2023 Sec 4NA Preliminary Examinations Paper 2

1(a)	$4^{x-3} = \frac{2^{x^2}}{8^{3x+2}}$ $2^{2(x-3)} = \frac{2^{x^2}}{2^{3(3x+2)}}$ $2^{2(x-3)} = 2^{x^2-3(3x+2)}$ $2x-6 = x^2-9x-6$ $x^2-11x=0$ $x=0$ or $x=11$	1(b)	Midpoint of PQ = (–2, 7.5) Gradient of PQ = $\frac{10-5}{2-(-6)} = \frac{5}{8}$ Gradient of ⊥ bisector = $-\frac{8}{5}$ Equation of ⊥ bisector $\frac{y-7.5}{x+2} = -\frac{8}{5}$ $y = -\frac{8}{5}x + \frac{43}{10}$
2	Area of square base $= \left(\sqrt{10}+\sqrt{2}\right)^2 = 10+2\sqrt{20}+2 = 12+4\sqrt{5}$ Height $= \frac{52+28\sqrt{5}}{12+4\sqrt{5}} = \frac{52+28\sqrt{5}}{12+4\sqrt{5}} \times \frac{12-4\sqrt{5}}{12-4\sqrt{5}} = \frac{64+128\sqrt{5}}{64} = 1+2\sqrt{5}$ cm		
3(a)	$\sin A = \frac{5}{7}$ Using Pythagoras Theorem: $x = \sqrt{24}$ $\cos A = \frac{\sqrt{24}}{7} = \frac{2\sqrt{6}}{7}$ $\sin 2A = 2\sin A\cos A$ $= 2(\frac{5}{7})(\frac{2\sqrt{6}}{7})$ $= \frac{20\sqrt{6}}{49}$	3b (i)	$\sec(2x-15°) = 2$ $\cos(2x-15°) = 0.5$ Basic angle = 60° $2x-15° = 60°, 300°$ $x = 37.5°, 157.5°$
		3b (ii)	$7\sec^2 A + 6\tan A - 23 = 0$ $7(1+\tan^2 A) + 6\tan A - 23 = 0$ $7\tan^2 A + 6\tan A - 16 = 0$ $(7\tan A - 8)(\tan A + 2) = 0$ $\tan A = \frac{8}{7}$ → $A = 0.852$ (3 sf) or $\tan A = -2$ → $A = 2.03$ (3 sf)
4a (i)	$N = -t^3 + \frac{41}{2}t^2 - 60t + 500$ $\frac{dN}{dt} = -3t^2 + 41t - 60$ To find max, let $\frac{dN}{dt} = 0$ $-3t^2 + 41t - 60 = 0$ $(-3t+5)(t-12) = 0$ $t = \frac{5}{3}$ (rej) or $t = 12$	4a (ii)	$N = -(12)^3 + \frac{41}{2}(12)^2 - 60(12) + 500$ $= 1004$
		4(b)	$y = \frac{3}{x^2+4}$ $\frac{dy}{dx} = \frac{-3(2x)}{(x^2+4)^2} = \frac{-6x}{\left(x^2+4\right)^2}$ When $x > 0, \left(x^2+4\right)^2 > 0$ Therefore, $\frac{dy}{dx} < 0$ for all $x > 0$. Hence y is a decreasing function.

5(i)	$\frac{d}{dx}(1-\frac{1}{x})^5 = 5x^{-2}(1-\frac{1}{x})^4$	**5(ii)**	From (i), rewrite as $\frac{d}{dx}(1-\frac{1}{x})^5 = 5x^{-2}(1-\frac{1}{x})^4 = 5x^{-6}(x-1)^4$ $\int_1^2 x^{-6}(x-1)^4 dx$ $=\frac{1}{5}\int_1^2 5x^{-6}(x-1)^4 dx$ $=\frac{1}{5}\left[(1-\frac{1}{x})^5\right]_1^2 = \frac{1}{5}(1-\frac{1}{2})^5 = \frac{1}{160}$
6(i)	$x^2+y^2-4x+6y-12=0$ $x^2-4x+4+y^2+6y+9=12+4+9$ $(x-2)^2+(y+3)^2=25$ Centre = (2, –3) & Radius = 5 units	**6 (iii)**	$y=-\frac{3}{4}x+\frac{19}{4}$ When $y = 0$, $x=\frac{19}{3}$ When $x = 0$, $y=\frac{19}{4}$ Area of triangle ABC $=\frac{1}{2}\begin{vmatrix} 0 & 2 & \frac{19}{3} & 0 \\ \frac{19}{4} & -3 & 0 & \frac{19}{4} \end{vmatrix}$ $=\frac{1}{2}(\frac{361}{12}-\frac{19}{2}+19)$ $=\frac{475}{24} \quad =19\frac{19}{24}$ or 19.8 (to 3sf)
6(ii)	Gradient $CP=\frac{1-(-3)}{5-2}=\frac{4}{3}$ Gradient of tangent at $P=-\frac{3}{4}$ Equation of tangent at P: $\frac{y-1}{x-5}=-\frac{3}{4}$ $y-1=-\frac{3}{4}x+\frac{15}{4}$ $y=-\frac{3}{4}x+\frac{19}{4}$		
7(i)	$x^3-3x^2-3x+12=x+12$ $x^3-3x^2-4x=0$ $x(x^2-3x-4)=0$ $x(x+1)(x-4)=0$ $x=0$ or -1 or 4 When $x = 4$, $y = 16$ A (4, 16) and B (2, 14)	**7 (ii)**	Area of ABC $=\frac{1}{2}(14+16)(2)-\int_2^4 x^3-3x^2-3x+12\,dx$ $=30-\left[\frac{x^4}{4}-x^3-\frac{3x^2}{2}+12x\right]_2^4$ $=30-[24-14]=30-10=20$ unit^2

8(i)	$\frac{x}{4} = \cos\theta$ $x = 4\cos\theta$ $\frac{y}{10} = \sin\theta$ $y = 10\sin\theta$ $AF = 10\sin\theta + 4\cos\theta$	**(ii)**	$AF = 10\sin\theta + 4\cos\theta$ $R = \sqrt{10^2 + 4^2}$ $= \sqrt{116} = 2\sqrt{29}$ $\alpha = \tan^{-1}\frac{4}{10}$ $= 21.8°$ $AF = 2\sqrt{29}\sin(\theta + 21.8°)$
8 (iii)	$AF = \sqrt{116}\sin(\theta + 21.8°)$ max length of $AF = \sqrt{116}$ when $\theta + 21.8° = 90°$ $\theta = 68.2°$		
8 (iv)	$CF = 10\cos 68.1986° + 4\sin 68.1986° = 7.4278$ $\frac{6}{h} = \tan 68.1986°$ $h = 2.4$ Max possible height of train $= 7.4278 - 2.40$ $= 5.03\,\text{m}$ (to 3 sf) > 3.6 m Yes, a train of height 3.6 m can pass through the tunnel.		