Candidate Index Number

Anglo - Chinese School

(Independent)

FINAL EXAMINATIONS 2018
YEAR 3 INTEGRATED PROGRAMME
CORE MATHEMATICS
PAPER 1

WEDNESDAY **3rd OCTOBER 2018** **1 h 30 min**

Candidates answer on the Question Paper.
No additional materials are required.

INSTRUCTIONS TO CANDIDATES

- Write your index number in the boxes above.
- Do not open this examination paper until instructed to do so.
- You are not permitted access to any calculator for this paper.
- Answer all questions in the spaces provided.
- Unless otherwise stated in the question, all numerical answers must be given exactly or correct to three significant figures.
- The maximum mark for this paper is 80.

For Examiner's Use

This paper consists of 16 printed pages.

[Turn over

Full marks are not necessarily awarded for a correct answer with no working. Answers must be supported by working and/or explanations. Where an answer is incorrect, some marks may be given for a correct method, provided this is shown by written working. You are therefore advised to show all working.

*Answer **all** the questions in the spaces provided.*

1 *[Maximum mark: 8]*

(a) Evaluate $3-\dfrac{5}{3-\dfrac{5}{3-\dfrac{5}{3}}}$. *[3 marks]*

(b) Make x the subject of the formula, $y=\dfrac{2-x}{x+1}$. *[2 marks]*

(c) Factorize $(x+y)^2-x^2+y^2$ completely. *[3 marks]*

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2 *[Maximum mark: 7]*

(a) Given that $-4 \le x \le -\frac{1}{2}$ and $-1 \le y \le 7$, and x and y are integers.

Find

(i) the smallest possible value of $y^3 - x$, *[1 mark]*

(ii) the greatest possible value of $\frac{x}{2y}$. *[1 mark]*

(b) Solve $x - \frac{1}{2} < \frac{6x-1}{4} \le x + \frac{3}{2}$ and state the largest integer that satisfies the inequality.

[5 marks]

3 *[Maximum mark: 6]*

Solve the simultaneous equations $4^{x-4} \div 32^{y} = 16^{\frac{1}{x}}$,

$7^{x} \times \sqrt{7^{y}} = 5^{\log_3 1}$.

4 *[Maximum mark: 6]*

(a) Simplify $\left[\left(2x^{-1}\right)^{-2}\right]^{-3} \div \left[\sqrt[3]{\frac{x^9}{64}}\right]^{-2}$. *[3 marks]*

(b) Solve for x if $29-3\left(10^{2x}\right)=3+10^{2x+1}$. Leave your answer in the form of $a \lg b$ where a and b are rational numbers. *[3 marks]*

5 *[Maximum mark: 6]*

It is given that x and y are related by the equation $x - q = \frac{px}{y}$ where p and q are constants.

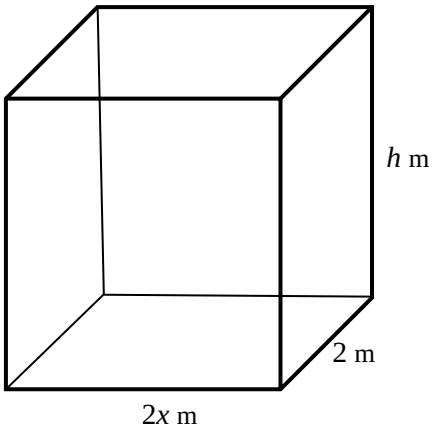
When y is plotted against $\frac{y}{x}$, a straight-line graph is obtained. The line has gradient 3 and it passes through $(1, 5)$.

(a) Find the value of p and of q. *[4 marks]*

(b) Find the value of y when $2x - y = 0$. *[2 marks]*

6 *[Maximum mark: 7]*

A metal wire, of length 64 m, is to be bent to form the frame of a rectangular box as shown below.



Given that the dimensions of the box are $2x$ m $\times$ 2 m $\times$ h m,

(a) express h in terms of x, *[2 marks]*

(b) express the volume V m^3 of the box in terms of x, *[2 marks]*

(c) find the maximum volume of the box and the value of x at which it occurs. *[3 marks]*

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7 *[Maximum mark: 9]*

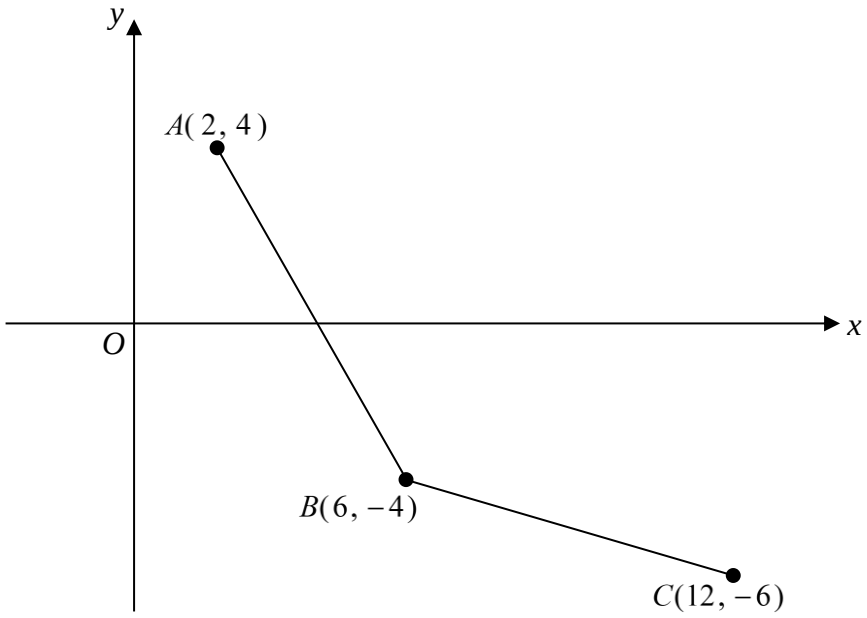
The points $A(2, 4)$, $B(6, -4)$ and $C(12, -6)$ are the three vertices of a parallelogram *ABCD*.
E is a point at the foot of the perpendicular from *B* to *AC*.
Find

(a) the coordinates of *D*, *[1 mark]*

(b) the equation of *AC*, *[2 marks]*

(c) the coordinates of *E*, *[3 marks]*

(d) the equation of the line, *l*, which is perpendicular to the line *AB* and whose points are each equidistant from *A* and from *B*. *[3 marks]*



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8 *[Maximum mark: 10]*

(a) Find the range of values of *k* if $4x^2 - 3kx > -9$ for all real values of *x*. *[4 marks]*

(b) The equation $2x^2 - x - 2 = 0$ has roots α and β. Find

(i) the value of $\alpha + \beta$ and of $\alpha\beta$, *[2 marks]*

(ii) the quadratic equation whose roots are α^2 *and* β^2. *[4 marks]*

9 *[Maximum mark: 8]*

Given that $\cos A = -\frac{1}{3}$ and $90^o \le A \le 180^o$, find the value of

(a) $\sin A$, *[2 marks]*

(b) $\tan\left(180^\circ - A\right)$, *[2 marks]*

(c) $\frac{3\sin A - \sqrt{2}\tan A}{3 + \tan A}$, leaving your answer in the form $a + b\sqrt{2}$ where *a* and *b* are constants. *[4 marks]*

10 *[Maximum mark: 13]*

(a) Given that $\log_y\left[\lg x+\left(\frac{6\lg 81}{\lg 27}\right)\right]=3\lg 10$, express x in terms of y. *[3 marks]*

(b) Solve the following equations

(i) $x^{1+\lg x}=10x$ *[5 marks]*

(ii) $\frac{\log_2\left(9-2^x\right)}{3-x}=1$ *[5 marks]*

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End of Paper 1

Answers:

1a. $\frac{29}{3}$ **1b.** $x=\frac{2-y}{y+1}$ **1c.** $(x+y)(2y)$

2ai. 0 **2aii.** 2 **2b.** 3

3. $(-\frac{1}{3}, \frac{2}{3})$ and $(1, -2)$

4a. 4 **4b.** $\frac{1}{2}\lg 2$

5a. $q=3$ and $p=2$ **5b.** 8

6a. $h=14-2x$ **6b.** $56x-8x^2$ **6c.** V_{max} = **98 m³** at $x = \frac{7}{2}$ m.

7a. $D(8, 2)$ **7b.** $y=-x+6$ **7c.** $E(8,-2)$ **7d.** $y=\frac{1}{2}x-2$

8a. $-4<k<4$ **8bi.** $\alpha+\beta=\frac{1}{2}$; $\alpha\beta=-1$ **8bii.** $4x^2-9x+4=0$

9a. $\frac{2\sqrt{2}}{3}$ **9b.** $2\sqrt{2}$ **9c.** $20+14\sqrt{2}$

10a. $x=10^{y^3-8}$ **10bi.** $x=10$ or $\frac{1}{10}$ **10bii.** $x=0$ or 3(NA)