2024 RI H3 Physics Prelims Solutions & Mark Scheme

1 **(a)** **(i)** By the conservation of linear momentum

$$m_A u_A + 0 = m_A v_A + m_B v_B \quad ......(1)$$

Since the collision is elastic

$$u_A - u_B = v_B - v_A$$

$$v_A = v_B - u_A \quad .....(2)$$ M1

Sub (2) into (1)

$$m_A u_A = m_A\left(v_B - u_A\right) + m_B v_B$$

$$m_A u_A + m_A u_A = m_A v_B + m_B v_B$$

$$v_B = \frac{2m_A u_A}{\left(m_A + m_B\right)} \quad .....(3)$$ M1

When m_B reduces to zero, $m_A \gg m_B$, $\left(m_A + m_B\right) \approx m_A$

This means that the term $\frac{2m_A}{\left(m_A + m_B\right)}$ tends towards $\frac{2m_A}{m_A} = 2$. A1

Hence v_B cannot exceed $2u_A$.

(ii)

$$f = \frac{\frac{1}{2}m_B v_B^2}{\frac{1}{2}m_A u_A^2} = \frac{m_B v_B^2}{m_A u_A^2} \quad .....(4)$$

Sub [3] into [4]

$$f = \frac{m_B}{m_A u_A^2}\left[\frac{2m_A u_A}{\left(m_A + m_B\right)}\right]^2 = \frac{m_B}{m_A u_A^2}\frac{4m_A^2 u_A^2}{\left(m_A + m_B\right)^2}$$

$$= \frac{4m_A m_B}{\left(m_A + m_B\right)^2}$$ B1

(iii) **1.** Fraction of kinetic energy of A transferred to B:

$$f_{A\to B} = \frac{4m_A m_B}{\left(m_A + m_B\right)^2}$$

Fraction of kinetic energy of B transferred to C:

$$f_{B\to C} = \frac{4m_B m_C}{\left(m_B + m_C\right)^2}$$

Fraction of kinetic energy of A transferred to C:

$$F = \left(f_{A\to B}\right)\left(f_{B\to C}\right)$$

$$= \frac{4m_A m_B}{\left(m_A + m_B\right)^2}\frac{4m_B m_C}{\left(m_B + m_C\right)^2}$$

$$= \frac{16 m_A m_B^2 m_C}{\left(m_A + m_B\right)^2\left(m_B + m_C\right)^2}$$ B1

2. For the largest F, $\frac{dF}{dm_B} = 0$

$$\frac{dF}{dm_B} = \frac{m_A m_C \left[32m_B (m_A + m_B)^2 (m_B + m_C)^2 - 2(m_A + m_B)(m_B + m_C)(m_A + 2m_B + m_C) 16m_B^2 \right]}{(m_A + m_B)^4 (m_B + m_C)^4}$$ C2

$$= 0$$

$$32m_B (m_A + m_B)^2 (m_B + m_C)^2 - 32m_B^2 (m_A + m_B)(m_B + m_C)(m_A + 2m_B + m_C) = 0$$

$$(m_A + m_B)(m_B + m_C) - m_B (m_A + 2m_B + m_C) = 0$$ M1

$$m_A m_B + m_A m_C + m_B^2 + m_B m_C - m_A m_B - 2m_B^2 - m_B m_C = 0$$

$$m_A m_C - m_B^2 = 0$$

$$m_B = \sqrt{m_A m_C}$$ A1

1. **(b)** **(i)**

$$v_{CM} = \frac{mu_1 - mu_2}{m + m} = \frac{u_1 - u_2}{2}$$ B1

$$v' = v - v_{CM}$$

$$u_1' = u_1 - \frac{u_1 - u_2}{2} = \frac{u_1 + u_2}{2}$$

$$u_2' = -u_2 - \frac{u_1 - u_2}{2} = -\frac{u_1 + u_2}{2}$$ B1

$$p_{total}' = p_1' + p_2' = m\left(\frac{u_1 + u_2}{2}\right) + m\left(-\frac{u_1 + u_2}{2}\right) = 0 \text{ (shown)}$$

(ii) **1.** In the laboratory frame, in the vertical direction, momentum must be zero after the collision. Hence,

$$v_1 \sin\theta = v_2 \sin\theta$$

$$\therefore v_1 = v_2 \text{ (shown)}$$ B1

2. For momentum to remain zero in the centre-of-mass frame, the two spheres must be moving in opposite directions in that frame, with the same speed $\frac{u_1 + u_2}{2}$ as before.

And since $v' = v - v_{CM}$ and therefore $v = v' + v_{CM}$, by drawing a vector diagram, the velocities of the two spheres in the centre of mass frame must be in the vertical direction. B1

Hence, since the vectors form a right-angled triangle, B1

$$\tan\theta = \frac{\frac{u_1+u_2}{2}}{\frac{u_1-u_2}{2}} = \frac{u_1+u_2}{u_1-u_2}$$

$$\theta = 60^\circ$$

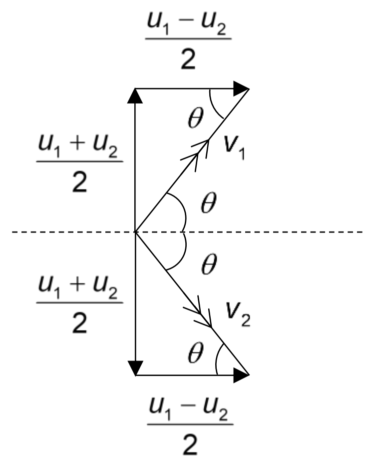
$$\tan 60^\circ = \frac{u_1+u_2}{u_1-u_2} = \sqrt{3}$$

$$u_1 + u_2 = \sqrt{3}\left(u_1 - u_2\right)$$

$$\left(\sqrt{3}-1\right)u_1 = \left(\sqrt{3}+1\right)u_2$$

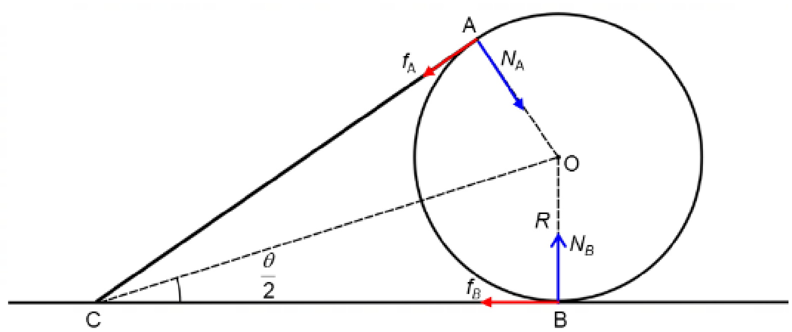
$$\frac{u_1}{u_2} = \frac{\sqrt{3}+1}{\sqrt{3}-1}$$

$$\therefore \frac{u_1}{u_2} = \frac{\alpha+1}{\alpha-1} \text{ where } \alpha = \sqrt{3}$$



B1

2 **(a)** Draw and label the normal contact forces N_A and N_B, and the frictional forces f_A and f_B correctly. B1



(b) Since the hoop is in equilibrium, taking moments about its centre O,

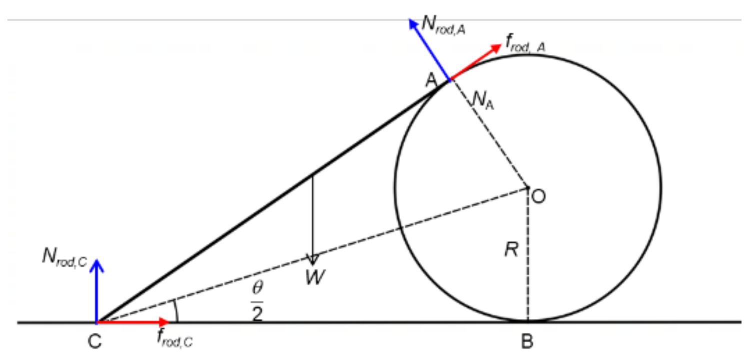
$$\sum \tau = 0$$

$$f_A R = f_B R$$ B1

$$\therefore f_A = f_B$$ A0

(c) Let $N_{rod,A}$ and $F_{rod,A}$ be the normal contact force and frictional force of the hoop on the stick.

Let $N_{rod,C}$ and $F_{rod,C}$ be the normal contact force and frictional force of the ground on the stick.



By Newton's third law, $|N_A| = |N_{rod,A}|$ and $|f_A| = |f_{rod,A}|$. B1

For the stick, taking moments about C,

$$W(\frac{L}{2}\cos\theta) = mg(\frac{L}{2}\cos\theta) = N_{rod,A} \times L$$ M1

$$N_{rod,A} = N_{\mathrm{A}} = \frac{1}{2}mg\cos\theta$$ A0

(d) Resolving horizontally for forces on the hoop,
$f_A\cos\theta + f_B = N_A\sin\theta$

Since $f_A = f_B$

$$f_B\cos\theta + f_B = \left(\frac{1}{2}mg\cos\theta\right)\sin\theta$$

$$f_B = \frac{\left(\frac{1}{2}mg\cos\theta\right)\sin\theta}{1+\cos\theta}$$ M1

Since $\tan\frac{\theta}{2} = \frac{R}{L}$ and using the identity $\tan\frac{\theta}{2} = \frac{\sin\theta}{(1+\cos\theta)}$

$$\frac{\sin\theta}{(1+\cos\theta)} = \frac{R}{L}$$

$$(1+\cos\theta) = \frac{L\sin\theta}{R}$$

Thus,

$$f_B = \frac{\left(\frac{1}{2}mg\cos\theta\right)\sin\theta}{\frac{L\sin\theta}{R}} = \frac{1}{2}\frac{mgR}{L}\cos\theta$$ A1

3 **(a)** Since the ring is smooth, the tensions in the left (of ring) and right (of ring) sections of the string are the same.

Resolving forces horizontally,

$$T\sin 73^0 - T\sin 51^0 = \frac{mv^2}{r} \quad \text{—(1)}$$ M1

Resolving forces vertically,

$$T\cos 73^0 + T\cos 51^0 = mg \quad \text{—(2)}$$ M1

$\frac{(1)}{(2)}$ $\frac{\sin 73^0 - \sin 51^0}{\cos 73^0 + \cos 51^0} = \frac{v^2}{250 \times 9.81}$

$v = 21.8 \text{ m s}^{-1}$ A1

(b) $a_c = \frac{v^2}{r} = \frac{21.83^2}{250} = 1.91 \text{ m s}^{-2}$ B1

(c) Although the speed of the car is constant, its velocity is not as its direction is constantly changing. Hence the car is accelerating and there is a resultant force acting on it. B1

The resultant force is directed towards the centre of the circle, hence it is always perpendicular to the velocity of the car.
As a result, no work is done by this force and hence there is no change in the kinetic energy of the car. B1

4 **(a)** When the platform is at displacement y from the equilibrium position, resultant force on the platform is

$$-(2k_1 y + k_2 y) = ma$$

$$a = -\left(\frac{2k_1 + k_2}{m}\right) y$$ M1

Since $\frac{2k_1 + k_2}{m}$ is constant, acceleration $a \propto -y$. M1

This satisfies the definition for simple harmonic motion where the angular frequency $\omega = \sqrt{\frac{2k_1 + k_2}{m}}$.

Hence the platform and the ball oscillate in simple harmonic motion.

(b) **(i)** At the equilibrium position of the three-springs system, extension of each spring is e.

$$2k_1 e = k_2 e + mg$$

$$e = \frac{mg}{2k_1 - k_2}$$ M1

After bottom spring breaks, at the equilibrium position of the two-springs system, extension of each spring above the platform is e'.

$$2k_1 e' = mg$$

$$e' = \frac{mg}{2k_1}$$ M1

The amplitude of oscillation of the two-springs system in SHM,

$y'_0 =$ distance from top support where spring breaks – distance from top support to equilibrium of two-springs system

$$y'_0 = (L + e + y_0) - (L + e')$$

$$= \frac{mg}{2k_1 - k_2} + y_0 - \frac{mg}{2k_1}$$

$$= mg\left(\frac{1}{2k_1 - k_2} - \frac{1}{2k_1} + \frac{y_0}{mg}\right)$$ A0

(ii) Maximum speed of the platform is at the equilibrium position.

$$|v_{\max}| = \omega' y'_0$$

$$= \sqrt{\frac{2k_1}{m}}\left(mg\left(\frac{1}{2k_1 - k_2} - \frac{1}{2k_1} + \frac{y_0}{mg}\right)\right)$$

$$= g\sqrt{2mk_1}\left(\frac{1}{2k_1 - k_2} - \frac{1}{2k_1} + \frac{y_0}{mg}\right)$$ A1

(iii)

$$|a_{\max}| = (\omega')^2 y'_0$$

$$= \frac{2k_1}{m}\left(mg\left(\frac{1}{2k_1 - k_2} - \frac{1}{2k_1} + \frac{y_0}{mg}\right)\right)$$

$$= 2k_1 g\left(\frac{1}{2k_1 - k_2} - \frac{1}{2k_1} + \frac{y_0}{mg}\right)$$

Ball will lose contact with the platform if

$$|a_{\max}| \geq g$$

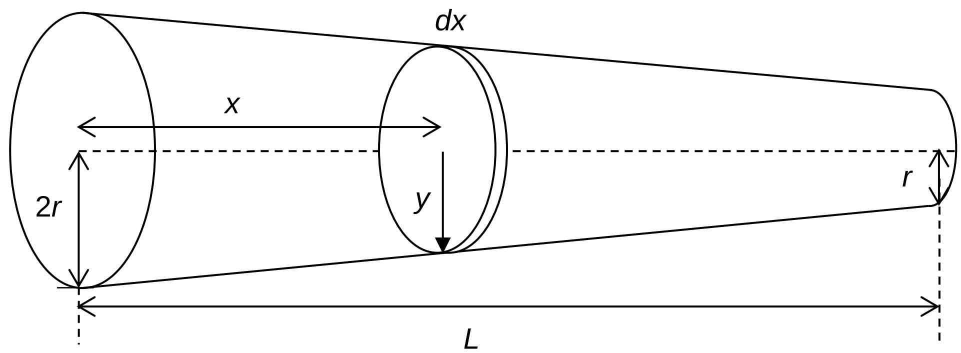
$$2k_1 g\left(\frac{1}{2k_1 - k_2} - \frac{1}{2k_1} + \frac{y_0}{mg}\right) \geq g$$ M1

$$\frac{2k_1}{2k_1 - k_2} - \frac{2k_1}{2k_1} + \frac{2k_1 y_0}{mg} \geq 1$$

$$\frac{2k_1}{2k_1 - k_2} + \frac{2k_1 y_0}{mg} \geq 2$$

$$\frac{k_1}{2k_1 - k_2} + \frac{k_1 y_0}{mg} \geq 1 \quad \text{(shown)}$$ A0

5 **(a)**



The radius of the cone decreases linearly with the length of the conductor.

Consider a section of the cone of radius *y*, and thickness *dx*, located a distance *x* from the left end of the cone as shown in the diagram above.

The radius *y* can be expressed as

$$y = 2r - \left(\frac{2r - r}{L}\right)x = 2r - \frac{r}{L}x$$ C1

The resistance across opposite sides of this small section is

$$dR = \frac{\rho\, dx}{\pi y^2} = \frac{\rho\, dx}{\pi\left(2r - \frac{r}{L}x\right)^2}$$ C1

Therefore, the total resistance across the truncated cone is

$$R = \frac{\rho}{\pi}\int_0^L \frac{dx}{\left(2r - \frac{r}{L}x\right)^2}$$

$$= \frac{\rho}{\pi}\left(\frac{L}{r}\right)\left[\frac{1}{2r - \frac{r}{L}x}\right]_0^L$$ M1

$$= \frac{\rho L}{\pi r}\left(\frac{1}{r} - \frac{1}{2r}\right)$$

$$= \frac{\rho L}{2\pi r^2}$$

(b)

Original volume of cone is $V = \frac{1}{3}\pi\left(2r\right)^2\left(2L\right) - \frac{1}{3}\pi r^2 L = \frac{7}{3}\pi r^2 L$

Volume of hollow cylinder is $V' = \pi\left(2r\right)^2 L' - \pi r^2 L' = 3\pi r^2 L'$

Since $V = V'$

$\frac{7}{3}\pi r^2 L = 3\pi r^2 L'$ C1

$L' = \frac{7}{9}L$

$R' = \frac{\rho L'}{\pi\left(2r\right)^2 - \pi r^2} = \frac{\rho L'}{3\pi r^2} = \frac{7\rho L}{27\pi r^2}$ B1

$\frac{R}{R'} = \frac{\frac{\rho L}{2\pi r^2}}{\frac{7\rho L}{27\pi r^2}} = \frac{27}{14} = 1.93$ A1

6 **(a)** To accelerate the particles, adjacent tubes must have opposite polarities. Their polarity must change continuously at a constant frequency to synchronize with the movement of particles from one tube to the next. B1

This means that the period T of the alternating voltage supply must be constant, and equal to twice the time interval that the particle takes to travel through each tube. B1

Since the velocity of the particle increases as it is being accelerated from one tube to the next, the tube length must increase proportionately to the increase in velocity of the particle, so as to keep the traverse time through each tube the same. B1

(b) By energy conservation,

gain in KE of particle = loss in electric PE of particle

$\frac{1}{2}mv^2 - \frac{1}{2}mu^2 = (10)Vq$ M1

$\frac{m}{q} = \frac{10(V)(2)}{v^2 - u^2} = \frac{(10)(5.0\times10^4)}{(1.0\times10^7)^2 - (2.0\times10^6)^2}$

$\frac{q}{m} = 9.6\times10^7\ \mathrm{C\,kg^{-1}}$ M1

For electron, $\dfrac{q}{m} = \dfrac{1.60 \times 10^{-19}}{9.11 \times 10^{-31}} = 1.76 \times 10^{11}\ \text{C kg}^{-1}$

For proton, $\dfrac{q}{m} = \dfrac{1.60 \times 10^{-19}}{1.67 \times 10^{-27}} = 9.58 \times 10^{7}\ \text{C kg}^{-1}$ B1

For alpha-particle, $\dfrac{q}{m} = \dfrac{2 \times 1.60 \times 10^{-19}}{4 \times 1.67 \times 10^{-27}} = 4.82 \times 10^{7}\ \text{C kg}^{-1}$

Hence the particle is a proton. A1

(c) Since the charged particles have the same charge, they will repel one another, and they will deviate from the (original straight line) direction of the beam. B1
Therefore, they need to be focussed back into the original (or intended) direction of the beam.

7 **(a)** Conservation of momentum:
$mv = h / \lambda$
$v = h / m\lambda$
B1

where h is the Planck constant.

(b)

$$K / E = \frac{\frac{1}{2}mv^2}{hc / \lambda}$$

$$= \frac{\frac{1}{2}m(h / m\lambda)^2}{hc / \lambda}$$

M1

$$= \frac{h}{2mc\lambda}$$

For K / E to be << 1, need <u>large mass</u> and <u>long wavelength</u>. A1

(b) $E = hc / \lambda$

$$\lambda = \frac{(6.63 \times 10^{-34})(3.00 \times 10^{8})}{(10.2)(1.60 \times 10^{-19})} = 1.22 \times 10^{-7}\ \text{m}$$

$$K / E = \frac{h}{2mc\lambda}$$ M1

$$= \frac{6.63 \times 10^{-34}}{2(1.67 \times 10^{-27})(3.00 \times 10^{8})(1.22 \times 10^{-7})}$$

A1

$$= 5.42 \times 10^{-9}$$

since K / E << 1, recoil of atom may be ignored. B1

8 **(a)** Since $E = \text{KE} + \text{GPE}$

$$\therefore E = \frac{1}{2}mv^2 - \frac{GMm}{r}$$ M1

But $v^2 = v_r^2 + v_t^2$

$$\therefore E = \frac{1}{2}mv_r^2 + \frac{1}{2}mv_t^2 - \frac{GMm}{r} = \frac{1}{2}mv_r^2 + U_{\text{eff}}$$ M1

$$\therefore U_{\text{eff}} = \frac{1}{2}mv_t^2 - \frac{GMm}{r}$$

$L = mv_t r$

$$\therefore v_t = \frac{L}{mr} \Rightarrow v_t^2 = \frac{L^2}{m^2r^2}$$ M1

$$\therefore U_{\text{eff}} = \frac{1}{2}m\left(\frac{L^2}{m^2r^2}\right) - \frac{GMm}{r} = \frac{1}{2}\frac{L^2}{mr^2} - \frac{GMm}{r}$$ A0

(b) At an apse, the velocity is perpendicular to the radius vector *r*, hence $v_r = 0$.

$\therefore E = U_{\text{eff}}$ B1

(c) Both total energy *E* and angular momentum *L* are constant.

At the apses,

$$E = U_{\text{eff}} = -\frac{GMm}{r_p} + \frac{L^2}{2mr_p^2} = -\frac{GMm}{r_a} + \frac{L^2}{2mr_a^2}$$ B1

$$\therefore \frac{L^2}{2mr_p^2} - \frac{L^2}{2mr_a^2} = \frac{GMm}{r_p} - \frac{GMm}{r_a} \Rightarrow \frac{L^2}{2m}\left(\frac{1}{r_p^2} - \frac{1}{r_a^2}\right) = GMm\left(\frac{1}{r_p} - \frac{1}{r_a}\right)$$

$$\Rightarrow \frac{L^2}{2m}\left(\frac{1}{r_p} - \frac{1}{r_a}\right)\left(\frac{1}{r_p} + \frac{1}{r_a}\right) = GMm\left(\frac{1}{r_p} - \frac{1}{r_a}\right) \Rightarrow \frac{L^2}{2m} = \frac{GMm}{\left(\frac{1}{r_p} + \frac{1}{r_a}\right)} = \frac{GMmr_pr_a}{r_a + r_p}$$ B1

$$\therefore \text{The total energy } E = -\frac{GMm}{r_p} + \frac{L^2}{2mr_p^2} = -\frac{GMm}{r_p} + \frac{GMmr_pr_a}{(r_a + r_p)r_p^2}$$

$$= -\frac{GMm}{r_p}\left(1 - \frac{r_a}{(r_a + r_p)}\right) = -\frac{GMm}{r_p}\left(\frac{r_p}{(r_a + r_p)}\right)$$

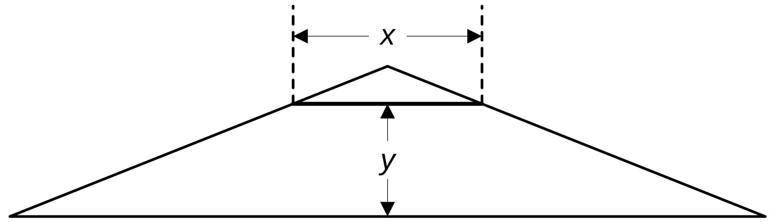
$$= -\frac{GMm}{(r_a + r_p)}$$ B1

$$= -\frac{GMm}{2a}$$ A0

9 **(a)** **(i)** Mass of plate

$$m = \frac{1}{2}\sigma bh$$ B1

(ii) Method 1



Clearly annotated diagram.

Consider a horizontal strip of length x and height dy.

By geometry,

$$\frac{x}{b} = \frac{h-y}{h}$$

$$x = \frac{b}{h}\cdot\left(h-y\right)$$ B1

Mass of strip

$$dm = \sigma x dy = \sigma \cdot \frac{b}{h}\cdot\left(h-y\right)dy$$ B1

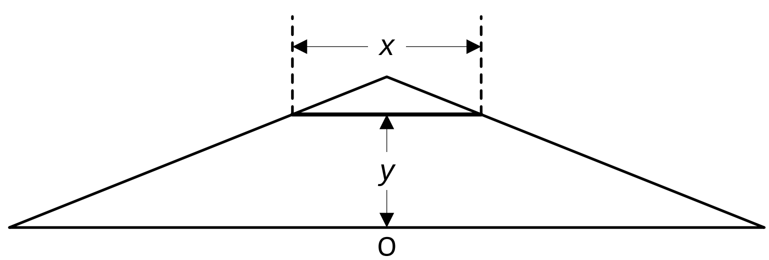
Centre of mass

$$y_{\text{CM}} = \frac{1}{m}\int_0^h y dm = \frac{1}{\frac{1}{2}\sigma bh}\cdot\sigma\cdot\frac{b}{h}\int_0^h y\left(h-y\right)dy = \frac{2}{h^2}\int_0^h hy - y^2 dy$$ M1

$$= \frac{2}{h^2}\left[\frac{1}{2}hy^2 - \frac{1}{3}y^3\right]_0^h = \frac{2}{h^2}\cdot\frac{h^3}{6} = \frac{h}{3}$$ A0

*Alternative methods such as considering strips in the vertical direction are acceptable.

(iii) **1.**



Moment of inertia of strip about its CM is

$$dI_C = \frac{1}{12} dm \cdot x^2$$ C1

Using parallel axis theorem, moment of inertia of strip about the axis perpendicular to the page and passing through the mid-point of its base is.

$$\begin{aligned}
dI &= \frac{1}{12} dm \cdot x^2 + dm \cdot y^2 \\
&= \frac{1}{12} \cdot \sigma \cdot \frac{b}{h} \cdot (h-y) dy \cdot \frac{b^2}{h^2} \cdot (h-y)^2 + \sigma \cdot \frac{b}{h} \cdot (h-y) dy \cdot y^2 \\
&= \frac{\sigma b^3}{12h^3} \cdot (h-y)^3 \, dy + \frac{\sigma b}{h} \cdot (h-y) y^2 dy \\
&= \frac{\sigma b^3}{12h^3} \cdot \left(h^3 - 3h^2 y + 3hy^2 - y^3\right) dy + \frac{\sigma b}{h} \cdot \left(hy^2 - y^3\right) dy
\end{aligned}$$ M1

Integrating from 0 to h,

$$I = \frac{\sigma b^3}{12h^3} \int_0^h \left(h^3 - 3h^2 y + 3hy^2 - y^3\right) dy + \frac{\sigma b}{h} \int_0^h \left(hy^2 - y^3\right) dy$$ M1

$$\begin{aligned}
&= \frac{\sigma b^3}{12h^3} \left[h^3 y - \frac{3}{2} h^2 y^2 + hy^3 - \frac{1}{4} y^4 \right]_0^h + \frac{\sigma b}{h} \left[\frac{1}{3} hy^3 - \frac{1}{4} y^4 \right]_0^h \\
&= \frac{\sigma b^3}{12h^3} \left[h^4 - \frac{3}{2} h^4 + h^4 - \frac{1}{4} h^4 \right] + \frac{\sigma b}{h} \left[\frac{1}{3} h^4 - \frac{1}{4} h^4 \right] \\
&= \frac{\sigma b^3}{12h^3} \left[\frac{1}{4} h^4 \right] + \frac{\sigma b}{h} \left[\frac{1}{12} h^4 \right] \\
&= \frac{\sigma b^3 h}{48} + \frac{\sigma b h^3}{12} = \frac{1}{24} m \cdot \left(b^2 + 4h^2\right)
\end{aligned}$$ A0

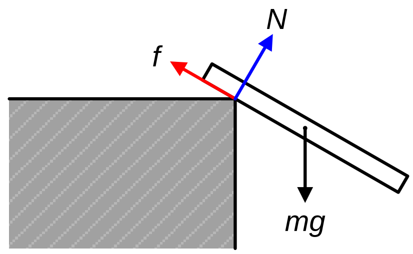
2. Using parallel axis theorem,

$$I = I_{CM} + m\left(\frac{h}{3}\right)^2$$ M1

$$I_{CM} = I - m\left(\frac{h}{3}\right)^2 = \frac{1}{24} m \cdot \left(b^2 + 4h^2\right) - \frac{1}{9} mh^2 = \frac{1}{72} m \cdot \left(3b^2 + 4h^2\right)$$ A1

*Alternative methods such as considering moment of inertia about x- and y-axes and perpendicular axis theorem are acceptable.

(b) **(i)**



B1

(ii) Moment of inertia of rod about A (axis through table edge)

$$I = \frac{1}{12}mL^2 + m\left(\frac{L}{4}\right)^2 = \frac{7}{48}mL^2$$ B1

(iii) After rotating by an angle θ, and applying the principle of conservation of energy,

loss in G.P.E. = gain in rotational K.E.

$$mg \times \frac{L}{4} \times \sin\theta = \tfrac{1}{2} I\omega^2 - 0$$

$$\Rightarrow \quad \omega = \sqrt{\frac{24g\sin\theta}{7L}}$$ B1

(iv) The angular acceleration α of the rod is given by

$$mg \times \frac{L}{4} \times \cos\theta = \frac{7}{48}mL^2\alpha$$

$$\Rightarrow \quad \alpha = \frac{12}{7} \cdot \frac{g}{L} \cdot \cos\theta$$ B1

(v) The required centripetal force $mr\omega^2$ is provided by the friction minus the component of the weight along the length of the rod.

$$\mu N - mg\sin\theta = m \cdot \frac{L}{4} \cdot \omega^2$$ ❶ B1

(r = L / 4)

(v) **1.** Resolving forces parallel to the rod,

$$F_{net} = \mu N - mg\sin\theta$$ ❶ B1

2. F_{net} is the required centripetal force $mr\omega^2$

$$F_{net} = m \cdot \frac{L}{4} \cdot \omega^2$$ ❶ B1

(r = L / 4)

(vi) Considering forces perpendicular to the rod,

$$mg\cos\theta - N = ma \qquad ❷$$ B1

(vii) Since the rod has yet to slip at A,

$$a = \frac{L}{4}\cdot\alpha$$ B1

From ❷:

$$mg\cos\theta - N = m\cdot\frac{L}{4}\cdot\alpha$$

$$\Rightarrow \quad N = mg\cos\theta - m\cdot\frac{L}{4}\cdot\alpha = mg\cos\theta - m\cdot\frac{L}{4}\cdot\frac{12}{7}\cdot\frac{g}{L}\cdot\cos\theta = \frac{4}{7}mg\cos\theta$$ B1

Substituting N into ❶,

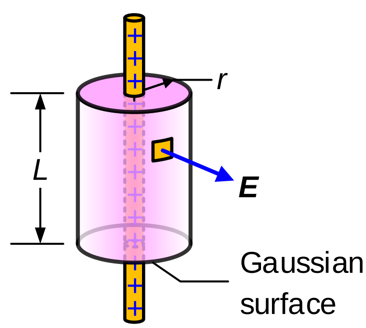
$$\mu\cdot\frac{4}{7}mg\cos\theta - mg\sin\theta = m\cdot\frac{L}{4}\cdot\frac{24g\sin\theta}{7L}$$ M1

$$\Rightarrow \quad \frac{4}{7}\mu\cos\theta - \sin\theta = \frac{6}{7}\sin\theta$$

$$\Rightarrow \quad \frac{4}{7}\mu\cos\theta = \frac{13}{7}\sin\theta$$

$$\Rightarrow \quad \tan\theta = \frac{4}{13}\mu$$ A1

10 **(a)** **(i)** Consider a cylindrical Gaussian surface of radius r and length L that is co-axial with the wire as shown.
Charge enclosed, $Q_{en} = \lambda L$
By Gauss's Law,



$$\frac{Q_{en}}{\varepsilon_o} = \oint \boldsymbol{E}\cdot d\boldsymbol{A}$$ B1

For the two flat surfaces, $\boldsymbol{E}$ is perpendicular to $d\boldsymbol{A}$. Hence, $\int \boldsymbol{E}\cdot d\boldsymbol{A} = 0$ B1

For the curved surface, $\boldsymbol{E}$ is parallel to $d\boldsymbol{A}$. Hence,

$$\int \boldsymbol{E}\cdot d\boldsymbol{A} = \int E\,dA$$ B1

$$= E\int dA \qquad (\text{since } E \text{ is constant over the curved surface})$$

$$= E(2\pi rL)$$ B1

Hence,

$$\frac{Q_{en}}{\varepsilon_0} = 0 + E\left(2\pi rL\right)$$

$$\frac{\lambda L}{\varepsilon_0} = E \cdot 2\pi rL$$

$$E = \frac{\lambda}{2\pi\varepsilon_0 r}$$ A1

(ii) **1.** $$E = \frac{\lambda}{2\pi\varepsilon_o r} + \frac{\lambda}{2\pi\varepsilon_o\left(d-r\right)}$$ A1

2. $$V = -\int_{3d/4}^{d/4}\left[\frac{\lambda}{2\pi\varepsilon_o r} + \frac{\lambda}{2\pi\varepsilon_o\left(d-r\right)}\right]dr$$ B1

$$= -\frac{\lambda}{2\pi\varepsilon_o}\left[\ln\left(\frac{r}{d-r}\right)\right]_{3d/4}^{d/4}$$ B1

$$= -\frac{\lambda}{2\pi\varepsilon_o}\left[\ln\left(\frac{d/4}{3d/4}\right) - \ln\left(\frac{3d/4}{d/4}\right)\right]$$

$$= \frac{\lambda}{2\pi\varepsilon_o}\ln 9$$ A1

Correct sign - [B1]

10 **(b)** **(i)** induced e.m.f. is equal / directly proportional to rate B1
of change of / cutting (magnetic) flux / flux linkage B1

(ii) **1.** $$\Phi = BA\cos\theta \quad \left(\begin{array}{l}\text{where } \theta \text{ is the angle between normal} \\ \text{to Earth's surface and magnetic field}\end{array}\right)$$

$$= BA\sin\alpha \quad \left(\alpha + \theta = 90^\circ\right)$$ M1

$$= \left(4.7\times10^{-5}\right)\pi\left(8.0^2\right)\sin 50^\circ$$

$$= 7.2\times10^{-3}\ \text{Wb}$$ A1

2. $$\varepsilon = -\frac{d\Phi}{dt}$$

$$\left|\varepsilon\right| = \left(7.2\times10^{-3}\right)\left(200\right) \quad \left(\because 200 \text{ revolutions per second}\right)$$ M1

$$= 1.4\ \text{V}$$ A1

3. correct reasoning, making appropriate use of Fleming's Left-Hand Rule, to conclude that the force acting on a positively charged particle is towards the outer end of the blade M1
outer end is at the higher potential A1

4. both rotor blades have the same e.m.f. induced across them OR both rotor blades have the same end at the higher potential M1
potential difference between the two ends is zero A1

11 **(a)** **(i)**

From $\omega = \frac{1}{\sqrt{LC}}$,

$$\frac{2\pi}{T} = \frac{1}{\sqrt{LC}}$$

$$T = 2\pi\sqrt{LC}$$

$$= 2\pi\sqrt{\left(10\times10^{-3}\right)\left(1.0\times10^{-6}\right)}$$

$$= 6.28\times10^{-4} \text{ s}$$ A1

(ii)

From $C = \frac{Q}{V}$,

$$Q_0 = CV$$

$$= \left(1.0\times10^{-6}\right)(50)$$ M1

$$= 5.0\times10^{-5} \text{ C}$$ A1

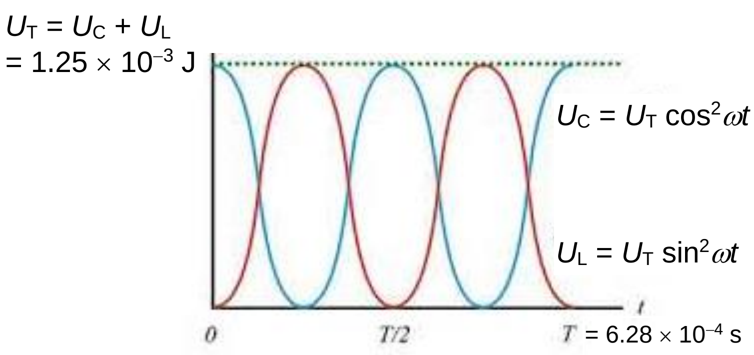
(iii)

$$U_C = \frac{1}{2}CV^2 \qquad\qquad U_C = \frac{1}{2}\frac{Q_0^{\,2}}{C}$$

$$= \frac{1}{2}\left(1.0\times10^{-6}\right)(50)^2 \qquad\qquad = \frac{1}{2}\frac{\left(50\times10^{-6}\right)^2}{1.0\times10^{-6}}$$

$$= 1.25\times10^{-3} \text{ J} \qquad\qquad = 1.25\times10^{-3} \text{ J}$$ A1

(b)



U_C starts from max and shape proportional to $\cos^2\omega t$, and correctly labelled as U_C. B1
U_L starts from 0 and shape proportional to $\sin^2\omega t$, and correctly labelled as U_L. B1
Both graphs show 2 cycles in 1 period, the same min / max values and values on the axes. B1

(c) **(i)** When the capacitor is fully discharged, all the energy is stored in the inductor. The maximum energy stored in inductor is equal to the maximum energy stored in the capacitor. Hence,

$$\frac{1}{2}LI_0^{\,2} = \frac{1}{2}\frac{Q_0^{\,2}}{C} \qquad\qquad \frac{1}{2}LI_0^{\,2} = \frac{1}{2}CV^2$$

$$I_0 = \sqrt{\frac{Q_0^{\,2}}{LC}} \qquad\qquad I_0 = \sqrt{\frac{CV^2}{L}}$$

$$= \frac{Q_0}{\sqrt{LC}} \qquad\qquad = V\sqrt{\frac{C}{L}}$$

$$= \frac{50\times10^{-6}}{\sqrt{\left(10\times10^{-3}\right)\left(1.0\times10^{-6}\right)}} \qquad\qquad = (50)\sqrt{\frac{\left(1.0\times10^{-6}\right)}{\left(10\times10^{-3}\right)}}$$ M1

$$= 0.50\text{ A} \qquad\qquad = 0.50\text{ A}$$ A1

Alternatively, maximum current I_0 can be found by applying

$$I_0 = \omega Q_0 \text{ just like } v_0 = \omega x_0 \text{ in shm}$$

$$\therefore I_0 = \omega Q_0 = \frac{1}{\sqrt{LC}}Q_0$$ M1

$$= \frac{1}{\sqrt{10\times10^{-3}\times1.0\times10^{-6}}}\times50\times10^{-6} = 0.50\text{ A}$$

A1

(ii) Maximum current occurs when the capacitor is fully discharged at time $t = T/4$. See Fig. 11.2.

Hence, time = $(6.23\times10^{-4}) / 4 = 1.57\times10^{-4}$ s. B1

(iii) When the capacitor is fully discharged, the current in the circuit is a maximum and it continues to flow to charge the capacitor.

As the <u>current through the inductor decreases, the magnetic flux density around the inductor also decreases</u>. B1

By the laws of electromagnetic induction, the <u>decrease in magnetic flux linkage of the inductor causes an e.m.f. to be induced in the inductor in a direction to prevent the current from decreasing</u>. B1

Hence a current continues to flow in the circuit until the capacitor becomes fully charged again and the current drops to zero.

(d) That same amount of energy stored in both the capacitor and the inductor is half of the initial energy stored in the capacitor.

$$\frac{1}{2}\frac{\left(Q_0 \cos \omega t\right)^2}{C} = \frac{1}{2}\left(\frac{1}{2}\frac{Q_0^{\ 2}}{C}\right)$$ M1

$$\cos \omega t = \pm\frac{1}{\sqrt{2}}$$

$$\omega t = \frac{\pi}{4}, \frac{3\pi}{4}, \frac{5\pi}{4}, \frac{7\pi}{4}$$

M1

$$\therefore t = \frac{T}{8}, \frac{3T}{8}, \frac{5T}{8}, \frac{7T}{8}$$

$$= 7.79\times10^{-5}, 2.34\times10^{-4}, 3.89\times10^{-4}, 5.45\times10^{-4}\ \text{s}$$

A1

Alternative method:

When $U_C = U_L$

$$\frac{1}{2}\frac{Q^2}{C} = \frac{1}{2}LI^2$$

$$\frac{1}{2}\frac{Q_0^{\ 2}}{C}\cos^2\omega t = \frac{1}{2}LI_0^{\ 2}\sin^2\omega t = \frac{1}{2}L\omega_0^{\ 2}Q_0^{\ 2}\sin^2\omega t$$ M1

$$\Rightarrow \tan^2\omega t = \frac{1}{LC}\times\frac{1}{\omega_0^{\ 2}} = 1$$

$$\Rightarrow \tan\omega t = \pm 1$$

M1

$$\Rightarrow \omega t = \frac{\pi}{4}, \frac{3\pi}{4}, \frac{5\pi}{4}, \frac{7\pi}{4}$$

$$\therefore t = \frac{T}{8}, \frac{3T}{8}, \frac{5T}{8}, \frac{7T}{8}$$

$$= 7.79\times10^{-5}, 2.34\times10^{-4}, 3.89\times10^{-4}, 5.45\times10^{-4}\ \text{s}$$

A1

Deduct 1m, if only 2 times were listed due to ecf from graph or otherwise.

(e) **(i)** A new resonant frequency is produced when the new capacitor is connected in series (C_S = ½C) or in parallel (C_P = 2C) to the original capacitor.

$$\text{Hence, ratio} = \frac{f_S}{f_P} = \frac{2\pi\sqrt{LC_P}}{2\pi\sqrt{LC_S}} = \sqrt{\frac{C_P}{C_S}}$$ M1

$$= \sqrt{\frac{2C}{\frac{1}{2}C}} = \sqrt{4} = 2 \quad (\text{accept ratio} = 0.5)$$

A1

(ii) With R = 200 Ω in the circuit, the new angular frequency is

$$\omega = \sqrt{\frac{1}{LC} - \frac{R^2}{4L^2}}$$
$$= \sqrt{\frac{1}{\left(10\times10^{-3}\right)\left(1.0\times10^{-6}\right)} - \frac{200^2}{4\left(10\times10^{-3}\right)^2}}$$
$$= 0$$

B1

Since ω = 0, the circuit is critically-damped. B1

Hence, sketch Q decreasing to 0 over a short duration of time t without undergoing any oscillation. B1

Show or state ω = 0 [B1], state critical damping [B1], sketch Q decay to 0 without undergoing oscillation [B1].