

2024 Sec 4/5 Prelims EM Marking Scheme

<b>1a</b>	$4m + 6n = 58 \text{-----} (1)$ $3m + 5n = 46 \text{-----} (2)$ $12m + 18n = 174 \text{----} (3) \quad \left. \vphantom{\begin{matrix} (3) \\ (4) \end{matrix}} \right\}$ $12m + 20n = 184 \text{----} (4)$ $(3) - (4)$ $-2n = -10$ $n = 5$ $m = 7$
<b>b</b>	$\frac{3x}{2x^2 - 50} - \frac{1}{x - 5}$ $= \frac{3x}{2(x - 5)(x + 5)} - \frac{1}{x - 5}$ $= \frac{3x - 2(x + 5)}{2(x - 5)(x + 5)}$ $= \frac{x - 10}{2(x - 5)(x + 5)}$
<b>ci</b>	-5
<b>ii</b>	$x = \sqrt[3]{\frac{25y}{7z + 2}}$ $x^3 = \frac{25y}{7z + 2}$ $x^3(7z + 2) = 25y$ $y = \frac{x^3(7z + 2)}{25}$
<b>d</b>	$\frac{2x + 3}{(3x + 1)(x - 1)} - 2 = 0$ $\frac{2x + 3}{(3x + 1)(x - 1)} = 2$ $2x + 3 = 2(3x + 1)(x - 1)$ $2x + 3 = 6x^2 - 4x - 2$ $6x^2 - 6x - 5 = 0$ $x = 1.54 \quad \text{or} \quad -0.541$
<b>2ai</b>	(a) 40.5
	(b) 35.5
	(c) $42 - 33 = 9$

<b>ii</b>	$\frac{90 - 69}{100} \times 100\%$ $= 21\%$
<b>bi</b>	$\left(\frac{5}{20} \times \frac{9}{19}\right) + \left(\frac{9}{20} \times \frac{5}{19}\right)$ $= \frac{9}{38}$
<b>ii</b>	$\left(\frac{6}{20} \times \frac{14}{19} \times \frac{13}{18}\right) + \left(\frac{14}{20} \times \frac{6}{19} \times \frac{13}{18}\right) + \left(\frac{14}{20} \times \frac{13}{19} \times \frac{6}{18}\right)$ $= \frac{91}{190}$
<b>3a</b>	$20000 \times 4 = 80000$ $\frac{80000}{40} \times 100$ $= \$200000$
<b>b</b>	$I = \frac{(55000)(0.97)(3)}{100}$ $= \$1600.50$ $1600.50 + \left(\frac{10}{100} \times 55000\right)$ $= \$7100.50$ $A = 55000 \left(1 + \frac{3.78}{100}\right)^3$ $= \$61475.72916$ $I = 61475.72916 - 55000$ $= \$6475.73$ <p>Wayne should choose package A as it will generate a higher interest.</p>
<b>c</b>	$(150 \times 7) + (380 \times 6)$ $= 3330 \text{AUD}$ $\left(\frac{3330}{0.91} \times 1.02\right)$ $= 3732.527$ $\approx \$3733$

<b>4a</b>	<p>Let <math>\angle QEC = \theta</math></p> <p><math>\angle QEC = \angle ECD = \theta</math> (Alternate angles are equal)</p> <p><math>\angle AEF = \angle QEC = \theta</math> (Angle bisector)</p> <p><math>\angle CED = 180 - 2\theta</math> (Sum of angles on a straight line)</p> <p><math>\angle EDC = 180 - (180 - 2\theta) - \theta = \theta</math> (Sum of angles in a triangle)</p> <p>Since <math>\angle ECD = \angle EDC</math>, <math>EC = ED</math></p>
<b>b</b>	<p><math>\angle AFE = \angle QFC</math> (Vertically opposite angles are equal)</p> <p><math>\angle ECF = 90 - \theta</math> (Angle in a semi-circle)</p> <p><math>\angle FCQ = 90 - (90 - \theta) = \theta</math> (Tangent perpendicular to radius)</p> <p><math>\angle AEF = \angle FCQ = \theta</math></p> <p>Triangle <math>AEF</math> and <math>QCF</math> are similar. (AA)</p>
<b>c</b>	<p><math>\angle CED = 360 - 2(61) - 180 = 58</math></p> <p>Area of sector <math>ECD = \frac{58}{360} \times \pi(14)^2</math>  <math>= 99.20451468</math></p> <p>Area of triangle <math>ECD = \frac{1}{2}(14)^2 \sin 58</math>  <math>= 83.10871342</math></p> <p>Area of segment <math>CD = 99.20451468 - 83.10871342</math>  <math>= 16.0958</math>  <math>\approx 16.1</math></p>
<b>5a</b>	<p><math>192 - 180 = 12</math></p> <p><math>\angle ABC = 137 - 12</math>  <math>= 125</math> (shown)</p>
<b>b</b>	<p><math>AC^2 = 35^2 + 68^2 - 2(35)(68) \cos 125</math></p> <p><math>AC^2 = 8579.223037</math></p> <p><math>AC = 92.62409966</math>  <math>\approx 92.6</math></p>
<b>c</b>	<p><math>\angle DBC = 125 - 85 = 40</math></p> <p><math>\angle BDC = 180 - 40 - 70 = 70</math></p> <p><math>\frac{DC}{\sin 40} = \frac{68}{\sin 70}</math></p> <p><math>DC = 46.5147</math>  <math>\approx 46.5</math></p>

<b>d</b>	$\text{Area of triangle} = \frac{1}{2}(68)^2 \sin 40$ $= 1486.124954$ $1486.124954 = \frac{1}{2} \times 46.5147 \times h$ $h = 63.89915247$ $\approx 63.9$
<b>e</b>	$\tan 40.5 = \frac{h}{68}$ $h = 58.07748661$ $\tan \theta = \frac{58.07748661}{63.89915247}$ $\theta = 42.3$
<b>6a</b>	$\overrightarrow{AD} = 6\mathbf{p}$ $\overrightarrow{DB} = \overrightarrow{DA} + \overrightarrow{AB}$ $= -6\mathbf{p} + 6\mathbf{p} + 4\mathbf{q}$ $= 4\mathbf{q}$ $\overrightarrow{DE} = 2\mathbf{q}$
<b>b</b>	$\overrightarrow{AE} = \overrightarrow{AD} + \overrightarrow{DE}$ $= 6\mathbf{p} + 2\mathbf{q}$ $\overrightarrow{AG} = 3\mathbf{p} + \mathbf{q}$
<b>c</b>	$\overrightarrow{BG} = \overrightarrow{BA} + \overrightarrow{AG}$ $= -6\mathbf{p} - 4\mathbf{q} + 3\mathbf{p} + \mathbf{q}$ $= -3\mathbf{p} - 3\mathbf{q}$
<b>d</b>	$\overrightarrow{GF} = \overrightarrow{GA} + \overrightarrow{AF}$ $= -3\mathbf{p} - \mathbf{q} + 2\mathbf{p}$ $= -\mathbf{p} - \mathbf{q}$ $\overrightarrow{BG} = 3\overrightarrow{GF}$ $\overrightarrow{BG} \text{ is parallel to } \overrightarrow{GF} \text{ and there is a common point } G;$ <p>therefore <math>B</math>, <math>G</math> and <math>F</math> lie on a straight line.</p>

<b>e</b>	$\frac{\text{the area of triangle } ABF}{\text{the area of triangle } ABD} = \frac{1}{3}$ $\frac{\text{the area of triangle } ABD}{\text{the area of parellogram } ABCD} = \frac{1}{2} = \frac{3}{6}$ $\frac{\text{the area of triangle } ABF}{\text{the area of parellogram } ABCD} = \frac{1}{6}$
<b>7a</b>	-1
<b>c</b>	From the graph, there is only one intersection point between the line $y = k$ and the curve for some values of $k$ hence there is only one solution for some values of $k$ .
<b>di</b>	$x^3 - 9x - 4 = 0$ $\frac{x^3}{4} - \frac{9x}{4} - 1 = 0$ $\frac{x^3}{4} - 2x + 1 = \frac{1}{4}x + 2$ $y = \frac{1}{4}x + 2$
<b>ii</b>	$x = -0.45$ or $-2.75$
<b>8a</b>	$T_5 = (5 \times 7) + 24 = 59$
<b>b</b>	$n(n+2) + 4n + 4$ $= n^2 + 2n + 4n + 4$ $= n^2 + 6n + 4$
<b>c</b>	$T_{p-1} = (p-1)^2 + 6(p-1) + 4$ $= p^2 - 2p + 1 + 6p - 6 + 4$ $= p^2 + 4p - 1$ $(p^2 + 4p - 1) + (p^2 + 6p + 4)$ $= 2p^2 + 10p + 3$
<b>d</b>	$2p^2 + 10p + 3 = 303$ $2p^2 + 10p - 300 = 0$ $p = 10 \text{ or } -15 \text{ (NA)}$
<b>9a</b>	$115 + 5500 + 4000$ $= \$9615$

<b>b</b>	$\frac{x}{70} + \frac{x+32}{80} = 4\frac{3}{20}$ $\frac{80x + 70(x+32)}{5600} = \frac{83}{20}$ $\frac{150x + 2240}{5600} = \frac{83}{20}$ $150x + 2240 = 23240$ $x = 140$ $\text{Total distance} = 140 + 140 + 32 = 312$ $\text{Amount of fuel used} = \frac{312}{100} \times 8.8 = 27.456$ $\text{Cost} = (27.456 \times 1.70) + 350 + 90 = 486.6752$ $\frac{486.6752}{100} \times 1.20 = \$584.01024$ <p>Jim is incorrect as the tips included is less than 20% of the amount he should be paying.</p>
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