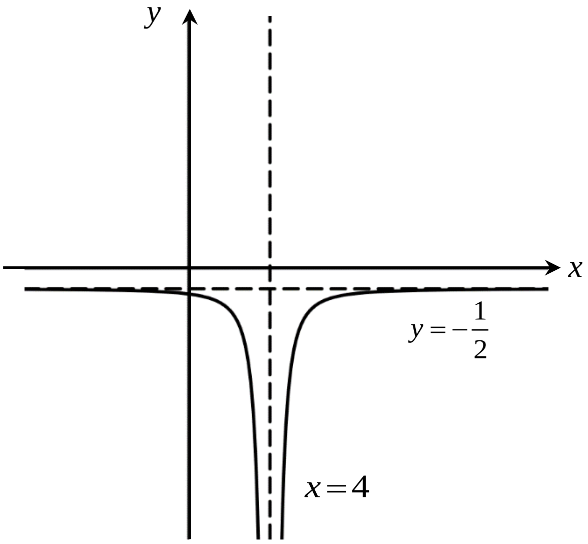
$= |\mathbf{a} \times \mathbf{a} + \mathbf{b} \times \mathbf{a}|$

$= |\mathbf{b} \times \mathbf{a}|$ since $\mathbf{a} \times \mathbf{a} = \mathbf{0}$ (Shown)

$|\mathbf{c} \times \mathbf{a}| = |\mathbf{b} \times \mathbf{a}| \Rightarrow |\mathbf{c}||\mathbf{a}|\sin \angle QPR = |\mathbf{b}||\mathbf{a}|\sin \angle PQR$

$\Rightarrow \frac{|\mathbf{c}|}{\sin \angle PQR} = \frac{|\mathbf{b}|}{\sin \angle QPR}$ (Shown)

6(a)



(b)

1. Translation of 4 units in the positive x-direction:

$$y^2-x^2=1 \xrightarrow{\text{replace } x \text{ by } x-4} y^2-(x-4)^2=1$$

2. Scale by scale factor k parallel to the y-axis:

$$y^2-(x-4)^2=1 \xrightarrow{\text{replace } y \text{ by } \frac{y}{k}} \frac{y^2}{k^2}-(x-4)^2=1$$

3. Translation of 1 unit in the negative y-direction:

$$\frac{y^2}{k^2}-(x-4)^2=1 \xrightarrow{\text{replace } y \text{ by } y+1} \frac{(y+1)^2}{k^2}-(x-4)^2=1$$

Alternative (manipulate y first then x)

1. Scale by scale factor k parallel to the y-axis
2. Translation of 1 unit in the negative y-direction
3. Translation of 4 units in the positive x-direction

Alternative (translation before scaling)

1. Translation of 4 units in the positive x-direction
2. Translation of $\frac{1}{k}$ unit in the negative y-direction

$$y^2-(x-4)^2=1 \xrightarrow{\text{replace } y \text{ by } y+\frac{1}{k}} \left(y+\frac{1}{k}\right)^2-(x-4)^2=1$$

3. Scale by scale factor k parallel to the y-axis

$$\left(y+\frac{1}{k}\right)^2-(x-4)^2=1 \xrightarrow{\text{replace } y \text{ by } \frac{y}{k}} \left(\frac{y}{k}+\frac{1}{k}\right)^2-(x-4)^2=1$$

$$\frac{(y+1)^2}{k^2}-(x-4)^2=1$$

(c)

C_1 has asymptotes $y=-\frac{x}{2}+1$ and $x=4$, which intersects at (4,-1)

C_2 is a hyperbola center at (4,-1) and has oblique asymptote $y=-1\pm k(x-4)$,

For C_1 and C_2 to cut exactly 2 times, the gradient of oblique asymptote $y=-1\pm k(x-4)$ must be greater or equals to than that of C_1.

$\therefore k \geq \frac{1}{2}$

7(a) $f^2(a)=5$

$f(\sqrt{5+2a})=5$

$\sqrt{5+2\sqrt{5+2a}}=5$

$\sqrt{5+2a}=\dfrac{25-5}{2}$

$a=47.5$

(b) largest $k=0$

(c)

Let $y=\left|\dfrac{x}{1-2x}\right|$

$y=-\dfrac{x}{1-2x} \quad \because x\le 0$

$y-2xy=-x$

$x=\dfrac{y}{2y-1}$

$g^{-1}(x)=\dfrac{x}{2x-1}$

Domain of g^{-1} = $\left[\dfrac{1}{3},\dfrac{1}{2}\right)$

(d) $R_g=\left[\dfrac{1}{3},\dfrac{1}{2}\right)$

$D_f=\left(-\dfrac{5}{2},\infty\right)$

Since $R_g \subseteq D_f$, so gf exists.

$(-\infty,-1] \xrightarrow{g} \left[\dfrac{1}{3},\dfrac{1}{2}\right) \xrightarrow{f} \left[\sqrt{\dfrac{17}{3}},\sqrt{6}\right)$

So $R_{fg}=\left[\sqrt{\dfrac{17}{3}},\sqrt{6}\right)$

8(a) When $x = 0$

$0 = 1 - 2\cos 2\theta \Rightarrow 2\theta = \cos^{-1}\left(\frac{1}{2}\right) = \frac{\pi}{3} \Rightarrow \theta = \frac{\pi}{6}$

$y = \sin\left(\frac{\pi}{3}\right) - 1 = \frac{\sqrt{3}}{2} - 1$

The coordinate of the point where C cuts the y-axis is $\left(0, \frac{\sqrt{3}}{2} - 1\right)$.

(b) $\frac{dx}{d\theta} = 4\sin 2\theta$; $\frac{dy}{d\theta} = 2\cos 2\theta$

$\frac{dy}{dx} = \frac{\frac{dy}{d\theta}}{\frac{dx}{d\theta}} = \frac{2\cos 2\theta}{4\sin 2\theta} \quad = \frac{1}{2}\cot 2\theta$

When $\theta = \frac{\pi}{4}$, $x = 1$ and $y = 0$.

$\therefore (1, 0)$

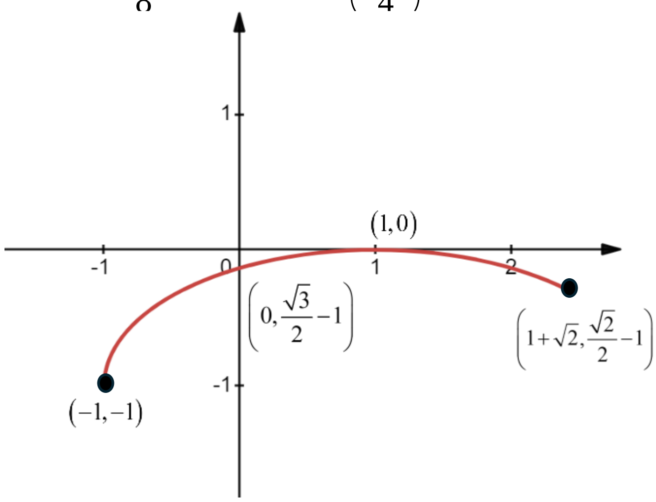
When $\theta = \frac{\pi}{4}$, $\frac{dy}{dx} = 0$

When $\theta \to 0$, the gradient will tend towards $+\infty$

The tangent of C as $\theta \to 0$ will get steeper and steeper until it tends towards a vertical line.

(c) When $\theta = 0$, $x = -1$ and $y = -1$. $\therefore (-1, -1)$

When $\theta = \frac{3\pi}{8}$, $x = 1 - 2\cos\left(\frac{3\pi}{4}\right) = 1 + \sqrt{2}$ $\quad y = \sin\left(\frac{3\pi}{4}\right) - 1 = \frac{\sqrt{2}}{2} - 1$



(d) $x = 1 - 2\cos 2\theta$ and $y = \sin 2\theta - 1$

$\cos 2\theta = \frac{1 - x}{2}$ and $\sin 2\theta = y + 1$

Since $\sin^2 A + \cos^2 A = 1$

Then $\left(\frac{1 - x}{2}\right)^2 + (y + 1)^2 = 1$

9(a)

$$\int \frac{x+2}{x^2+2x-3}\,\mathrm{d}x = \int \frac{1}{2}\left(\frac{2x+2}{x^2+2x-3}\right)+\frac{1}{(x+1)^2-(2)^2}\,\mathrm{d}x$$

$$= \frac{1}{2}\ln\left|x^2+2x-3\right|+\frac{1}{2(2)}\ln\left|\frac{(x+1)-2}{(x+1)+2}\right|+C$$

$$= \frac{1}{2}\ln\left|x^2+2x-3\right|+\frac{1}{4}\ln\left|\frac{x-1}{x+3}\right|+C$$

<u>Alternative Method</u>

$$\int \frac{x+2}{x^2+2x-3}\,\mathrm{d}x = \frac{1}{4}\int \frac{3}{x-1}+\frac{1}{x+3}\,\mathrm{d}x$$

$$= \frac{3}{4}\ln|x-1|+\frac{1}{4}\ln|x+3|+C$$

(b)

$$\int 2x\sin x\,\mathrm{d}x = 2x(-\cos x)-\int 2(-\cos x)\,\mathrm{d}x$$

$$= -2x\cos x+\int 2\cos x\,\mathrm{d}x$$

$$= -2x\cos x+2\sin x+C$$

(c)

For $x=3\sin\theta$

$$\frac{\mathrm{d}x}{\mathrm{d}\theta}=3\cos\theta$$

$$\int_0^{\frac{3}{2}}\sqrt{9-x^2}\,\mathrm{d}x = \int_0^{\frac{\pi}{6}}\sqrt{9-(3\sin\theta)^2}\left(\frac{\mathrm{d}x}{\mathrm{d}\theta}\right)\mathrm{d}\theta$$

$$= \int_0^{\frac{\pi}{6}} 3\sqrt{1-\sin^2\theta}\,(3\cos\theta)\,\mathrm{d}\theta$$

$$= \int_0^{\frac{\pi}{6}} 9\cos^2\theta\,\mathrm{d}\theta$$

$$= 9\int_0^{\frac{\pi}{6}}\frac{\cos 2\theta+1}{2}\,\mathrm{d}\theta$$

$$= \frac{9}{2}\left[\frac{\sin 2\theta}{2}+\theta\right]_0^{\frac{\pi}{6}}$$

$$= \frac{9}{4}\sin\frac{\pi}{3}+\frac{9}{2}\left(\frac{\pi}{6}\right)-0-0$$

$$= \frac{9}{4}\left(\frac{\sqrt{3}}{2}\right)+\frac{3}{4}\pi$$

$$= \frac{9}{8}\sqrt{3}+\frac{3}{4}\pi$$

10(i) The lines are coplanar ⇒ The lines are intersecting lines since they are not parallel.

Let $\begin{pmatrix} 1-\lambda \\ 2 \\ 5+3\lambda \end{pmatrix} = \begin{pmatrix} a+\mu \\ 9+7\mu \\ 9+b\mu \end{pmatrix}$

By comparing rows, $2 = 9 + 7\mu \quad \Rightarrow \quad \mu = -1$

$1 - \lambda = a + \mu \quad \Rightarrow \quad \lambda = 2 - a$ …Eq(1)

$5 + 3\lambda = 9 + b\mu \quad \Rightarrow \quad \lambda = \dfrac{4-b}{3}$ …Eq(2)

Eq(1) = Eq(2): $2-a = \dfrac{4-b}{3}$

$\Rightarrow \quad 6 - 3a = 4 - b$

$\Rightarrow \quad 3a = b + 2$ (Shown)

(ii) Angle between the two lines is $\cos^{-1}\dfrac{11}{\sqrt{660}}$

$$\Rightarrow \theta = \cos^{-1}\left|\frac{\underset{\sim}{a}\cdot\underset{\sim}{b}}{|\underset{\sim}{a}||\underset{\sim}{b}|}\right|$$

$$\Rightarrow \cos^{-1}\frac{11}{\sqrt{660}} = \cos^{-1}\frac{\left|\begin{pmatrix} -1 \\ 0 \\ 3 \end{pmatrix}\cdot\begin{pmatrix} 1 \\ 7 \\ b \end{pmatrix}\right|}{\left|\begin{pmatrix} 1 \\ 0 \\ -3 \end{pmatrix}\right|\left|\begin{pmatrix} 1 \\ 7 \\ b \end{pmatrix}\right|}$$

$$\therefore \frac{11}{\sqrt{660}} = \frac{|3b-1|}{\sqrt{1+9}\sqrt{1+49+b^2}}$$

$$\frac{11}{6} = \frac{1-6b+9b^2}{50+b^2}$$

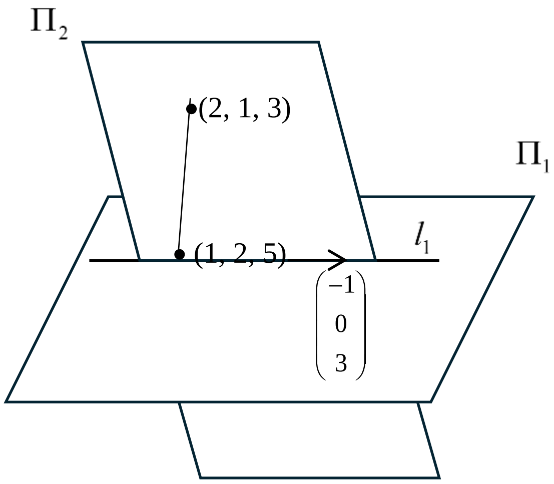
$43b^2 - 36b - 544 = 0$

$b = \dfrac{36 \pm \sqrt{1296+93568}}{86} = 4$ (reject negative value of b)

(iii) A normal for Π_2 is $\begin{pmatrix}1\\0\\-3\end{pmatrix}\times\left[\begin{pmatrix}2\\1\\3\end{pmatrix}-\begin{pmatrix}1\\2\\5\end{pmatrix}\right]$

$$=\begin{pmatrix}1\\0\\-3\end{pmatrix}\times\begin{pmatrix}1\\-1\\-2\end{pmatrix}$$

$$=\begin{pmatrix}-3\\-1\\-1\end{pmatrix}=-\begin{pmatrix}3\\1\\1\end{pmatrix}$$



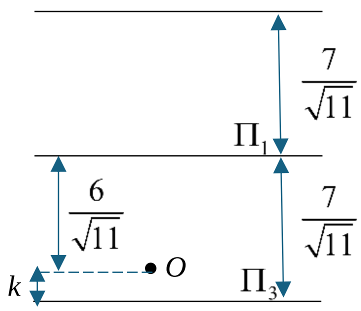
$$\mathbf{r}\cdot\begin{pmatrix}3\\1\\1\end{pmatrix}=\begin{pmatrix}1\\2\\5\end{pmatrix}\cdot\begin{pmatrix}3\\1\\1\end{pmatrix}=3+2+5=10$$

$$\therefore\ \Pi_2:\ \mathbf{r}\cdot\begin{pmatrix}3\\1\\1\end{pmatrix}=10$$

(iv) Angle required $=\cos^{-1}\left|\dfrac{\begin{pmatrix}3\\-1\\1\end{pmatrix}\cdot\begin{pmatrix}3\\1\\1\end{pmatrix}}{\sqrt{11}\sqrt{11}}\right|$

$$=\cos^{-1}\left|\frac{9}{11}\right|=0.613 \text{ rad (3 sf) or } 35.1^\circ \text{ (nearest } 0.1^\circ)$$

(v) $\Pi_1:\ \mathbf{r}\cdot\dfrac{\begin{pmatrix}3\\-1\\1\end{pmatrix}}{\sqrt{11}}=\dfrac{6}{\sqrt{11}}<\dfrac{7}{\sqrt{11}}$



Since Π_1 and Π_3 are parallel, then

$\Pi_3:\ \mathbf{r}\cdot\dfrac{\begin{pmatrix}3\\-1\\1\end{pmatrix}}{\sqrt{11}}=k$ where k is a real constant.

Since Π_3 is closer to the origin to Π_1, then $k = -\left(\frac{7}{\sqrt{11}} - \frac{6}{\sqrt{11}}\right) = \frac{-1}{\sqrt{11}}$

$$\Rightarrow \Pi_3 : \mathbf{r} \cdot \frac{\begin{pmatrix} 3 \\ -1 \\ 1 \end{pmatrix}}{\sqrt{11}} = \frac{-1}{\sqrt{11}}$$

Hence the cartesian equation for Π_3 is $3x - y + z = -1$

11(a) (i) $V = \frac{4}{3}\pi r^3 \Rightarrow \frac{\mathrm{d}V}{\mathrm{d}r} = 4\pi r^2$

$$S = 4\pi r^2 \Rightarrow \frac{\mathrm{d}S}{\mathrm{d}r} = 8\pi r$$

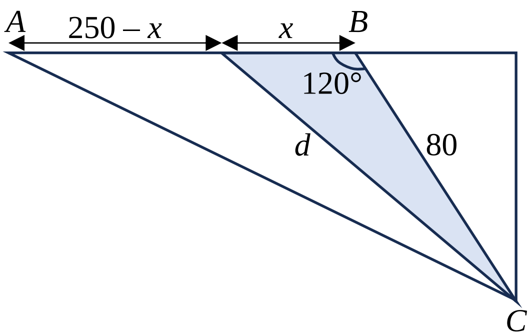
$$\frac{\mathrm{d}V}{\mathrm{d}S} = \frac{\mathrm{d}V}{\mathrm{d}r} \div \frac{\mathrm{d}S}{\mathrm{d}r} = \frac{4\pi r^2}{8\pi r} = \frac{r}{2}$$

(ii) $\frac{\mathrm{d}V}{\mathrm{d}t} = \frac{\mathrm{d}V}{\mathrm{d}S} \cdot \frac{\mathrm{d}S}{\mathrm{d}t} = \frac{r}{2}(3) = \frac{3r}{2}$

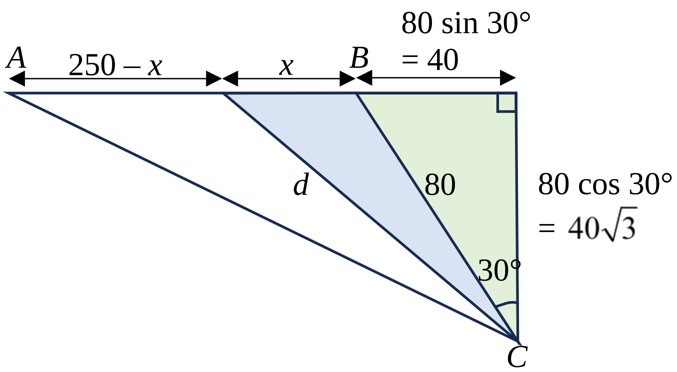
When $\frac{\mathrm{d}V}{\mathrm{d}t} = 9$,

$$9 = \frac{3r}{2} \Rightarrow r = 6$$

Method 1



Method 2



(b) **(i)** Time taken on straight road = $\frac{250 - x}{130}$

Distance from straight road to Town C, d

(Method 1): $= \sqrt{x^2 + 80^2 - 2x(80)\cos\frac{2\pi}{3}}$ (cosine rule)

(Method 2): $= \sqrt{(x + 40)^2 + (40\sqrt{3})^2}$ (Pythagoras' Thm)

Distance from straight road to Town C $= \sqrt{x^2 + 80x + 6400}$

Time taken on desert = $\dfrac{\sqrt{x^2+80x+6400}}{110}$

Total time taken by competitor P, $T=\dfrac{250-x}{130}+\dfrac{\sqrt{x^2+80x+6400}}{110}$

$$T=11\left(\frac{250-x}{1430}\right)+13\left(\frac{\sqrt{x^2+80x+6400}}{1430}\right)$$

$$T=\frac{1}{1430}\left(2750-11x+13\sqrt{x^2+80x+6400}\right) \quad \text{(shown)}$$

(ii) $\dfrac{\mathrm{d}T}{\mathrm{d}x}=\dfrac{1}{1430}\left(-11+13\left(\dfrac{1}{2}\right)\left(x^2+80x+6400\right)^{-\frac{1}{2}}(2x+80)\right)$

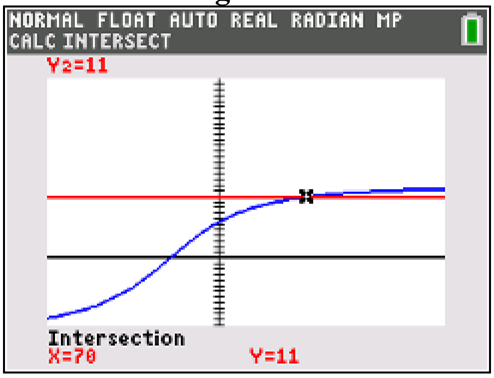
$$\frac{\mathrm{d}T}{\mathrm{d}x}=\frac{1}{1430}\left(-11+13(x+40)\left(x^2+80x+6400\right)^{-\frac{1}{2}}\right)$$

To find minimum time, $\dfrac{\mathrm{d}T}{\mathrm{d}x}=0$

$$\frac{1}{1430}\left(-11+13(x+40)\left(x^2+80x+6400\right)^{-\frac{1}{2}}\right)=0$$

$$\frac{13(x+40)}{\sqrt{x^2+80x+6400}}=11$$

Method 1: Using GC



$x=70$

or

Method 2: Algebra

$13(x+40)=11\sqrt{x^2+80x+6400}$

$169(x+40)^2=121(x^2+80x+6400)$

$169(x^2+80x+1600)-121(x^2+80x+6400)=0$

$169x^2+13520x+270400-121x^2-9680x-774400=0$

$48x^2+3840x-504000=0$

$x=70$ or $x=-150$ (rej as $x\geq 0$)

Sub $x=70$ into T

$$T=\frac{1}{1430}\left(2750-11(70)+13\sqrt{(70)^2+80(70)+6400}\right)$$
$$=\frac{3670}{1430}$$
= 2.566 hr

= 154 min or 2 hr 34 min (nearest minute)

(iii) Time taken by Competitor $Q=\frac{\sqrt{250^2+80^2-2(250)(80)\cos\frac{2\pi}{3}}}{M}=\frac{\sqrt{88900}}{M}$

Time taken by Competitor P = $\frac{250}{130}+\frac{80}{M}$

Since Competitor P is faster than Competitor Q

$$\frac{\sqrt{88900}}{M}>\frac{250}{130}+\frac{80}{M}$$

$$\frac{\sqrt{88900}-80}{M}>\frac{25}{13}$$

$$M<113.44$$

Also, $M>0$

Hence M lies between 0 and 113.

12(a) $10000(1.009)^2 + 810(1.009) = \$10998.10$

(b)

Month	Start of month	End of month
1	10000	10000(1.009)
2	10000(1.009) + 810	$10000(1.009)^2 + 810(1.009)$
3	$10000(1.009)^2 + 810(1.009) + 810$	$10000(1.009)^3 + 810(1.009)^2 + 810(1.009)$
	$\vdots$	$\vdots$
n		$10000(1.009)^n + 810(1.009)^{n-1} + \ldots + 810(1.009)$

At the end of nth month,

$$S_n = 10000(1.009)^n + 810(1.009)^{n-1} + \ldots + 810(1.009)$$

$$= \$10000(1.009)^n + \$810\left(\frac{(1.009)\left[(1.009)^{n-1} - 1\right]}{(1.009) - 1}\right)$$

$$= \$10000(1.009)^n + \$90810\left[(1.009)^{n-1} - 1\right]$$

$$= \$10000(1.009)(1.009)^{n-1} + \$90810(1.009)^{n-1} - \$90810$$

$$= \$100900(1.009)^{n-1} - \$90810 \quad \text{(Shown)}$$

(c)

$$100900(1.009)^{n-1} - 90810 > 30000$$

$$100900(1.009)^{n-1} > 90810 + 30000$$

$$n - 1 > \frac{\lg 1.197324}{\lg 1.009}$$

$$n > 21.0998$$

<u>OR</u> using GC

	Amt at the end of the month
n	$100900(1.009)^{n-1} - 90810$
21	29892.01
22	30978.32
23	32074.42

When $n = 21$, at the end of the 21st month, amount in account = \$29892.01

At the start of the 22nd month, amount in account = \$29892.01 + \$810 = \$30702.01

$\therefore$ on 1 Oct 2025, the account will first exceed \$30000.

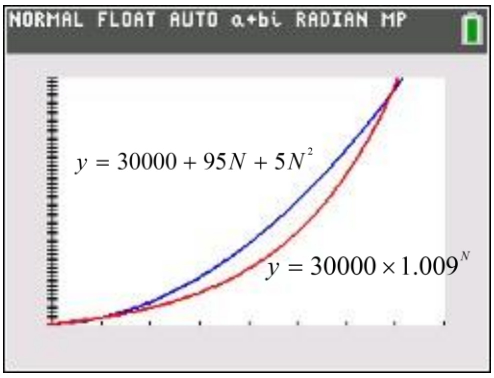
(d) Amount Mr. P will have at the end of N months

$$= 30000 + \left[100 + 110 + ... + \left(100 + 10(N-1)\right)\right]$$

$$= 30000 + \frac{N}{2}\left[2(100) + (N-1)(10)\right]$$

$$= 30000 + 95N + 5N^2$$

(e) $30000 + 95N + 5N^2 > 30000 \times 1.009^N$



	$30000 + 95N + 5N^2$	30000×1.009^N
⋮	⋮	⋮
48	46080	46120.84
49	46660	46535.93
50	47250	46954.75
⋮	⋮	⋮
343	650830	648322.77
344	654360	654157.67
345	657900	660045.09

From GC, $49 \le N \le 344$