Candidate Index Number

Anglo - Chinese School (Independent)

FINAL EXAMINATIONS 2016
YEAR 3 INTEGRATED PROGRAMME
CORE MATHEMATICS
PAPER 1

WEDNESDAY **5th OCTOBER 2016** **1 h 30 min**

Additional Material
Graph Paper (1 sheet)

INSTRUCTIONS TO CANDIDATES

- Write your index number in the boxes above.
- Do not open this examination paper until instructed to do so.
- You are not permitted access to any calculator for this paper.
- Answer all questions in the spaces provided.
- Unless otherwise stated in the question, all numerical answers must be given exactly or correct to three significant figures.
- The maximum mark for this paper is 80.

For Examiner's Use

This paper consists of 13 printed pages.

[Turn over

Full marks are not necessarily awarded for a correct answer with no working. Answers must be supported by working and/or explanations. Where an answer is incorrect, some marks may be given for a correct method, provided this is shown by written working. You are therefore advised to show all working.

*Answer **all** the questions in the spaces provided.*

1 *[Maximum mark: 8]*

(a) Simplify $\frac{a-b}{ab}-\frac{c-b}{bc}+\frac{c-a}{ac}$.

[2 marks]

(b) Simplify $\frac{x^3y^3\times\sqrt[3]{x}}{\sqrt{x^2y^4}}$, giving your answer in the from $x^n y^m$, where n and m are rational numbers.

[3 marks]

(c) Factorise completely $(x+y)^2-(x+y)-6$.

[3 marks]

.....

$\frac{c(a-b)-a(c-b)+b(c-a)}{abc}$

$=\frac{ca-cb-ac+ab+bc-ba}{abc}$

$=0$

.....

$\frac{x^3\frac{1}{3}y^3}{xy^2}$

$=x^{\frac{7}{3}}y$

.....

$(x+y-3)(x+y+2)$

.....

2 *[Maximum mark: 8]*

Mrs. Lim imported some olive oil for \$500. She paid $\$x$ for each liter of the olive oil.

(a) Find, in terms of x, an expression for the amount of olive oil she bought.

[1 mark]

During transportation, 30 liters of olive oil was spilled. She sold the remaining olive oil for \$1 more per liter than what she paid initially.

(b) Write down an expression, in terms of x, for the sum of money she received.

[2 marks]

Mrs. Lim made a loss of \$25.

(c) Write down an equation in x to represent this information, and show that it can be reduced to $6x^2 + x - 100 = 0$.

[2 marks]

(d) Solve the equation $6x^2 + x - 100 = 0$, and hence, find the amount of olive oil she bought.

[2 marks]

.....

(a)

Amount = $\frac{500}{x}$

(b)

Amount left = $\frac{500}{x} - 30$

Amount received = $\left(\frac{500}{x} - 30\right)(x + 1)$

(c)

$500 - \left(\frac{500}{x} - 30\right)(x + 1) = 25$

$500 - 500 - \frac{500}{x} + 30x + 30 = 25$

$-500 + 30x^2 + 5x = 0$

$6x^2 + x - 100 = 0$

(d)

$(6x + 25)(x - 4) = 0$

$x = 4$

Amount = $\frac{500}{4} = 125$

3 *[Maximum mark: 8]*

(a) Given that $-a\ln b$ is a solution to $25-2e^{-x}=9$, find the value of a and of b, where a and b are integers.

[4 marks]

(b) Solve $\log_2(x+1)=\log_4(13-2x)$

[4 marks]

$25-2e^{-x}=9$

$16=2e^{-x}$

$16e^{x}=2$

$e^{x}=\frac{1}{8}$

$x=\ln\frac{1}{8}$

$x=-3\ln 2$

$\log_2(x+1)=\log_4(13-2x)$

$\log_2(x+1)=\frac{\log_2(13-2x)}{\log_2 4}$

$\log_2(x+1)=\frac{\log_2(13-2x)}{2}$

$2\log_2(x+1)=\log_2(13-2x)$

$\log_2(x+1)^2=\log_2(13-2x)$

$(x+1)^2=13-2x$

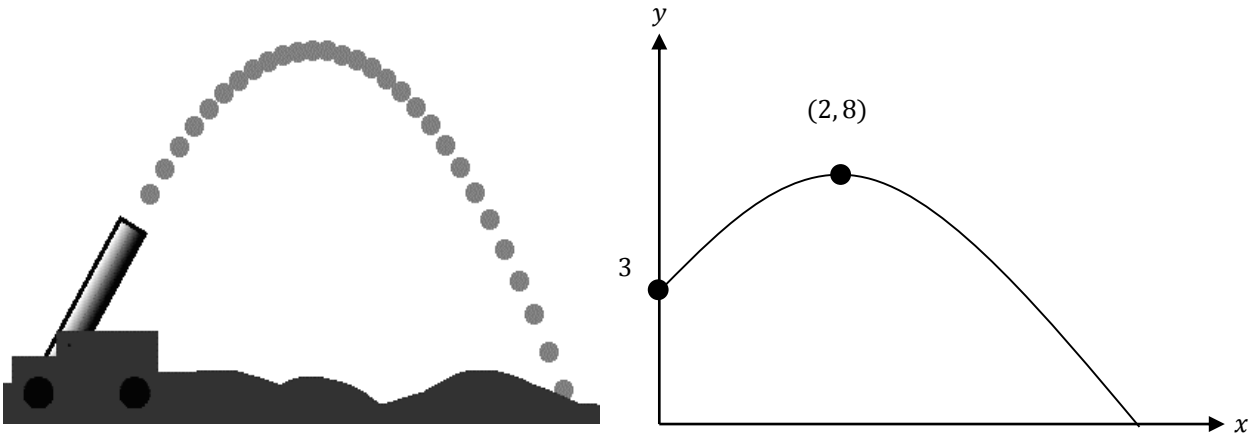
$x^2+4x-12=0$

$(x+6)(x-2)=0$

$x=-6$ (NA) or $x=2$

4 *[Maximum mark: 5]*

A cannonball was fired and is modelled by a quadratic graph as shown in the diagram below. The cannonball was fired from a height of 3 m. The trajectory of the cannonball reached a maximum point at (2, 8).



(a) Express the equation of the trajectory in the form $y = a(x-h)^2 + k$, where a, h and k are constants.

[3 marks]

(b) Find the range of values of x for which $a(x-h)^2 + k > 3$.

[2 marks]

.....

$y = a(x-2)^2 + 8$

$3 = 4a + 8$

$4a = -5$

$a = -\frac{5}{4}$

.....

Using symmetry property of quadratic curve,

$0 < x < 4$

.....

.....

.....

5 *[Maximum mark: 5]*

Given that $5 \le a \le 10$ and $-6 \le b \le -1$, where a and b are integers.
Find

(a) the largest possible value of $a - b$.

[1 mark]

(b) the smallest possible value of $b^2 - a$.

[1 mark]

(c) the smallest possible value of $\dfrac{-a^2 + 12a - 31}{b}$.

[3 marks]

(a)

$10 - (-6)$

$= 16$

(b)

$(-1)^2 - 10$

$= -9$

(c)

$\frac{-a^2+12a-31}{b}$

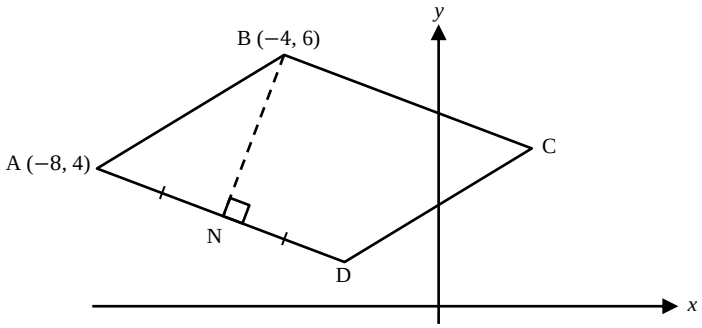
$= \frac{-(a-6)^2+5}{b}$

$= \frac{5}{-1}$

$= -5$

6 *[Maximum mark: 12]*

The diagram shows a parallelogram ABCD where the coordinates of A and B are $(-8, 4)$ and $(-4, 6)$ respectively. The equation of AD is $3y + x - 4 = 0$. N is the foot of the perpendicular from B to AD, and *AN = ND*.



(a) Find the equation of *BN*.

[3 marks]

(b) Find the coordinates of *N*.

[3 marks]

(c) State the coordinates of C and D.

[3 marks]

(d) Calculate the area of the parallelogram ABCD.

[3 marks]

$3y + x - 4 = 0$
$y = -\frac{1}{3}x + \frac{4}{3}$
$m_{BN} = 3$
$y - 6 = 3(x + 4)$
$y = 3x + 18$

$-\frac{1}{3}x + \frac{4}{3} = 3x + 18$
$3\frac{1}{3}x = -16\frac{2}{3}$
$x = -5$
$y = 3$

$D(-2, 2)$
$C(2, 4)$

$BN = \sqrt{(6-3)^2 + (-4+5)^2} = \sqrt{10}$
$AD = \sqrt{(4-2)^2 + (-8+2)^2} = \sqrt{40}$

Area = $\sqrt{10}\sqrt{40} = 20$

7 *[Maximum mark: 6]*

Solve the simultaneous equations

$2^{2+m}+3\left(3^{n}\right)=5$

$2^{m}+3^{n+1}=2$

$4(2^m) + 3(3^n) = 5$

$2^m + 3(3^n) = 2$

$3(2^m) = 3$

$2^m = 1$

$m = 0$

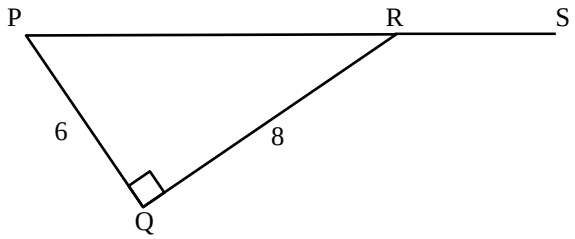
$1 + 3(3^n) = 2$

$3^n = 3^{-1}$

$n = -1$

8 *[Maximum mark: 8]*

(a) In the diagram below, PRS is a straight line and $\angle PQR = 90^o$. Find the value of



(i) $\tan \angle QRP$

[1 mark]

(ii) $\cos \angle QRS$

[2 marks]

(b) Given that θ is obtuse and that $\sin\theta = \frac{2}{3}$, find the value of $\frac{\tan\theta}{1-\cos\theta}$. Leave your answer in the from $a\sqrt{b}+c$, where a, b and c are constants.

[5 marks]

.....

$\tan QRP = \frac{6}{8}$

$PR^2 = 6^2 + 8^2$
$PR = 10$

$\cos QRP = \frac{8}{10}$

$\cos QRS = -\frac{4}{5}$

$\tan\theta = -\frac{2}{\sqrt{5}}$ $\quad\quad$ $\cos\theta = -\frac{\sqrt{5}}{3}$

$\frac{-\frac{2}{\sqrt{5}}}{1+\frac{\sqrt{5}}{3}} = \frac{-\frac{2}{\sqrt{5}}}{\frac{3+\sqrt{5}}{3}} = -\frac{2}{\sqrt{5}} \times \frac{3}{3+\sqrt{5}} = \frac{-6}{3\sqrt{5}+5}$

$= \frac{-6}{3\sqrt{5}+5} \times \frac{3\sqrt{5}-5}{3\sqrt{5}-5}$

$= \frac{3}{2} - \frac{9}{10}\sqrt{5}$

.....

9 *[Maximum mark: 6]*

(a) It is given that the graph of $y = -x^2 + (k+2)x + (k-1)$ touches the $x-$ axis at only one point. Find the possible values of k.

[3 marks]

(b) Given that $\frac{3}{x^2 + 5x - 14} < 0$, find the range of values of x.

[3 marks]

$b^2 - 4ac = 0$

$(k+2)^2 - 4(-1)(k-1) = 0$

$k^2 + 4k + 4 + 4k - 4 = 0$

$k^2 + 8k = 0$

$k = 0$ or $k = -8$

$x^2 + 5x - 14 < 0$

$(x+7)(x-2) < 0$

$-7 < x < 2$

10 *[Maximum mark: 5]*

If $\alpha^2\beta$ and $\beta^2\alpha$ are the roots of the equation $x^2 - 10x - 8 = 0$, find the quadratic equation with the roots, α and β.

$\alpha^2\beta + \beta^2\alpha = 10$

$\alpha\beta(\alpha + \beta) = 10$

$\alpha^3\beta^3 = -8$

$\alpha\beta = -2$

$-2(\alpha + \beta) = 10$

$\alpha + \beta = -5$

$x^2 + 5x - 2 = 0$

11 *[Maximum mark: 9]*

Answer the whole of this question on a sheet of graph paper.

Given that the variables x and y are connected by the equation $y = x + \frac{3}{x} - 1$. The table below shows some values of x and the corresponding values of y, correct to 1 decimal place.

x	0.5	1	2	3	4	5	6	7
y	5.5	3	2.5	n	3.8	4.6	5.5	6.4

(a) Calculate the value of n.

[1 mark]

(b) Using a scale of 2 cm to represent 1 unit on the x – axis and 1 cm to represent 1 unit on the y – axis, draw the graph of $y = x + \frac{3}{x} - 1$ for $0.5 \le x \le 7$.

[3 marks]

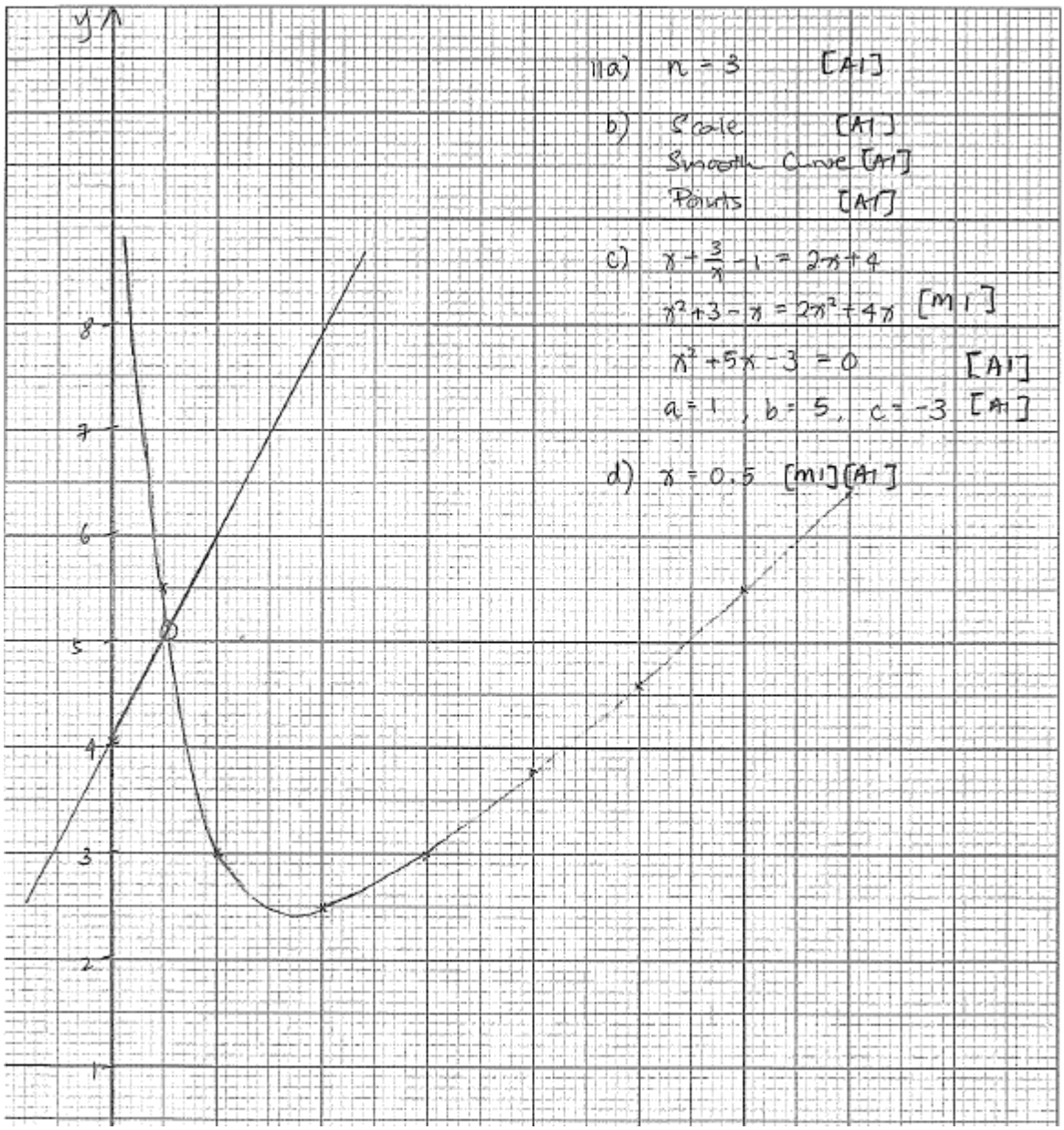
(c) Using the same axes, draw the graph of $y = 2x + 4$. The roots of the equation $ax^2 + bx + c = 0$ is given by the x – coordinate of the point of intersection of the curve $y = x + \frac{3}{x} - 1$ and the line $y = 2x + 4$. Find the values of a, b and c.

[3 marks]

(d) Use your graph to write down the solutions of the equation $ax^2 + bx + c = 0$ in (c).

[2 marks]

********** END OF PAPER 1 **********


y
8
7
6
5
4
3
2
1
11a) n = 3 [A1]
b) Scale [A1]
Smooth Curve [A1]
Points [A1]
c) $x + \frac{3}{x} - 1 = 2x + 4$
$x^2 + 3 - x = 2x^2 + 4x$ [M1]
$x^2 + 5x - 3 = 0$ [A1]
a = 1, b = 5, c = -3 [A1]
d) x = 0.5 [m1][A1]