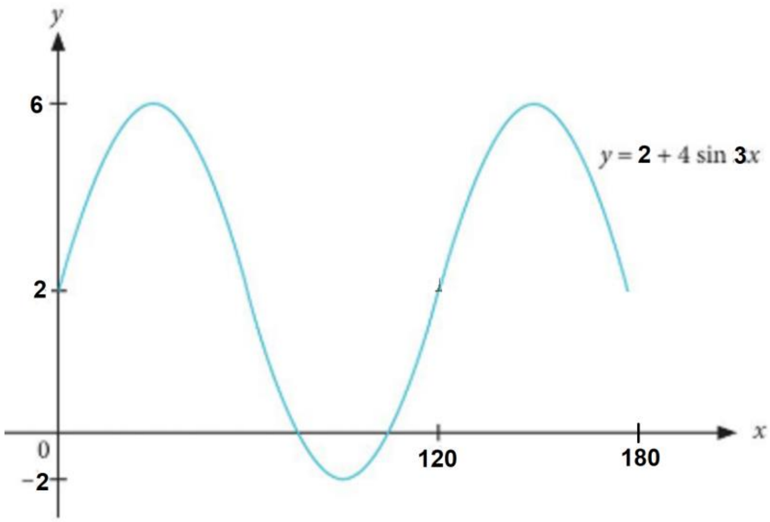
1	$4y = 2x + 1$ (1) $\frac{3}{x} - \frac{1}{y} = 4$ (2) From (1), $y = \frac{2x+1}{4}$ (3) Subst. (3) into (2): $\frac{3}{x} - \frac{4}{2x+1} = 4$ $6x + 3 - 4x = 8x^2 + 4x$ $8x^2 + 2x - 3 = 0$ $(2x-1)(4x+3) = 0$ $x = 0.5 / -0.75$ Subst. $x = 0.5$ into (3): $y = \frac{2(0.5)+1}{4} = 0.5$ Subst. $x = -0.75$ into (3): $y = \frac{2(-0.75)+1}{4} = -0.125$ $Midpoint_{AB}\left(\frac{0.5-0.75}{2}, \frac{0.5-0.125}{2}\right)$ $= (-0.125, 0.1875)$	
2	(a) Amplitude, $b = \frac{6-(-2)}{2} = 4$ Maximum value = $a + 4(1) = 6$ $a = 2$ (b) a shifts the graph vertically from the x-axis by a units	

	(c)  1.5 cycle x-intercepts between 60° and 120°; y-intercept at 2	
3	(a) $(1-x+x^2)^5 = 1^5 + 5(-x+x^2) + \binom{5}{2}(-x+x^2)^2 + \ldots$ $= 1-5x+5x^2+10(x^2-2x^3+x^4)+\ldots$ $= 1-5x+15x^2+\ldots$ A1 (b) $T_{n+1} = \binom{11}{n}(mx)^{11-n}(-x^{-2})^n = \binom{11}{n}m^{11-n}(-1)^n x^{11-3n}$ $T_4 = \binom{11}{3}m^8x^2 = -165m^8x^2$ $T_8 = \binom{11}{7}m^4x^{-10} = -330m^4x^{-10}$ $165m^8 = 128(330m^4)$ $m^4(m^4-256) = 0$ $m = 0$ or ± 4 $m = \pm 4$	
4	(a) $f(x) = 0.05[11+(3x+1)^3]$ $f'(x) = 0.05(3)(3x+1)^2(3)$ $= 0.45(3x+1)^2$ Since $f'(x) \geq 0$ and $x \neq -\frac{1}{3}$, f is an increasing function.	

	(b) When $V = 0.95$, $0.95 = 0.05[11+(3x + 1)^3]$ $(3x + 1)^3 = 8$ $3x + 1 = 2$ $x = \frac{1}{3}$ When $x = \frac{1}{3}$, $\frac{dV}{dx} = 1.8$ $\frac{\mathrm{d}x}{\mathrm{d}t} = \frac{1}{\frac{\mathrm{d}V}{\mathrm{d}x}} \times \frac{\mathrm{d}V}{\mathrm{d}t}$ $= \frac{1}{1.8} \times 0.081$ $= 0.045$ m/s **(c)** When the volume of liquid in the container = 0.95 m^3, the height of liquid in the container is increasing at 0.045 m/s.	
5	**(a)** When $t = 0$, $50 = \frac{p}{q+2^7}$ $p - 50q = 6400$ (1) When $t = 2$, $250 = \frac{p}{q+2^4}$ $p - 250q = 4000$ (2) $p - 50q = 6400$ (1) $p - 250q = 4000$ (2) (1) – (2): $200q = 2400$ $q = 12$ Subst. $q = 12$ into (1): $p = 7000$ (b) $550 = \frac{7000}{12+2^{7-1.5t}}$ $2^{7-1.5t} = \frac{8}{11}$ $t = \frac{7-\log_2\frac{8}{11}}{1.5}$	

	= 4.9729 (5 fig) = 5 years (round up) (c) $t = 10$ $N = \frac{7000}{12+2^{7-1.5(10)}}$ = 583.14 (5 fig) = 583 clients (round down) No it will have **583** clients after 10 years.	
6	$\frac{d^2y}{dx^2} = 5\cos\frac{1}{6}x + 2\sin\frac{1}{3}x$ $\frac{dy}{dx} = \int 5\cos\frac{1}{6}x + 2\sin\frac{1}{3}x\ dx$ $\frac{dy}{dx} = 30\sin\frac{1}{6}x - 6\cos\frac{1}{3}x + c$ $8 = 30\sin\frac{1}{6}\pi - 6\cos\frac{1}{3}\pi + c$ $8 = 30\left(\frac{1}{2}\right) - 6\left(\frac{1}{2}\right) + c$ $8 = 15 - 3 + c$ $c = -4$ $\frac{dy}{dx} = 30\sin\frac{1}{6}x - 6\cos\frac{1}{3}x - 4$ $y = \int 30\sin\frac{1}{6}x - 6\cos\frac{1}{3}x - 4\ dx$ $y = -180\cos\frac{1}{6}x - 18\sin\frac{1}{3}x - 4x + c$ $-99\sqrt{3} = -180\cos\frac{1}{6}\pi - 18\sin\frac{1}{3}\pi - 4\pi + c$ $-99\sqrt{3} = -180\left(\frac{\sqrt{3}}{2}\right) - 18\left(\frac{\sqrt{3}}{2}\right) - 4\pi + c$ $c = 4\pi$ $y = -180\cos\frac{1}{6}x - 18\sin\frac{1}{3}x - 4x + 4\pi$	

7	(a) $h = -0.1d^2 + 4d + 1.8$ $h = -0.1d^2 + 4d + 1.8$ $= -0.1[d - 40d - 18]$ $= -0.1[d^2 - 40d + (-20)^2 - (-20)^2 - 18]$ $= -0.1[(d-20)^2 - 418]$ $= -\frac{1}{10}(d-20)^2$ $+41.8$ (b) The coordinates of the turning point are (20, 41.8). The greatest height reached by the object is 41.8 m and its corresponding horizontal distance is 20 m. (c) When $h = 0$, $-0.1(d-20)^2 + 41.8 = 0$ $-0.1(d-20)^2 = -41.8$ $(d-20)^2 = 418$ $d - 20 = \pm\sqrt{418}$ $d = 20 \pm \sqrt{418}$ = 40.445 or –0.44504 (5 fig) Since the ball travelled 40.4 m (> 40m) when it hits the ground. Therefore the canon is not precise.	
8	$\cos A = \frac{1}{2}$; $\sin A = \frac{\sqrt{3}}{2}$ $\sin B = -\frac{1}{\sqrt{2}}$; $\cos B = -\frac{1}{\sqrt{2}}$ **(a)** cos (*A* – *B*) = cos *A* cos *B* + sin *A* sin *B* $= \left(\frac{1}{2}\right)\left(-\frac{1}{\sqrt{2}}\right) + \left(\frac{\sqrt{3}}{2}\right)\left(-\frac{1}{\sqrt{2}}\right)$ $= -\frac{1}{2\sqrt{2}} - \frac{\sqrt{3}}{2\sqrt{2}}$ $= \frac{-1-\sqrt{3}}{2\sqrt{2}}$ $= \frac{-\sqrt{2}-\sqrt{6}}{4}$ $= -\frac{1}{4}\sqrt{2} - \frac{1}{4}\sqrt{6}$	

	(b) $\cos A = 2\cos^2\frac{A}{2} - 1$ $2\cos^2\frac{A}{2} - 1 = \frac{1}{2}$ $\cos\frac{A}{2} = \pm\sqrt{\frac{3}{4}}$ $\cos\frac{A}{2} = \frac{\sqrt{3}}{2}$	
9	(a) Volume of gift box, $V = 2.5x^2h$ When $V = 27000$, $2.5x^2h = 27000$ $h = \frac{10800}{x^2}$ (b) $A = 2(2.5x^2) + 2(hx) + 2(2.5hx) + 2\left(\frac{1}{4}hx\right) + 2(2.5)\left(\frac{1}{4}hx\right)$ $A = 5x^2 + 8.75hx$ $A = 5x^2 + 8.75\left(\frac{10800}{x^2}\right)x$ $A = 5x^2 + \frac{94500}{x}$ (shown) (c) $\frac{\mathrm{d}A}{\mathrm{d}x} = 10x - \frac{94500}{x^2}$ For stationary points, $\frac{\mathrm{d}A}{\mathrm{d}x} = 0$ $10x - \frac{94500}{x^2} = 0$ $x^3 = \frac{94500}{10}$ $x = 21.1$ (to 3 fig) $\frac{\mathrm{d}^2A}{\mathrm{d}x^2} = 10 + \frac{189000}{x^3}$ When $x = 21.141$, $\frac{\mathrm{d}^2A}{\mathrm{d}x^2} > 0$. Thus when $x = 21.1$, A is a minimum.	

10	(a) Remainder = f(–5) = 0 $(-5)^3 + a(-5) + b = 0$ $b - 5a = 125$ (1) Remainder = f(3) = 24 $(3)^3 + a(3) + b = 24$ $3a + b = -3$ (2) (2) – (1): $8a = -128$ $a = -16$ Subst. $a = -16$ into (1): $b = 45$ (b) $f(x) = x^3 - 16x + 45 = 0$ $(x + 5)$ is a factor of f(x). $\frac{x^3 - 16x + 45}{x+5}$ $f(x) = (x + 5)(x^2 - 5x + 9)=0$ $(x+5)[(x-2.5)^2+2.75]=0$ Since $\left(x-\frac{5}{2}\right)^2 + 2.75 > 0$ for all real values of x (or discriminant = -20 < 0), f(x) has only 1 real root where $x = -5$.	
11	(a) $m_{AB} = \frac{8-2}{3-1}$ $= 3$ $m_{AD} = -\frac{1}{3}$ since $m_{AB} \times m_{AD} = -1, \angle BAD = 90°$ and AB parallel to CD, $ABCD$ is a right-angled trapezium.	

	(b) Let M be midpoint of AC $M=\left(\frac{3+5}{2},\frac{8+4}{2}\right)=(4,6)$ E is the midpoint of MC $E=\left(\frac{4+5}{2},\frac{6+4}{2}\right)=\left(\frac{9}{2},5\right)$	
	(c) eqn. of side DC: $\frac{y-4}{x-5}=3$ $y=3x-11-----(1)$ $3y+x=27----(2)$ subs. (1) into (2) $x=6, y=7$ $D(6,7)$ Area = $\frac{1}{2}\begin{vmatrix}3 & 4.5 & 6 & 3\\ 8 & 5 & 7 & 8\end{vmatrix}$ $=\frac{1}{2}(94.5-87)$ $=3.75\text{ units}^2$	
12	(a) $\frac{1-\tan x}{\tan x+1}-\frac{\tan x+1}{\tan x-1}=2\sec 2x$ LHS $=\frac{1-\tan x}{\tan x+1}-\frac{\tan x+1}{\tan x-1}$ $=\frac{(tan x-1)^2+(\tan x+1)^2}{1-\tan^2 x}$ $=\frac{2\tan^2 x+2}{1-\tan^2 x}$ $=\frac{2(\tan^2 x+1)}{1-\tan^2 x}$ $=\frac{2\left(\frac{\sin^2 x+cos^2 x}{cos^2 x}\right)}{\frac{cos^2 x-\sin^2 x}{cos^2 x}}$ or $\frac{2\sec^2 x}{2-1\sec^2 x}$ $=\frac{\frac{2}{cos^2 x}}{\frac{cos^2 x-\sin^2 x}{cos^2 x}}$ or $\frac{2}{\cos^2 x}\div\frac{2\cos^2 x-1}{\cos^2 x}$	

	$=\frac{2}{cos^2x-\sin^2 x}$ $=\frac{2}{cos2x}$ = 2 sec 2x = RHS (proven)	
	(b)$2sec2x = 5$ $cos2x = \frac{2}{5}$ $\propto= cos^{-1}\frac{2}{5} = 66.421°$ $2x = 66.421,293.579,426.421,653.579$ $x = 33.2,146.8,213.2,326.8(rej)$	
	(c) Principal value of $\cos^{-1}\left(-\frac{1}{2}\right) = \frac{2\pi}{3}$.	
13	(a) $v=18-6e^{3(0)}$ = 12 ms^{-1} (b) $0=18-6e^{3t}$ $-18=-6e^{3t}$ $e^{3t}=3$ $3t=\ln 3$ $t=\frac{\ln 3}{3}=0.36620$ $t=0.366$ s A1 (c) $v=18-6e^{3t}$ $a=\frac{dv}{dt}=-18e^{3t}$ $a<0$, Decreasing velocity (d) $v=18-6e^{3t}$	

	$s=\int v\ \mathrm{d}t=\int 18-6e^{3t}\ \mathrm{d}t$ $s=18t-2e^{3t}+c$ $s=0; t=0$ $0=0-2e^{3(0)}+c$ $c=2$ $s=18t-2e^{3t}+2$ (e) $s_{0.36620}=18(0.36620)-2e^{3(0.36620)}+2=2.5916$ m $s_2=18(2)-2e^{3(2)}+2=-768.85$ m Total distance = 2(2.5916) + 768.85 = 774.03 m Average speed = $\frac{774.03}{2}=387.01=387\ \mathrm{m}s^{-1}$	