2019 H2 Math A Level Paper 2 (Suggested Solutions)

Q1

(i)

$$I = \int x(1-x)^{\frac{1}{2}}\,dx \qquad u = x \qquad \frac{dv}{dx} = (1-x)^{\frac{1}{2}}$$

$$\frac{du}{dx} = 1 \qquad v = \frac{(1-x)^{\frac{3}{2}}}{(-1)\frac{3}{2}} = -\frac{2}{3}(1-x)^{\frac{3}{2}}$$

$$= -\frac{2}{3}x(1-x)^{\frac{3}{2}} - \int -\frac{2}{3}(1-x)^{\frac{3}{2}}(1)\,dx$$

$$= -\frac{2}{3}x(1-x)^{\frac{3}{2}} + \frac{2}{3}\left[\frac{(1-x)^{\frac{5}{2}}}{(-1)\frac{5}{2}}\right] + C_1$$

$$= -\frac{2}{3}x(1-x)^{\frac{3}{2}} - \frac{4}{15}(1-x)^{\frac{5}{2}} + C_1$$

* If you had chosen $u = (1-x)^{\frac{1}{2}}$ & $\frac{dv}{dx} = x$

$\frac{du}{dx} = -\frac{1}{2}(1-x)^{-\frac{1}{2}}$ $\quad v = \frac{x^2}{2}$

then $I = \frac{x^2}{2}(1-x)^{\frac{1}{2}} - \underbrace{\int \frac{x^2}{2}\left(-\frac{1}{2}\right)(1-x)^{-\frac{1}{2}}\,dx}_{}$ → not able to solve!

(ii)

$$I = \int x(1-x)^{\frac{1}{2}}\,dx \qquad u^2 = 1-x$$

$$= \int (1-u^2)(u^2)^{\frac{1}{2}}(-2u)\,du \qquad 2u\frac{du}{dx} = -1$$

$$= \int (1-u^2)\,u\,(-2u)\,du \qquad \frac{du}{dx} = -\frac{1}{2u}$$

$$= \int -2u^2 + 2u^4\,du$$

$$= -\frac{2u^3}{3} + \frac{2u^5}{5} + C_2$$

$$= -\frac{2}{3}(1-x)^{\frac{3}{2}} + \frac{2}{5}(1-x)^{\frac{5}{2}} + C_2$$

(iii)

$$-\frac{2}{3}x(1-x)^{\frac{3}{2}} - \frac{4}{15}(1-x)^{\frac{5}{2}} + C_1 - \left[-\frac{2}{3}(1-x)^{\frac{3}{2}} + \frac{2}{5}(1-x)^{\frac{5}{2}} + C_2\right]$$

$$= \frac{2}{3}(1-x)^{\frac{3}{2}}(-x+1) + (1-x)^{\frac{5}{2}}\left(-\frac{4}{15}+\frac{2}{5}\right) + C_1 - C_2$$

$$= \frac{2}{3}(1-x)^{\frac{3}{2}}(1-x) - \frac{2}{3}(1-x)^{\frac{5}{2}} + C_1 - C_2$$

$$= \frac{2}{3}(1-x)^{\frac{3}{2}}\left(1-x-(1-x)\right) + C_1 - C_2$$

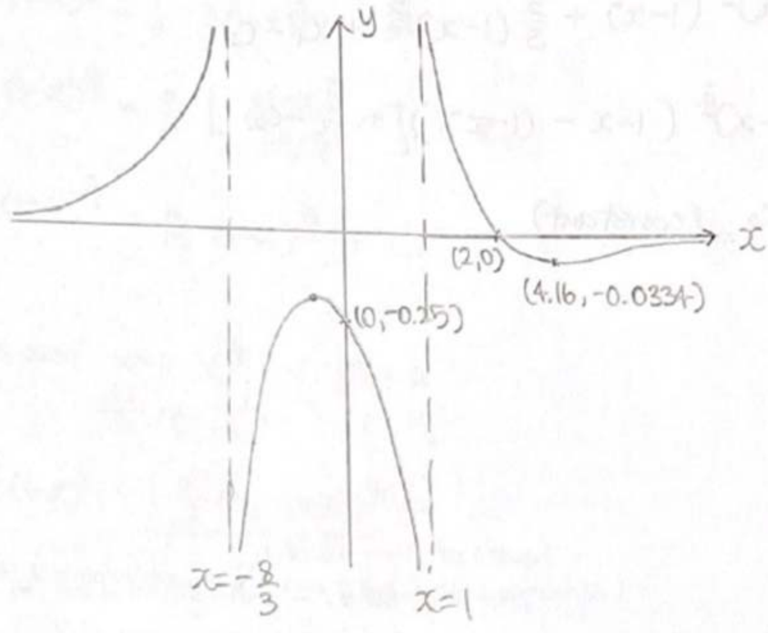
$$= C_1 - C_2 \quad \text{(constant)}.$$

Q2

(i)

$$y = \frac{2-x}{(3x+8)(x-1)}$$

asymptotes : $x = -\frac{8}{3}$, $x = 1$, $y = 0$



* How do we know there's a min point at (4.16, -0.0334) ?

(ii)

From graph,

$$x < -\frac{8}{3} \text{ or } 1 < x < 2$$

(iii)

multiply by (-1) : $\dfrac{2-x}{3x^2+5x-8} < 0$

$\therefore -\frac{8}{3} < x < 1$ or $x > 2$ (from graph).

Q3

Total Surface area $= 2\pi r^2 + 2\pi rh = 900$

$$2\pi rh = 900 - 2\pi r^2$$

$$h = \frac{900 - 2\pi r^2}{2\pi r}$$

$$= \frac{450 - \pi r^2}{\pi r}$$

Volume $V = \pi r^2 h$

$$= \pi r^2 \left(\frac{450 - \pi r^2}{\pi r} \right)$$

$$= 450r - \pi r^3$$

$$\frac{dV}{dr} = 450 - 3\pi r^2$$

At stationary point, $\dfrac{dV}{dr} = 0$

$$450 - 3\pi r^2 = 0$$

$$r^2 = \frac{450}{3\pi} = \frac{150}{\pi}$$

$$r = \sqrt{\frac{150}{\pi}} \quad (\text{since } r \geq 0)$$

$$\frac{d^2V}{dr^2} = -6\pi r = -6\pi\sqrt{\frac{150}{\pi}} < 0$$

$\therefore r = \sqrt{\frac{150}{\pi}}$ gives max volume.

$$\max V = 450\sqrt{\frac{150}{\pi}} - 3\pi\left(\frac{150}{\pi}\right)^{\frac{3}{2}} = \sqrt{\frac{150}{\pi}}\left(450 - \pi\left(\frac{150}{\pi}\right)\right)$$

$$= 300\sqrt{\frac{150}{\pi}} \text{ cm}^3$$

$$\frac{r}{h} = \frac{r}{\left(\frac{450-\pi r^2}{\pi r}\right)} = \frac{\pi r^2}{450-\pi r^2}$$

$$= \frac{\pi\left(\frac{150}{\pi}\right)}{450-\pi\left(\frac{150}{\pi}\right)}$$

$$= \frac{150}{300}$$

$$= \frac{1}{2}$$

$\therefore r:h = 1:2$

Q4

(i)

$f(x) = \sec 2x$

$f'(x) = 2\sec 2x \tan 2x$

$f''(x) = 2\sec 2x\,(2\sec^2 2x) + \tan 2x\,(2)(2\sec 2x \tan 2x)$

$= 4(\sec^3 2x + \sec 2x \tan^2 2x)$

When $x=0$,

$f(0) = 1$

$f'(0) = 0$

$f''(0) = 4$

$\therefore f(x) = 1 + 0x + \frac{4}{2!}x^2 + \cdots$

$= 1 + 2x^2 + \cdots$

(ii)

$$\int_0^{0.02} \sec 2x \, dx \approx \int_0^{0.02} 1 + 2x^2 \, dx$$

$$= 0.02001 \text{ (to 5d.p.)}$$

(iii)

By GC, $\int_0^{0.02} \sec 2x \, dx \approx 0.02001$ (to 5d.p.)

(iv)

The answers for (ii) and (iii) are the same up to 5 d.p.

The approximation is accurate because the values of x were small.

(v)

Since g(0) is undefined, Maclaurin series cannot be used.

Q5

(i)

X is the point of intersection between lines AC & BD.

$$l_{BD}: \underset{\sim}{r} = \overrightarrow{OB} + \lambda \overrightarrow{BD}$$

$$= \underset{\sim}{b} + \lambda(\underset{\sim}{b} + 5\underset{\sim}{a} - \underset{\sim}{b})$$

$$= \underset{\sim}{b} + \lambda(5\underset{\sim}{a}), \quad \lambda \in \mathbb{R}$$

$l_{AC}: \underline{r} = \overrightarrow{OA} + \mu \overrightarrow{AC}$

$$= \underline{a} + \mu(2\underline{a} + 4\underline{b} - \underline{a})$$

$$= \underline{a} + \mu(\underline{a} + 4\underline{b})$$

To find X :

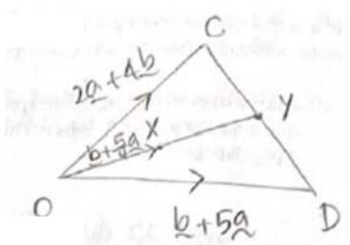
$$\underline{b} + \lambda(5\underline{a}) = \underline{a} + \mu(\underline{a} + 4\underline{b})$$

$$5\lambda\underline{a} + \underline{b} = (1+\mu)\underline{a} + 4\mu\underline{b}$$

$$4\mu = 1 \Rightarrow \mu = \frac{1}{4} \quad , \quad 5\lambda = 1 + \mu \Rightarrow \lambda = \frac{1}{4}$$

$$\therefore \overrightarrow{OX} = \underline{b} + \frac{5}{4}\underline{a}$$

(ii)



$l_{CD}: \underline{r} = \overrightarrow{OC} + \gamma\, \overrightarrow{CD}$

$$= 2\underline{a} + 4\underline{b} + \gamma(\underline{b} + 5\underline{a} - 2\underline{a} - 4\underline{b})$$

$$= 2\underline{a} + 4\underline{b} + \gamma(3\underline{a} - 3\underline{b})$$

$$= 2\underline{a} + 4\underline{b} + \gamma'(\underline{a} - \underline{b}), \quad \gamma' \in \mathbb{R}$$

Since Y lies on l_{CD}: $\overrightarrow{OY} = 2\underline{a} + 4\underline{b} + \gamma'(\underline{a} - \underline{b})$ for some $\gamma' \in \mathbb{R}$

Since O, X, Y collinear, $\overrightarrow{OY} = k\overrightarrow{OX}$ for some k.

$$(2+\gamma')\underset{\sim}{a} + (4-\gamma')\underset{\sim}{b} = k\left(\underset{\sim}{b} + \frac{5}{4}\underset{\sim}{a}\right)$$

$$\begin{cases} 2+\gamma' = k \\ 4-\gamma' = \frac{5}{4}k \end{cases}$$

$$\Rightarrow \gamma' = -\frac{4}{3}, \quad k = \frac{8}{3}$$

$$\therefore \overrightarrow{OY} = \frac{8}{3}\left(\underset{\sim}{b} + \frac{5}{4}\underset{\sim}{a}\right)$$

$$= \frac{8}{3}\underset{\sim}{b} + \frac{10}{3}\underset{\sim}{a}$$

$$\frac{OX}{OY} = \frac{1}{k} = \frac{1}{\left(\frac{8}{3}\right)} = \frac{3}{8}$$

$$\therefore OX : OY = 3 : 8$$

Q6

(i)

These 22 clubs form a population since they are <u>all</u> the clubs Alice is interested in.

(ii)

Dilip should take a sample of the clubs and the sample should be random to avoid bias.

* explicit knowledge on how to obtain the sample is not required.

(iii)

$$\binom{22}{5} \times \binom{24}{5} \times \binom{26}{5} \times \binom{28}{5}$$

$= 7.24 \times 10^{18}$ samples

Q7

(i)

① The probability of each chosen mug being faulty is constant.

② The event that a randomly chosen mug is faulty is independent of another randomly chosen mug being faulty.

(ii)

$F \sim B(50, 0.08)$

$P(F \geq 7) = 1 - P(F \leq 6)$

$= 0.101872$

≈ 0.102

(iii)

Let X be the no. of days out of 5 days where at least 7 faulty mugs are found.

$X \sim B(5, 0.101872)$

$P(X \leq 2) \approx 0.991$

(iv)

Let Y be the no. of faulty saucers out of 10.

$Y \sim B(10, p)$

$P(Y=2) = \binom{10}{2} p^2 (1-p)^8 = 45p^2(1-p)^8$

(v)

P(Set contains at most 1 faulty item)

$= P(0 \text{ faulty}) + P(1 \text{ faulty})$

$= (1-p)^2(0.92)^2 + P(1 \text{ mug faulty}) + P(1 \text{ saucer faulty})$

$= (1-p)^2 \times (0.92)^2 + \binom{2}{1}(0.08)(0.92) \times (1-p)^2 + \binom{2}{1} p(1-p) \times 0.92^2$

$= 0.8464(1-p)^2 + 0.1472(1-p)^2 + 1.6928\, p(1-p)$

$= (1-p)\left(0.9936(1-p) + 1.6928p\right)$

$= (1-p)(0.6922p + 0.9936)$

So $(1-p)(0.6922p + 0.9936) = 0.97$

$\Rightarrow (1-p)(0.6922p + 0.9936) - 0.97 = 0$

By GC, $p = 0.0689$

Q8(i)(a)

$P(\text{Horse or Rider}) = \dfrac{9+14}{56}$

$= \dfrac{23}{56}$

(i)(b)

$P(\text{Dog or Bird but not White}) = \dfrac{11+15}{56}$

$= \dfrac{26}{56}$

$= \dfrac{13}{28}$

(ii)(a)

P(both are Horses but neither orange)

$$= \frac{\binom{8}{2}}{\binom{56}{2}} \quad \text{or} \quad \frac{8}{56} \times \frac{7}{55}$$

$$= \frac{1}{55}$$

(b)

P(exactly 1 Dog & exactly 1 yellow)

56 − 14 − 10 = 32 possible

= P(1 Yellow Dog & other non-yellow, not dog) +

P(1 non-yellow Dog & 1 yellow, not dog)

$$= \frac{\binom{7}{1}\binom{32}{1} + \binom{10}{1}\binom{7}{1}}{\binom{56}{2}}$$

$$= \frac{21}{110}$$

Alternatively,

$$\frac{7}{56} \times \frac{32}{55} \times 2! + \frac{10}{56} \times \frac{7}{55} \times 2!$$

$$= \frac{21}{110}$$

(iii)

Let x and y be the no. of characters that are Gerri's 2 favourites.

$$\frac{x}{56} \times \frac{y}{55} \times 2! = \frac{1}{77}$$

$$\Rightarrow xy = 20$$

From table, these are the cases that will give $xy = 20$:

① Orange Bird & Yellow Bird

② Orange Bird & White Rider

③ White Horse & White Rider

④ White Horse & Yellow Bird

Q9(i)

A 2-tail test should be carried out as he wants to find out if the mean resistance differs from 750 ohms.

Let μ be the population mean resistance of the resistors.

Let H_0, H_1 be null hypothesis & alternative hypothesis respectively.

$H_0 : \mu = 750$

$H_1 : \mu \neq 750$

(ii)

By GC, $\bar{x} = 756$.

Under H_0, $\bar{X} \sim N\left(750, \frac{100}{8}\right)$

Test statistic $Z = \dfrac{\bar{X} - 750}{\frac{10}{\sqrt{8}}} \sim N(0,1)$

By GC, p-value $\approx 0.0897 > 0.05$

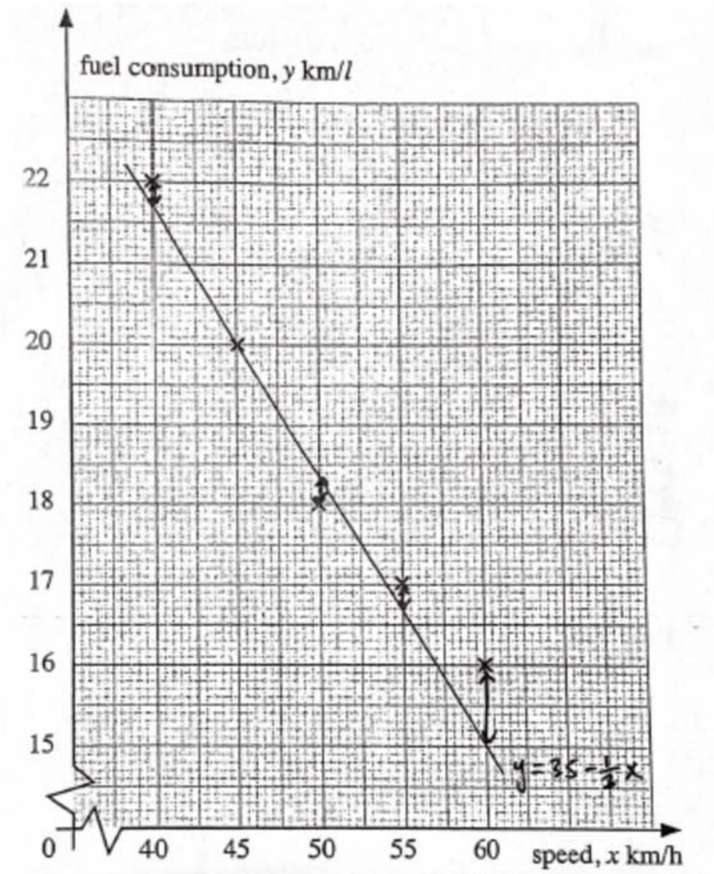
$\therefore$ do not reject H_0

There is insufficient evidence at 5% level of significance to conclude that the mean resistance is not 750 ohms.

(iii)

Since the distribution of the resistances is unknown, a large sample of at least 30 (or 50) should be taken so that Central Limit Theorem can be applied to approximate the sample mean distribution to a normal distribution.

Q10(i)(a), (b)



(c)

x	40	45	50	55	60
$y' = 35 - \frac{1}{3}x$	$\frac{65}{3}$	20	$\frac{55}{3}$	$\frac{50}{3}$	15

$\therefore$ sum of squares of residuals $= \Sigma e_i^2$

$= (22 - \frac{65}{3})^2 + (20-20)^2 + (18 - \frac{55}{3})^2 + (17 - \frac{50}{3})^2 + (16-15)^2$

$= \frac{4}{3}$

or By GC, $\Sigma e_i^2 = 1.33$ (to 3sf)

(d)

Residuals might result in negative values, thus when finding the sum of residuals, we might have values cancelling each other out. By squaring the residuals, we ensure that the values will always be positive.

(ii)

Bhani's model will give a better fit since it has a smaller value for the sum of squares of residuals.

(iii)

$(\bar{x}, \bar{y}) = (50, 18.6)$

(iv)

By GC, $y = -0.3x + 33.6$

$r = -0.985$ (to 3sf)

(v)

When $x = 30$, $y = 24.6$ km/l

As $x = 30$ is outside the data range of 40 to 60, the estimate is not reliable. (extrapolation).

*Note: we cannot say y is out of the data range!

(vi)

Her data points are perfectly linear and lie on the best fit line.

Q11

(i)

let W & B be the mass of a white & a black ball respectively.

$W \sim N(110, 4^2)$, $B \sim N(55, 2^2)$

$W_1 + W_2 + W_3 + W_4 \sim N(4(110), 4(4^2))$
$= N(440, 64)$

$P(W_1 + W_2 + W_3 + W_4 > 425) = 0.970$ (to 3 sf)

(ii)

$W + B \sim N(110 + 55, 4^2 + 2^2)$
$= N(165, 20)$

$P(161 < W + B < 175) = 0.802$ (to 3sf)

(iii)

$W_1 + W_2 + B_1 + B_2 + B_3 \sim N(2(110) + 3(55), 2(4^2) + 3(2^2))$

$= N(385, 44)$

$P(W_1 + W_2 + B_1 + B_2 + B_3 < M) = 0.271$

$M = 380.955$
≈ 381 (to 3sf)

(iv)

let R be the mass of a rod.

$R \sim N(20, 0.9^2)$

let T be the mass of Arif's model.

$T = 0.7W + 0.9(B_1 + B_2 + B_3 + B_4) + R_1 + R_2 + R_3 + R_4$

$E(T) = 0.7(110) + 0.9(4)(55) + 4(20) = 355$

$Var(T) = 0.7^2(4^2) + 0.9^2(4)(2^2) + 4(0.9^2) = 24.04$

$T \sim N(355, 24.04)$

$P(T > 350) = 0.846$ (to 3sf).