Anderson Junior College
Preliminary Examination 2007
H2 Mathematics Paper 2

Section A: Pure Mathematics (40 marks)

1a) Differentiate $\tan^{-1}(\ln x^3)$ with respect to *x*. [2]

b) The curve *C* has parametric equations $x = \frac{2}{t} + t^2, \quad y = t^2 - t + 1$, where *t* is a non-zero parameter.

(i) Show that the gradient of the curve at any point (*x*,*y*) satisfies the equation $\frac{dy}{dx} = \frac{(2t-1)t^2}{2t^3 - 2}$. [2]

(ii) The line $x = p$ is a tangent to the curve *C*. By using the result in (i), find the exact value of *p*. [2]

2. Express $(2r+3)$ in the form $2(r+1) + Ar + B(r-1)$, where *A* and *B* are constants.

Using the method of difference, find $\sum_{r=1}^{n}(2r+3)2^r$ in terms of *n*. [5]

Hence, or otherwise, find an expression for $\sum_{r=n}^{2n}(2r+3)2^{r-1}$ in terms of *n*. [2]

3. In an Argand diagram, the point *A* represents the fixed complex number *a*, where $0 < \arg(a) < \frac{\pi}{2}$. The complex numbers *z* and *w* are such that $|z - 2ia| = |a|$ and $|w| = |w + ia|$. Sketch, in a single diagram, the loci of the points representing *z* and *w* [3]

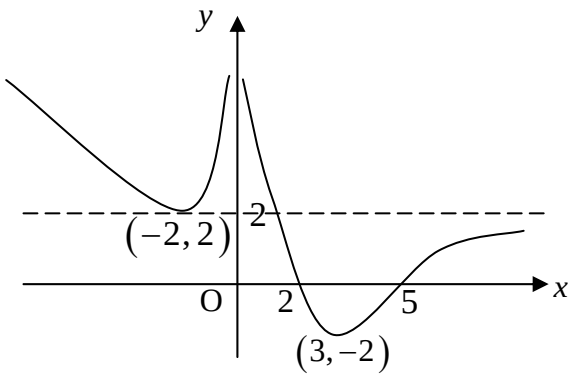
Find

a) the minimum value of $|z - w|$ in terms of $|a|$, [1]

b) the range of values of $\arg\left(\frac{1}{z}\right)$ in terms of arg(*a*). [3]

4a) By completing the square, or otherwise, describe the geometrical transformation by which the curve $x^2 - y^2 - 4y - 5 = 0$ can be obtained from the curve $x^2 - y^2 = 1$. [2]

b) The diagram below shows the graph of $y = \mathrm{f}(x)$ with asymptotes $y = 2$ and $x = 0$. The curve has turning points at (-2, 2) and (3, -2).



On separate diagrams, sketch the graphs of

(i) $y^2 = \mathrm{f}(-x)$ [3]

(ii) $y = \mathrm{f}'(x)$ [3]

Show all intercepts, asymptotes and turning points clearly on your diagrams if they can be found.

5. Relative to an origin O, the point A has position vector $\begin{pmatrix} 6 \\ 2 \\ -6 \end{pmatrix}$, the line ℓ_1 has equation $\boldsymbol{r} = \begin{pmatrix} 5 \\ 2 \\ -8 \end{pmatrix} + \lambda \begin{pmatrix} 1 \\ 2 \\ -10 \end{pmatrix}$, $\lambda \in \mathbb{R}$ and the plane Π_1 has Cartesian equation $5x - 4y + 3z = 15$.

(i) Find the shortest distance from A to the plane Π_1 and determine whether A and the origin O are on the same side or on opposite sides of Π_1. [3]

(ii) Π_2 is the plane that passes through point A and contains the line ℓ_1. Find the acute angle between the planes Π_1 and Π_2. [4]

(iii) The plane Π_3 has Cartesian equation $x + 8y + az = b$. Find the values of a and b if the planes Π_1, Π_2 and Π_3 intersect along a common line. [5]

Section B: Statistics (60 marks)

6. From the 10 letters of the word "LEGITIMISE", find

(i) the number of possible arrangements if no two vowels are next to each other, [2]

(ii) the number of 3-letter code-words that can be formed. [3]

7. In a certain country, 16% of the total population are aged 60 years or more, and 18% of the total population have myopia. Furthermore, 65% of those aged 60 years or more have myopia. Given that a person is chosen at random from the population, find the probability that the chosen person is

(i) less than 60 years old and has myopia, [2]

(ii) aged 60 years or more, given that the person does not have myopia, [3]

(iii) either aged 60 years or more, or has myopia, or both. [2]

8. A manufacturer produces pens in two colours, red and blue. On average, 35% of the pens produced are red pens.

(a) Find the probability that in a sample of 10 pens, more than half of them are red pens. [2]

(b) The manufacturer offers a special price on red pens by selling them in packs of six. Pens are selected at random to fill each pack. Given that five red pens have been selected, find the least value of n such that the probability that at most n more pens need to be selected to complete a pack of 6 red pens is greater than 0.98. [4]

9. An IT services company has two help-lines which receive calls independently and at random times. The help-line manned by Alice receives calls at a rate of 1 call per hour and the help-line manned by Brenda receives calls at a rate of 3.6 calls per hour.

(a) Find the probability that both help-lines receive a total of less than 5 calls in each of 3 successive 30-minute periods. [2]

(b) Find the probability that Alice receives at most 1 call in a randomly chosen 30-minute period, given that both help-lines receive a total of less than 5 calls. [3]

(c) The help-lines are open for 8 hours a day. By using a suitable approximation, find the probability that, in a random sample of 60 days, Alice receives less than 6 calls on at least ten but less than twenty days. [4]

10. A manufacturer claims that his slimming diet helps people lose weight. A random sample of 20 people whose original weights were 80kg took the diet for a month and their new weights, x kg, were summarized as follows:

$$\sum(x-80) = -18.7, \quad \sum(x-80)^2 = 102.5$$

(a) Assume that the population variance is not known.

(i) Test whether or not the manufacturer's claim is justified at the **5%** level of significance. You should state any assumptions that you need to make. [6]

(ii) Explain what is meant by the expression "at the 5% level of significance" in the context of this question. [1]

(b) Suppose the population variance is now known to be 10 kg. Determine the set of values of the mean weight, $\bar{x}$ kg of a random sample of 20 people which will lead to the rejection of the null hypothesis at the 5% level of significance. [2]

If the test is now conducted at the 8% level of significance, explain briefly what can be concluded about the manufacturer's claim if a random sample of 20 people yielded a sample mean weight of 78 kg. [1]

11. Large beer cans contain a volume of beer which is normally distributed with mean 500 ml and standard deviation 3.3 ml, while small beer cans contain a volume of beer which is also normally distributed with mean 340 ml and standard deviation 2.4. The volume of beer in any can is independent of the volume of beer in any other can.

(a) If 5% of the small cans produced contain more than k ml of beer each, find the value of k. [2]

(b) Find the probability that the volume of beer in two large cans differ by not more than 10 ml. [3]

(c) Beer is also sold in bottles, each of which contains four times the volume of a large can. Find the probability that a crate of six bottles contain in total more than 12040 ml of beer. [3]

A sample of n large cans and a sample of n small cans are selected. If there is a probability of at most 0.01 that the average volume of beer in a small can exceeds half the average volume of beer in a large can by more than 92 ml, find the least value of n. [4]

12(a) The marks of two short quizzes in Mathematics, x and y, of 10 students are shown in the table:

Student	**1**	**2**	**3**	**4**	**5**	**6**	**7**	**8**	**9**	**10**
Marks for 1st quiz (x)	6	5	8	8	9	6	10	4	9	6
Marks for 2nd quiz(y)	8	7	8	10	1	8	10	5	8	7

(i) Find the product-moment correlation coefficient between x and y and comment on the relationship between x and y. [2]

(ii) Plot the data on a scatter diagram of y against x. [2]

(iii) By using (ii), state with a reason whether or not your interpretation in (i) should be amended. Justify your answer. [2]

(b) Drums of hair shampoo are kept in storage for a number of weeks before being rebottled for retail sale.
To investigate the relationship between the number of weeks(x), $0 \le x \le 15$, that each drum is kept in storage and the amount of water content loss (y ml) due to evaporation, 6 drums are examined and the following results are obtained:

$$\sum x = 48, \quad \sum x^2 = 472, \quad \sum y = 408, \quad \sum y^2 = 29584, \quad \sum xy = 3664$$

(i) Find the least squares line of regression of y on x [3]

(ii) Estimate to the nearest integer, the water content loss due to evaporation for a drum kept in storage for eight weeks. [1]

(iii) Explain why you would not expect to get good estimates for evaporation loss from the line of regression when the storage time is more than a year. [1]

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