Candidate Index Number

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# Anglo - Chinese School (Independent)

## FINAL EXAMINATIONS 2016
## YEAR 3 INTEGRATED PROGRAMME
## CORE MATHEMATICS
## PAPER 1

**WEDNESDAY** **5th OCTOBER 2016** **1 h 30 min**

**Additional Material**
Graph Paper (1 sheet)

**INSTRUCTIONS TO CANDIDATES**

- Write your index number in the boxes above.
- Do not open this examination paper until instructed to do so.
- You are not permitted access to any calculator for this paper.
- Answer all questions in the spaces provided.
- Unless otherwise stated in the question, all numerical answers must be given exactly or correct to three significant figures.
- The maximum mark for this paper is 80.

| For Examiner's Use |
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**This paper consists of 14 printed pages.**

**[Turn over**

*Full marks are not necessarily awarded for a correct answer with no working. Answers must be supported by working and/or explanations. Where an answer is incorrect, some marks may be given for a correct method, provided this is shown by written working. You are therefore advised to show all working.*

*Answer **all** the questions in the spaces provided.*

**1** *[Maximum mark: 8]*

(a) Simplify $\frac{a-b}{ab}-\frac{c-b}{bc}+\frac{c-a}{ac}$.

*[2 marks]*

(b) Simplify $\frac{x^3y^3\times\sqrt[3]{x}}{\sqrt{x^2y^4}}$, giving your answer in the form $x^n y^m$, where $n$ and $m$ are rational numbers.

*[3 marks]*

(c) Factorise completely $(x+y)^2-(x+y)-6$.

*[3 marks]*

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**2** *[Maximum mark: 8]*

Mrs. Lim imported some olive oil for \$500. She paid $\$x$ for each liter of the olive oil.

(a) Find, in terms of $x$, an expression for the amount of olive oil she bought.

*[1 mark]*

During transportation, 30 liters of olive oil was spilled. She sold the remaining olive oil for \$1 more per liter than what she paid initially.

(b) If she sold all the oil, write down an expression, in terms of $x$, for the sum of money she received from the sale.

*[2 marks]*

Mrs. Lim made a loss of \$25.

(c) Write down an equation in $x$ to represent this information, and show that it can be reduced to $6x^2 + x - 100 = 0$.

*[2 marks]*

(d) Solve the equation $6x^2 + x - 100 = 0$, and hence, find the amount of olive oil she bought.

*[3 marks]*

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**3** *[Maximum mark: 8]*

(a) Given that $-a\ln b$ is a solution to $25-2e^{-x}=9$, find the value of $a$ and of $b$, where $a$ and $b$ are integers.

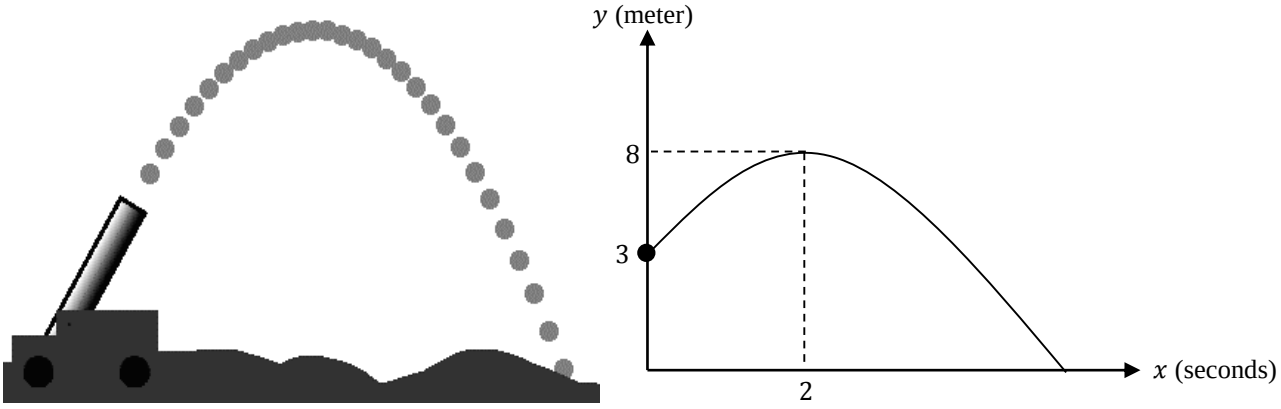
*[4 marks]*

(b) Solve $\log_2(x+1)=\log_4(13-2x)$.

*[4 marks]*

4 *[Maximum mark: 5]*

The quadratic graph below represents the motion of a cannonball which was fired from a height of 3 meter. The trajectory of the cannonball reached a maximum height of 8 m after 2 seconds.




(a) Given that $y$ is the vertical distance in meters and $x$ is the time in seconds, express the equation of the trajectory in the form $y = a(x-h)^2 + k$, where $a$, $h$ and $k$ are constants.

*[3 marks]*

(b) Hence, find the range of values of $x$ for which $a(x-h)^2 + k > 3$.

*[2 marks]*

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**5** *[Maximum mark: 5]*

Given that $5 \le a \le 10$ and $-6 \le b \le -1$, where $a$ and $b$ are integers.
Find

(a) the largest possible value of $a - b$,

*[1 mark]*

(b) the smallest possible value of $b^2 - a$,

*[1 mark]*

(c) the smallest possible value of $\dfrac{-a^2 + 12a - 31}{b}$.

*[3 marks]*

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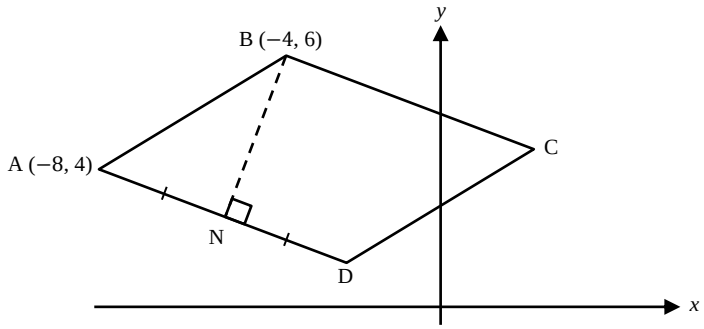
**6** *[Maximum mark: 12]*

The diagram shows a parallelogram ABCD where the coordinates of A and B are $(-8, 4)$ and $(-4, 6)$ respectively. The equation of AD is $3y + x - 4 = 0$. N is the foot of the perpendicular from B to AD, and *AN = ND*.




(a) Find the equation of *BN*.

*[3 marks]*

(b) Find the coordinates of *N*.

*[3 marks]*

(c) State the coordinates of C and D.

*[3 marks]*

(d) Calculate the area of the parallelogram ABCD.

*[3 marks]*

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**7** *[Maximum mark: 6]*

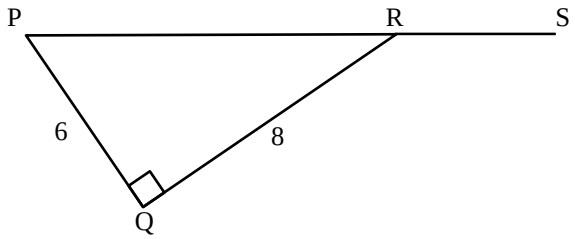
Solve the simultaneous equations

$2^{2+m} + 3\left(3^{n}\right) = 5$

$2^{m} + 3^{n+1} = 2$

**8** *[Maximum mark: 8]*

(a) In the diagram below, PRS is a straight line and $\angle PQR = 90^o$. Find the value of




(i) $\tan \angle QRP$,

*[1 mark]*

(ii) $\cos \angle QRS$.

*[2 marks]*

(b) Given that $\theta$ is obtuse and that $\sin\theta = \frac{2}{3}$, find the value of $\frac{\tan\theta}{1-\cos\theta}$. Leave your answer in the from $a\sqrt{b}+c$, where $a$, $b$ and $c$ are constants.

*[5 marks]*

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**9** *[Maximum mark: 6]*

(a) It is given that the graph of $y = -x^2 + (k+2)x + (k-1)$ touches the $x$ – axis at only one point. Find the possible values of $k$.

*[3 marks]*

(b) Given that $\frac{3}{x^2 + 5x - 14} < 0$, find the range of values of $x$.

*[3 marks]*

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**10** *[Maximum mark: 5]*

If $\alpha^2\beta$ and $\beta^2\alpha$ are the roots of the equation $x^2 - 10x - 8 = 0$, find the quadratic equation with the roots, $\alpha$ and $\beta$.

**11** *[Maximum mark: 9]*

**Answer the whole of this question on a sheet of graph paper.**

The variables $x$ and $y$ are connected by the equation $y = x + \frac{3}{x} - 1$, $x > 0$. The table below shows some values of $x$ and the corresponding values of $y$, correct to 1 decimal place.

| $x$ | 0.5 | 1 | 2 | 3 | 4 | 5 | 6 | 7 |
|---|---|---|---|---|---|---|---|---|
| $y$ | 5.5 | 3 | 2.5 | $n$ | 3.8 | 4.6 | 5.5 | 6.4 |

(a) Calculate the value of $n$.

*[1 mark]*

(b) Using a scale of 2 cm to represent 1 unit on the $x$ – axis and 2 cm to represent 1 unit on the $y$ – axis, draw the graph of $y = x + \frac{3}{x} - 1$ for $0.5 \le x \le 7$.

*[3 marks]*

(c) The roots of the equation $ax^2 + bx + c = 0$ is given by the x – coordinate of the point of intersection of the curve $y = x + \frac{3}{x} - 1$ and the line $y = 2x + 4$. Find the values of $a$, $b$ and $c$.

*[3 marks]*

(d) Using the same axes, draw the graph of $y = 2x + 4$ and hence write down the solution(s) of the equation $ax^2 + bx + c = 0$ in (c).

*[2 marks]*

********** END OF PAPER 1 **********