Chapter 17 ELECTROMAGNETIC INDUCTION

Content

- Magnetic flux
- Laws of electromagnetic induction

Learning Outcomes

Candidates should be able to:

(a) define magnetic flux as the product of an area and the component of the magnetic flux density perpendicular to that area

(b) recall and solve problems by using $\phi = BA$

(c) define magnetic flux linkage

(d) infer from appropriate experiments on electromagnetic induction:

(i) that a changing magnetic flux can induce an e.m.f.
(ii) that the direction of the induced e.m.f. opposes the change producing it
(iii) the factors affecting the magnitude of the induced e.m.f.

(e) recall and solve problems using Faraday's law of electromagnetic induction and Lenz's law

(f) explain simple applications of electromagnetic induction

Introduction

We began our study on the relationship between electricity and magnetism in the previous chapter where we learned that an electric current (or moving charges) produces a magnetic field. As a result, when a current-carrying conductor or a moving charge is within a magnetic field, it will experience a force.

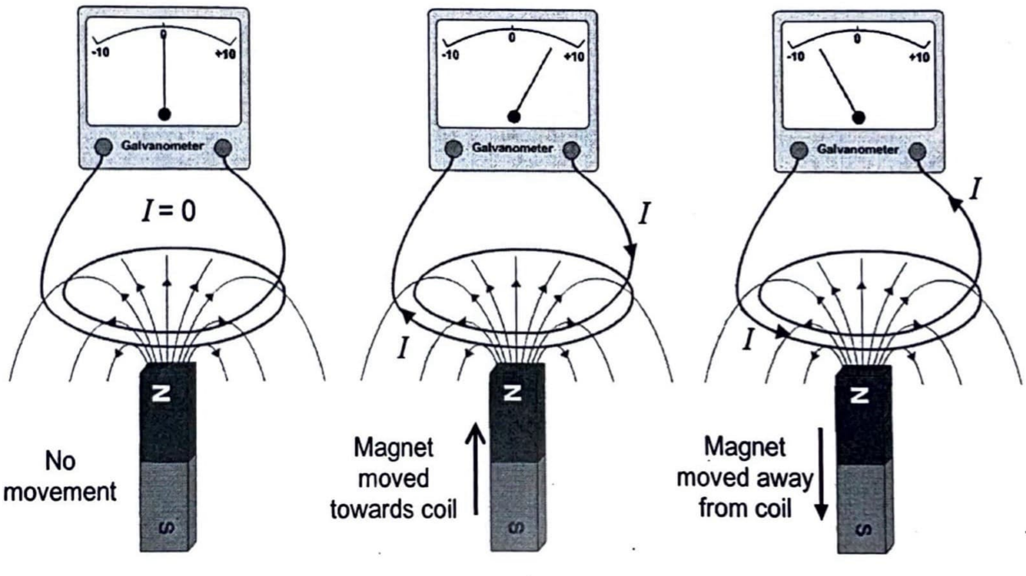
In this chapter, we will see that the converse happens. Experiments conducted by Michael Faraday in England in 1831 and independently by Joseph Henry in the United States the same year demonstrated that an e.m.f. could be induced by a changing magnetic field. This effect is known as **electromagnetic induction.**

This discovery greatly revolutionized the production of electricity and has a significant impact on our daily lives. The electrical power in our homes today is generated based on this principle and many modern day conveniences that we take for granted would not be possible without the understanding of this phenomenon.

17.1 Faraday's Law of Electromagnetic Induction

Observing Electromagnetic Induction

Electromagnetic induction can be demonstrated easily using the laboratory apparatus shown in Fig. 1 below.



(a) (b) (c)

Fig. 1

A bar magnet is moved towards or away from a coil of wire which is connected to a sensitive galvanometer. It was found that:

- **no current** is generated when the magnet is kept stationary relative to the coil (Fig. 1a)
- **a current is induced** in the coil when the magnet is moved towards (Fig. 1b) or moved away (Fig. 1c) from the coil
- the **directions** of the currents in the two cases are opposite.
- the **magnitude** of the induced current increases as the magnet moves faster.

It was also found that

- a current is induced when the coil is moved towards or away from the magnet.
- a current is induced when the coil is pulled at 2 ends such that its cross sectional area decreases.

Conclusion

A current is induced whenever there is some relative motion between the magnetic field and the coil.

Faraday's Law

The observations in the simple demonstration in Fig. 1 show that a current is induced as long as there is some change in the magnetic field through the coil. Such an induced current must be produced by an induced e.m.f. This forms the basis of Faraday's law of electromagnetic induction.

Definition

Faraday's law of electromagnetic induction states that the induced e.m.f. is proportional to the rate of change of magnetic flux linkage.

The law can be expressed mathematically as:

Formula

$$\text{Induced e.m.f.} = -\frac{d\Phi}{dt}$$

where Φ is the **magnetic flux linkage** of a coil or circuit.

To understand the law, we need to know the meaning of magnetic flux and magnetic flux linkage.

17.2 Magnetic Flux and Magnetic Flux Linkage

Magnetic Flux

Consider an area A where a uniform magnetic field of magnetic flux density B passes through at an angle θ to the normal.

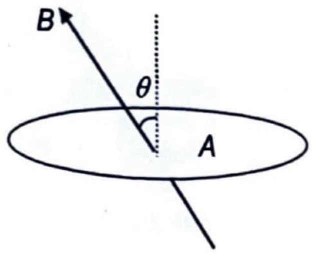


Fig. 2

Definition

Magnetic flux defined as the product of an area and the component of the magnetic flux density perpendicular to that area.

Magnetic flux can be thought of as the number of magnetic field lines passing through the area. This clearly depends on

(i) B, how strong the magnetic flux density is (stronger fields are represented by closer lines),
(ii) A, how large the area is, and
(iii) θ, the angle between the area and the magnetic field.

The expression for magnetic flux ϕ is

Formula

$$\phi = BA\cos\theta$$

where θ is the angle that the magnetic flux density vector B makes with the normal to the area A.

This relation can also be expressed in vector notation as $\phi = \vec{B} \cdot \vec{A}$, where $\vec{A}$ is a vector that has a magnitude that is given by the area A and a direction that is normal to the area.

Note:

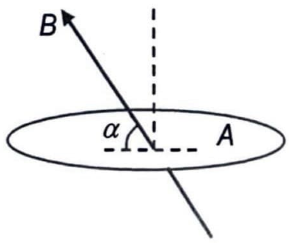


Fig. 3

If the angle which B makes with the plane of A is given instead,
ϕ = $BA\sin\alpha$

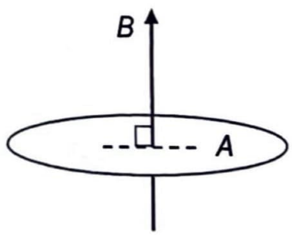


Fig. 4

If B passes perpendicularly through A, i.e. θ = $0°$,
ϕ = BA

Magnetic Flux Linkage

Suppose we have a coil of N turns and uniform cross-sectional area,

Definition

The **magnetic flux linkage** of a coil is the product of the magnetic flux through the coil and the number of turns of the coil.

The expression for magnetic flux linkage is $\Phi = N\phi$

The S.I. unit for both ϕ and Φ is the weber (Wb).

Note:

(i) Since $\Phi = N\phi =$ $NBA\cos\theta$, changing any of the quantities N, B, A and θ will change the magnetic flux linkage in the coil.

(ii) Making reference to Faraday's law, we can see that an e.m.f. can be induced in a coil/circuit when one or more of the following changes are made:

- The magnitude and direction of B varies with time.
- The area A of the coil (or circuit) varies with time.
- The angle θ between B and the normal to the plane varies with time.

Example 1

A coil A experiences a change in magnetic flux linkage from 0 Wb to 0.20 Wb at a constant rate in 2.0 seconds. Another coil B experiences a change in magnetic flux linkage from 0 Wb to 0.50 Wb at a constant rate in 10.0 seconds. Calculate the magnitude of induced e.m.f. in each coil.

Solution

Magnitude of induced e.m.f. in coil A = $\left|-\frac{d\Phi}{dt}\right| = \frac{0.20 - 0}{2.0} = 0.1\text{ V}$

Magnitude of induced e.m.f. in coil B = $\left|-\frac{d\Phi}{dt}\right| = \frac{0.50 - 0}{10.0} = 0.05\text{ V}$

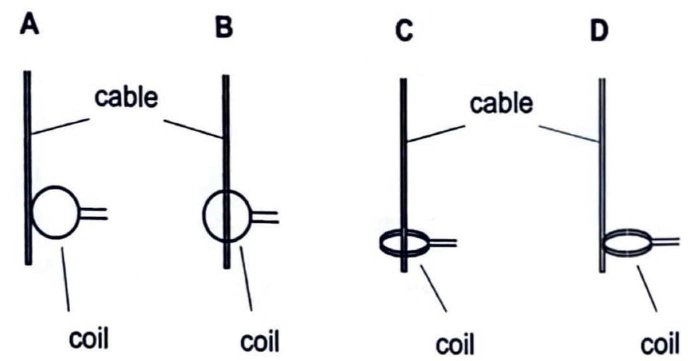
It can be seen that although coil B experiences a larger change in magnetic flux linkage, it has a lower induced e.m.f.

The size of the induced e.m.f. is determined by the rate of change of magnetic flux, not how large the change in magnetic flux is.

Example 2

Large alternating currents in a cable can be measured by monitoring the e.m.f. induced in a small coil situated near the cable. This e.m.f. is induced by the varying magnetic field set up around the cable.

In which arrangement of coil and cable will the e.m.f. induced be a maximum?



Solution

The coil in __A__ has the largest amount of magnetic flux passing through it. Hence it will experience the __largest change in magnetic flux linkage__ when the direction of current flowing through the cable changes.

The net magnetic flux through __B__ is zero because the magnetic field in one half is flowing __out__ of the page while that in the other half is flowing __into__ the page.

The magnetic flux through C and D is also __zero__ as the magnetic field is __parallel__ to the plane of the coil.

Hence, the answer is __A__.

17.3 Direction of the Induced e.m.f. (or current)

Faraday's law allows us to calculate the magnitude of the induced e.m.f. The direction of this induced e.m.f. can be determined in 3 ways:

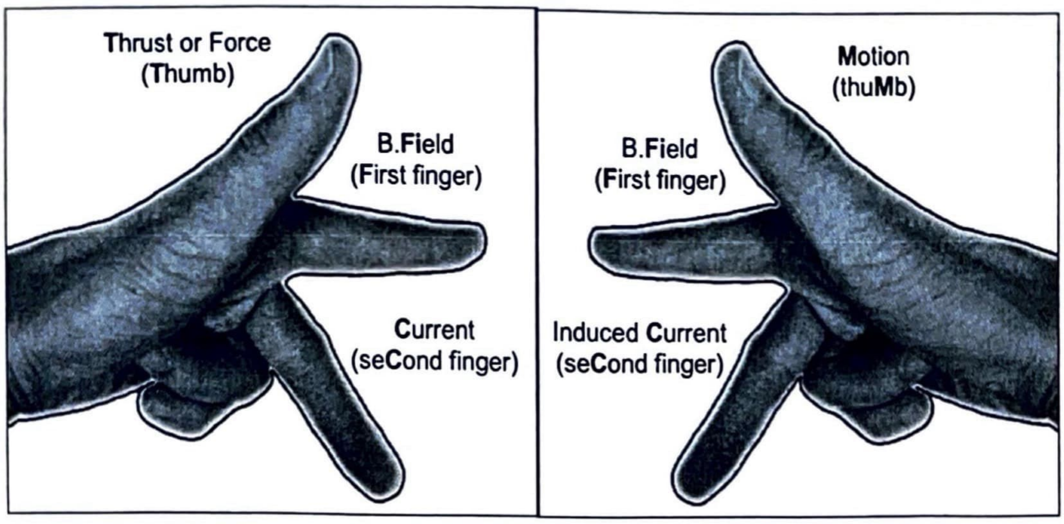
1. Fleming's right-hand rule *(this is typically used as a quick check and should not be used to explain the direction of induced e.m.f.)*
2. Lenz's Law
3. From first principles *(to be covered in section 17.4)*

IMPORTANT Note:

- Faraday's law states that there will be an induced e.m.f. when there is a change in magnetic flux linkage.
- This does not always mean that there will be an induced current. There will be an induced current only when there is a __Closed Circuit__.

1. Fleming's Right-Hand Rule

This is similar to Fleming's Left-Hand Rule except that you use your RIGHT HAND. The first three fingers are used to denote the directions of motion (of conductor), magnetic field and induced current respectively.



Left-hand rule
(to find direction of force)

Right-hand rule
(to find direction of induced current)

Fig. 5

2. Lenz's Law

In the demonstration in Fig. 1, the direction of the induced current was observed to depend on the direction of movement of the magnet.

- In Fig. 1b, when the **north** pole of the magnet is pushed **towards** the coil, the induced current in the coil flows in a direction so that a **north** pole is facing the magnet.
- In Fig. 1c, when the **north** pole of the magnet is pulled **away** from the coil, the induced current in the coild flows in a direction so that a **south** pole is facing the magnet.

It appeared that *the induced current always flows in a direction so as to oppose the change that produces it.*

Definition

Lenz's law states that the direction of induced e.m.f. is such as to cause effects to __opposee__ the change producing it.

Lenz's Law is a consequence of the **principle of conservation of** __energy__.
(See example 3)

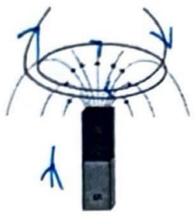
Example 3

Referring to Fig. 1b and Fig. 1c,

(i) use Lenz's Law to determine the direction of the induced current in the coil.

(ii) explain how the direction of current is consistent with the principle of conservation of energy.

Solution



(i) Fig. 1 (b): as the North pole of the magnet approaches the coil, a north pole should be induced to face the incoming magnet (repulsion). This is so as to oppose the increasing magnetic flux linkage through the coil. Using the right hand grip rule for the coil, the induced current flows clockwise as shown.

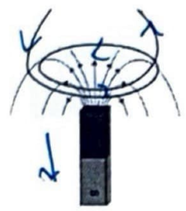


Fig. 1 (c): as the North pole of the magnet moves away from the coil, a South pole should be induced to face the receding magnet (attraction). This is to oppose the decreasing magnetic flux through the coil. Then using right-hand grip rule for the coil, the induced current flows anti-clockwise as shown.

(ii) In both cases, an applied force does mechanical work to move the magnet towards or away from the coil. Due to Lenz's law, the induced current in the coil flows in a direction such that a magnetic force is produced in the coil to oppose the movement of the magnet (either to repel or attract). The work done by the applied force in overcoming this magnetic force gives rise to the electrical energy to drive the current. In other words, mechanical energy is converted to electrical energy. This is a consequence of the principle of conservation of energy.

Example 4

In Fig. 6, an induced current is momentarily produced in the secondary coil when current in the primary coil is switched on or off. This is similar to Example 3, except that the magnetic field is varied by changing the current of the primary coil.

(a) Using **Faraday's law**, explain why

(i) an induced current is produced in the secondary coil,

(ii) this induced current only exists momentarily.

(b) Using **Lenz's law**, determine the directions of the induced current through the galvonometer in the secondary coil when

(i) the switch connected to the primary coil is **closed**,

(ii) the switch connected to the primary coil is **opened**.

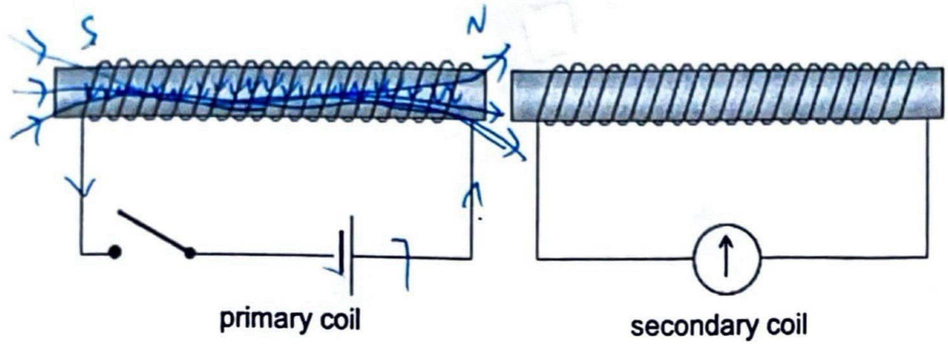


Fig. 6

Solution

(a)(i) When the switch is closed or opened, an increasing or decreasing current flows through the primary coil, producing a change in the magnetic flux of the primary coil. The secondary coil experiences a change in magnetic flux linkage and by Faraday's Law, a current is induced in it.

(ii) After the magnitude of the current (and hence magnetic flux) in the primary coil reaches a steady value, the secondary coil will no longer experience a change in magnetic flux linkage. The induced current will be zero.

Based, on the direction of current through the primary coil, can you figure out what pole is on the right side of the primary coil?

(b)(i) When the switch is **closed**, the secondary coil experiences an increase in magnetic flux linkage. The induced current must flow in such a way to produce a North pole on its left to repel the primary coil so as to oppose this increase. Hence, the induced current flows leftwards through the galvanometer.

(ii) When the switch is **opened**, the secondary coil experiences a decrease in magnetic flux linkage. The induced current must flow in such a way to produce a South pole on its left to attract the primary coil so as to oppose the decrease. Hence, the induced current flows rightwards through the galvanometer.

3. Determining direction of induced e.m.f. from first principles

The third method to determine the direction of the induced e.m.f. requires an understanding of the actual cause of the motion of charge carriers in a conductor when there is relative motion between the conductor and a magnetic field.

This is described in detail in the next section.

17.4 A metal rod moving across a uniform magnetic field

Induced e.m.f.

Fig. 7 shows a metal rod PQ of length L sliding along two frictionless metal rails with constant velocity v to the right across a uniform magnetic field B perpendicular to the plane of the paper. The metal rails are connected by a connecting wire such that the rod, the rails and wire form a **closed circuit**.

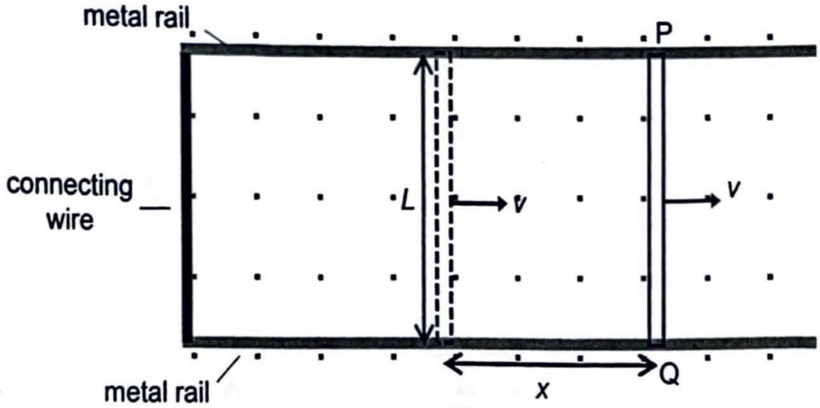


Fig. 7

The distance travelled by the rod in time t is x. As the rod cuts across the magnetic field, the magnetic flux linkage through the circuit in time t is given by

$$\Phi = BA = \underline{BLx}$$

> **? THINK**
> *If PQ was moving similarly across the field, but not in contact with any metal rail, would there still be induced e.m.f.? How about current?*

By Faraday's Law, the magnitude of the induced e.m.f. across the rod is given by

$$|E| = \left| -\frac{d\Phi}{dt} \right| = \underline{BL\frac{dx}{dt} = BLv}$$

The above expression tells us that the magnitude of the induced e.m.f., and hence current, depends on the <u>speed</u> at which the rod is moving across the magnetic field, as well as the magnetic flux density and the length of the rod.

Direction of induced current

We can use the methods stated in section 17.3 to determine the direction of the induced current.

1. Fleming's Right Hand Rule

- Thumb – points to the right to indicate direction of motion of the rod
- First finger – points out of the paper to represent direction of B
- Second finger – points in the direction <u>PQ</u> along the rod to represent direction of induced current

As the rod is a **source of e.m.f.**, end Q should be at <u>higher</u> potential with respect to end P.

**Note that in exams, right-hand rule cannot be used to <u>explain</u> the direction of induced e.m.f. or induced current.*

2. Lenz's Law

As the rod moves to the right, there is an increase in flux linkage through the circuit (area is increasing). By Lenz's law, the induced current would give rise to a magnetic force (to the left) on the rod that tries to oppose its rightward movement. Using Fleming's **Left** Hand rule, the induced current should flow in the direction PQ along the rod (so as to produce a magnetic force to the left).

3. From first principles

As the rod moves to the right, the free electrons inside would also be moving in the same direction. (Fig. 8).

Since the electrons are moving towards the right, the direction of _Conventional_ current due to the electrons is towards the _left_. (Fig. 9)

(Recall from the previous chapter that a charged particle moving in a magnetic field will experience a magnetic force acting at right angles to both the velocity of the particle and the magnetic field.)

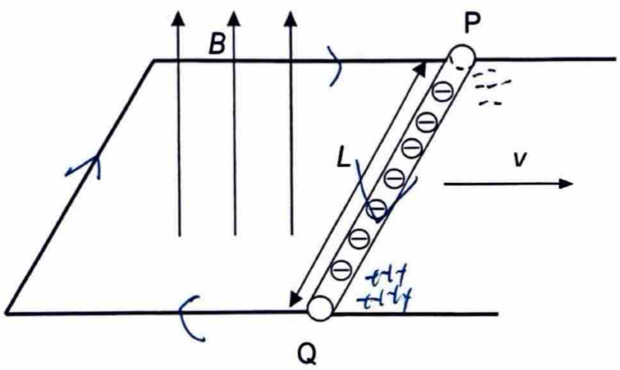


Fig. 8

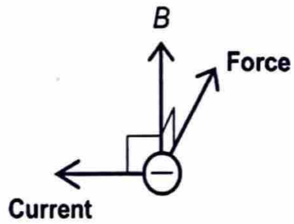


Fig. 9

As shown in Fig. 9, using Fleming's Left-Hand Rule, we can deduce that the electrons in the metal rod will experience a force upwards along the rod (i.e. from Q to P).

? THINK
What would happen to the motion of PQ after a while? Does it require an external force to keep it moving at constant velocity?

As a result, negative charge will tend towards end _P_ while positive charge will tend towards end _Q_. This sets up an _electric field_ across the ends of the rod, with Q at a higher potential than P. The induced current moves in _Clockwise_ direction around the circuit.

This is consistent with that predicted by Fleming's Right-Hand Rule and Lenz's Law.

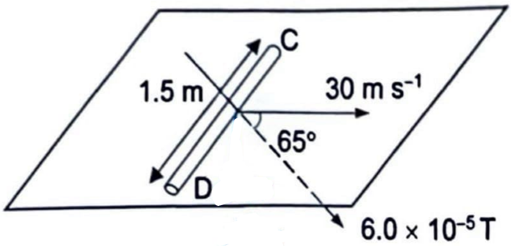
Note:
Current flows from _high_ to _low_ potential _outside_ an e.m.f. source, but from _low_ to _high_ potential _inside_ the e.m.f. source.

Example 5

A 1.5 m rod CD is moving at 30 m s^{-1} horizontally to the right through a region where the Earth's magnetic field of flux density 6.0×10^{-5} T acts downwards at 65° to the horizontal,

(i) Calculate the induced e.m.f. across the rod.

(ii) Which end of CD is at a higher potential?



Solution

(i) $E = (B \sin 65^\circ) LV$ or $(B \cos 25^\circ) LV$

$= (6.0 \times 10^{-5})(\sin 65^\circ)(1.5)(30)$

$= 2.45 \times 10^{-3}$ V

(ii) Induced current would flow from D to C. Hence, C would be at a higher potential.

17.5 A rotating disc in a uniform magnetic field

A metal disc of radius r is mounted on an axle and rotates in a clockwise direction as shown in Fig. 10. With magnets installed beneath the axle, the bottom part of the disc is like a straight conductor of length r that continuously cuts the magnetic field between the poles of the magnet.

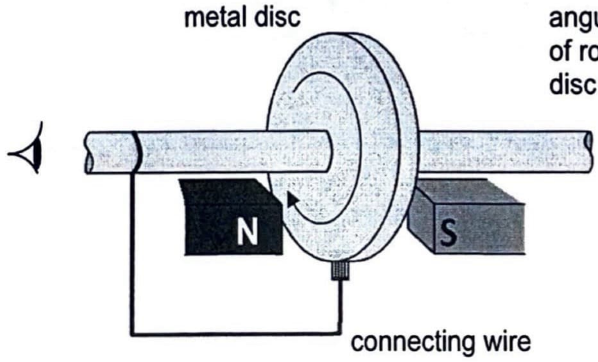


Fig. 10(a)

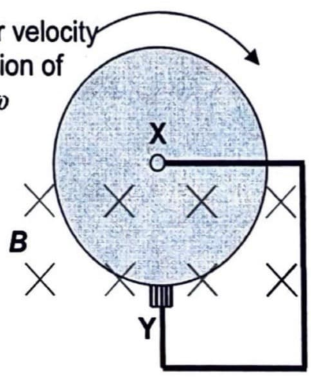


Fig. 10(b)
Eye's view

By Faraday's law, the magnitude of the induced e.m.f. between the axle (X) and rim (Y) of the disc is

$$|E| = \left|-\frac{d\Phi}{dt}\right| = B\frac{dA}{dt} = B\frac{\pi r^2}{T} = BAf$$

? THINK
What would happen to the motion of the disc after a while?

where A is the area of the disc and f is the frequency of rotation.

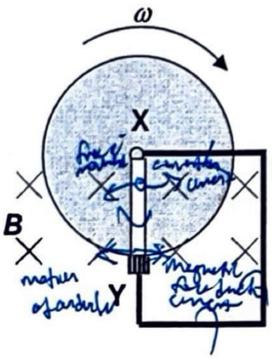
Alternatively, it can be shown that

$$|E| = \frac{1}{2}Br^2\omega$$

The direction of this induced e.m.f. can be deduced using the 3 methods explained earlier (see example 5).

Example 6

Refering to Fig. 10(b), determine the direction of the induced current through the rotating disc.



Solution:

Assume that the disc is made up of radial straight conductors each of length r.

Consider one such conductor connected from X to Y as shown. At this instant, this conductor is rotating to the left (disc is rotating clockwise).

1. Fleming's Right Hand Rule
Using Fleming's Right Hand Rule, the induced current flows through the conductor in the direction __XY__, in an __anti-clockwise__ direction through the connecting wire.

2. Lenz's Law
The induced current would flow in a direction so as to produce a __force__ which __opposes__ the clockwise rotation of the disc. Using Fleming's Left Hand Rule, for a force to be induced to the right, the current must flow through the conductor in the direction XY.

3. First principles
The free electrons in the conductor would have linear velocities to the left. Using Fleming's Left Hand Rule, there would be a __force__ pushing the electrons __towards X__. An electric field is set up across XY, with __Y__ being at a higher potential with respect to __X__. Hence, induced current flows in the direction __XY__ within the conductor.

17.6 Rotating coil in a uniform magnetic field (AC Generator)

A coil of N turns each of area A is being rotated at a steady angular speed ω in a uniform magnetic field of flux density B as shown in Fig. 11.

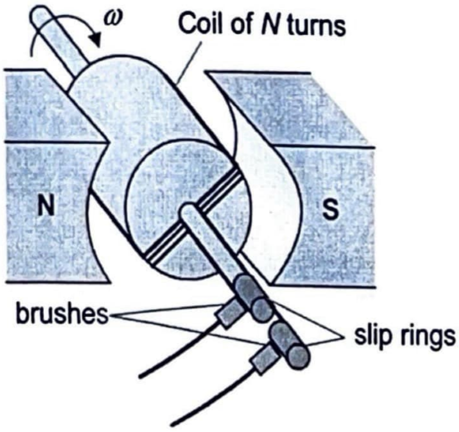


Fig. 11

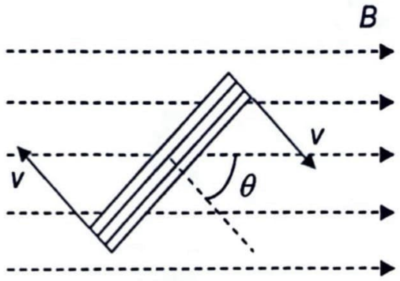


Fig. 12

In Fig. 12, the normal to the plane of the coil makes an angle θ with the magnetic field at time t (measured from the position where $\theta = 0°$). The magnetic flux linkage through the coil would be

$$\Phi = NBA\cos\theta = \underline{NBA\cos\omega t}$$

By Faraday's law,

$$E = -\frac{d\Phi}{dt}$$

$$= -\frac{d(NBA\cos\omega t)}{dt}$$

$$= -NBA\frac{d(\cos\omega t)}{dt}$$

$$= \underline{NBA\omega\sin\omega t}$$

This expression shows that the induced e.m.f. in the coil varies sinusoidally with time, i.e. the variation would be a sine or cosine function.

It can be seen that the maximum induced e.m.f. can be given by

$$E_{max} = \underline{NBA\omega}$$

Alternating current is generated by electromagnetic induction using this method.

Fig. 13 below shows the variation with time t of the magnetic flux linkage and induced e.m.f. in the coil as the coil rotates through 1 complete cycle.

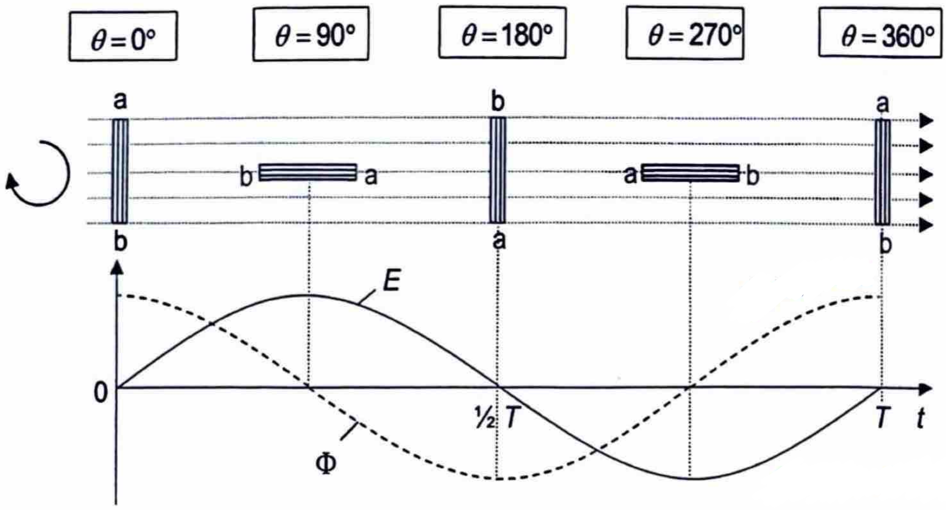


Fig. 13

Example 7

An AC generator consists of eight turns of wire each of area 0.090 m^2 and total resistance 12.0 Ω. The loop rotates in a magnetic field of 0.500 T at a constant frequency of 60.0 Hz. Fig. 14 shows the position of the loop with respect to the field at t = 0 s.

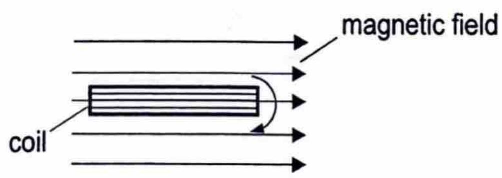


Fig. 14

(a) Determine
- **(i)** the angular frequency of rotation ω
- **(ii)** the maximum induced e.m.f. E_{max}
- **(iii)** the maximum induced current I_{max}

(b) Using the values in **(a)**, state expressions to represent the variation with time t of the induced e.m.f. E and current I through the loop.

Solution

(a)(i) $\omega = 2\pi f = 2\pi(60.0) = 377 \text{ rad s}^{-1}$

(ii) $E_{max} = NBA\omega = (8)(0.500)(0.090)(377)$
$= 136 \text{ V}$

(iii) $I_{max} = \frac{E_{max}}{R} = \frac{136}{12.0} = 11.3 \text{ A}$

(b) At $t = 0$ s, there is zero magnetic flux linkage. Hence, the variation of flux linkage with time is given by $\Phi = NBA \sin(377t)$

Since $E = -\frac{d\Phi}{dt}$,

$E = -NBA\omega \cos(\omega t) = -136 \cos(377t)$

$I = -I_{max} \cos(\omega t) = -11.3 \cos(377t)$

Back e.m.f.

In the previous chapter, we saw how electrical energy is converted into mechanical energy in an electric motor. The torque acting on a current-carrying coil in a uniform magnetic field causes it to rotate.

However, the magnetic flux linkage of the coil changes as it rotates. According to Faraday's law, an e.m.f. would be induced in the coil. By Lenz's law, this induced e.m.f. always acts to reduce the current supplied to the coil (or motor). This is known as the **back e.m.f.**, and its magnitude increases as the rotational speed of the coil increases.

As such, the effective current I_{net} flowing through the coil would decrease as the rotational speed increases.

$$I_{net} = I_{\text{supplied to motor}} - I_{\text{due to back e.m.f.}}$$

Example 8

A motor having coils of resistance 10 Ω is supplied by a p.d. of 120 V. When the motor is running at its maximum speed, the back e.m.f. is 70 V. Find the current in the coils when
(a) the motor is first turned on,
(b) the motor has reached maximum speed.

Solution
(a) when the motor is first turned on, there is no back e.m.f

$I = \frac{V}{R} = \frac{120}{10} = 12 \text{ A}$

(b) When the motor has reached maximum speed,

$I = \frac{V - V_{back\ emf}}{R} = \frac{120 - 70}{10} = 5.0 \text{ A}$

17.7 Practical Applications of Electromagnetic Induction

AC Generators

As discussed earlier, alternating current (AC) is generated by rotating a coil in a uniform magnetic field. Today, almost all power stations generate electricity using AC generators. The mechanical energy required to rotate the coil can be derived from a variety of sources (e.g. water, burning coal etc).

Alternating current is preferred to direct current (DC) in the distribution of electricity as using AC is much more efficient. You will learn more about AC in the next topic.

Transformers

Example 4 illustrated how an e.m.f. (and hence current) could be induced in the secondary coil by changing the current (and hence magnetic flux) of the primary coil. This is the working principle behind transformers, where electrical energy can seemingly be 'transferred' from one coil to another, even though both coils are not in physical contact.

In fact, it was through a similar set up that Faraday discovered the phenomenon of electromagnetic induction in 1831.

A practical and more efficient arrangement of the coils is shown in Fig. 15 below.

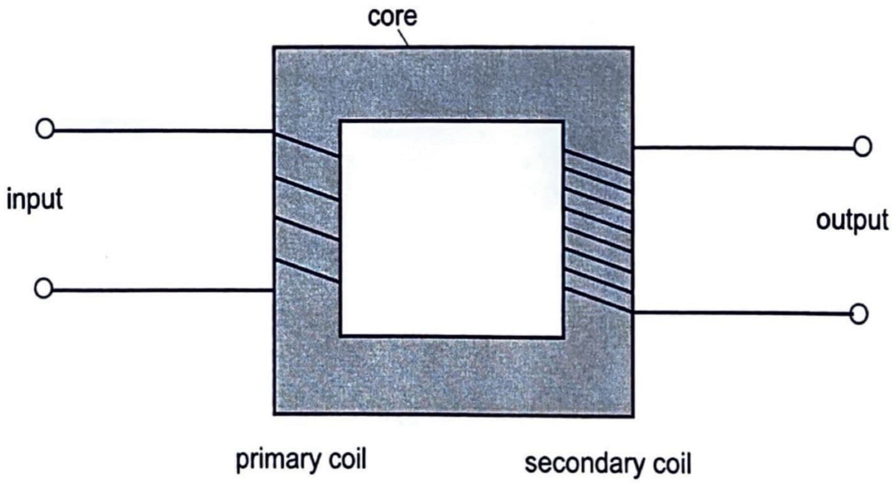


Fig. 15

Transformers are used to either raise (step up) or lower (step down) voltages. In electrical distribution systems, stepping up voltage during transmission helps to reduce power losses during transmission. Domestically, we use transformers to convert electricity from 240 V to 120 V or vice versa so that we can use our electrical appliances internationally.

Wireless Charging

Many of our portable devices (e.g. smartwatches, smartphones, electric toothbrush, medical devices, portable vacuum cleaner etc) make use of wireless charging. This is made possible by the same principle behind transformers.

The wireless charging station consists of a primary coil. When an alternating current passes through the primary coil, a changing magnetic flux induces a current in a secondary coil which is situated within the device to be charged.

Eddy Currents

Eddy currents are loops of induced current that circulate in a conductor of extended size when there is a change in the magnetic flux linkage through the conductor.

By Lenz's law, eddy currents flow in a direction that oppose the change that gives rise to them.

For example, in Fig, 16a, if a rectangular aluminum metal plate is set swinging between the poles of a strong magnet, it soon comes to rest because the eddy currents circulating inside the metal oppose the motion of the plate.

But if many slots are cut into the plate, as seen in Fig. 16b, the plate oscillates for a longer time before coming to rest. The eddy currents are considerably reduced in this case as they cannot flow across the many air gaps formed by the slots.

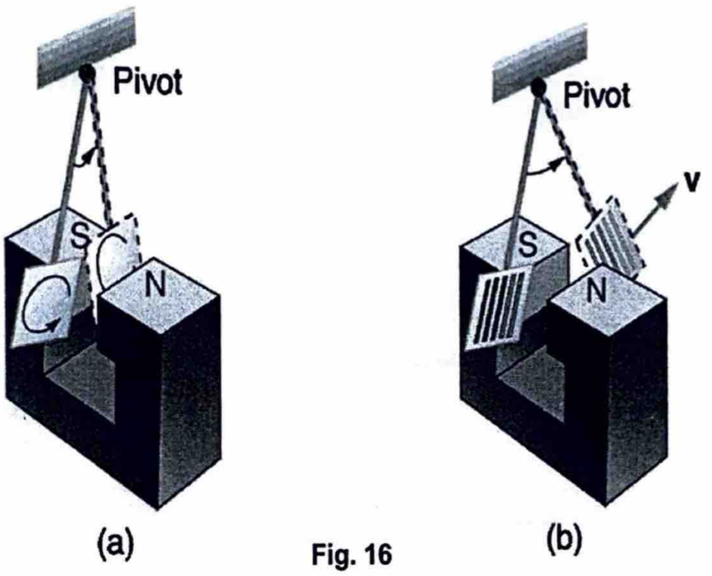


Fig. 16

Induction Heating

Eddy currents lead to the heating of the conductor ($P = I^2R$) in which they are circulating. This is sometimes undesirable as the heating represents power loss.

On the other hand, eddy currents provide a useful means of heating a metallic substance (which is inaccessible to other ways of heating) to a very high temperature. The sample is placed in the centre of a coil that carries a high frequency current. This is known as induction heating.

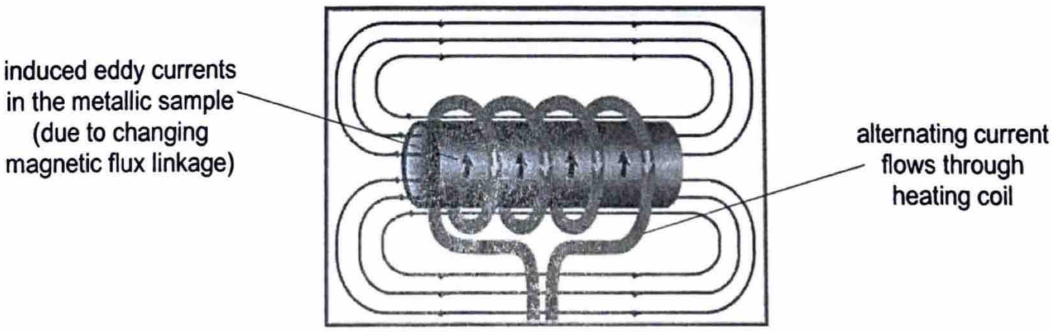


Fig. 17

Induction Cookers

An induction cooker consists of a coil of copper wire underneath the ceramic surface. When an alternating current flows through the coil, it produces an alternating magnetic field all around it.

When a frying pan made of ferromagnetic material is placed on the cooking zone, the changing magnetic flux linkage through the base of the pan induces eddy currents in the pan. Heat from the pan then transfers to the liquid or food in it.

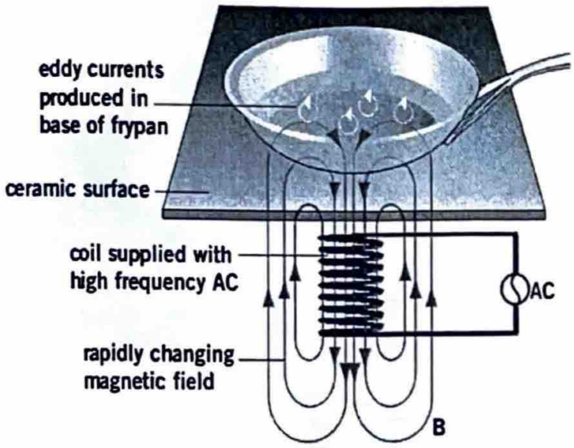


Fig. 18

Some advantages of using induction cookers include:

- It is deemed safer as there is no direct heat or naked flame used.
- More energy efficient hence more cost savings.
- More even cooking as the eddy currents are induced throughout the base of the pot, eliminating the occurrence of hot spots which may in turn burn or scorch the food.

Electromagnetic Braking

Electromagnetic braking in vehicles is an application of the case of a rotating disc in a magnetic field which was discussed in Section 17.5.

The set up is similar to that shown in Fig. 10, except that the magnetic field is produced by an electromagnet. When the vehicle is moving, the disc rotates. When the brakes are applied, the electromagnet is activated. As the disc now cuts the magnetic field, eddy currents are induced in the disc. These eddy currents generate a magnetic force which act as the braking force. This is an example of an application of Lenz's law.

This frictionless braking system has the following advantages over conventional brakes:

- Little or no heat produced
- Easier maintenance as there is little or no wear and tear of parts
- Retarding force is not affected by wet conditions

Electromagnetic brakes are very useful for slowing down high speed vehicles such as trains. However, as the disc slows down, the induced eddy currents would be smaller, making the braking less efficient. Hence, it is not used in the parking brake of a car.

Metal Detectors

A metal detector consists of a coil of wire known as the transmitter coil. When an alternating current passes through the transmitter coil, a changing magnetic field is set up. When the coil is placed over a metal object, eddy currents are induced in the object. As a result, the metal object produces its own magnetic field.

Inside the metal detector lies another coil known as the receiver coil. When the detector moves over the metal object, the magnetic field of the object induces a current in the receiver coil, which is connected to a circuit containing a loudspeaker.

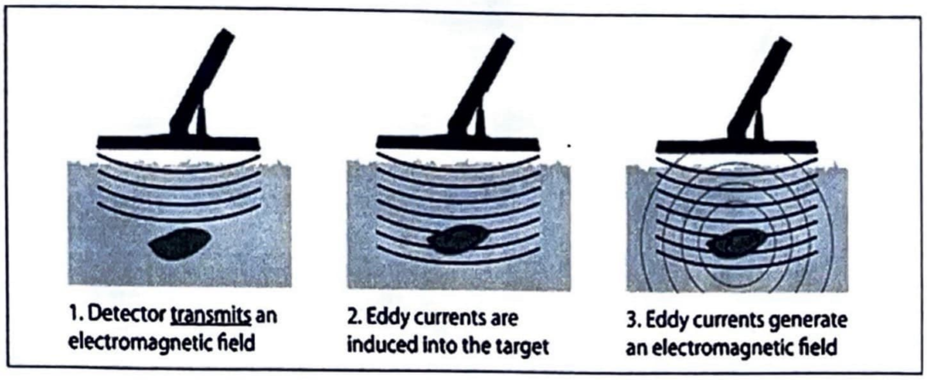


Fig. 19

17.8 Miscellaneous Examples

Example 9

A square wire frame PQRS with sides 0.10 m, moves with a constant velocity v of 0.20 m s^{-1} into a region of uniform magnetic flux density B of 1.0×10^{-2} T acting into the plane of the paper as shown in Fig.20. The resistance of the frame is 5.0 Ω.

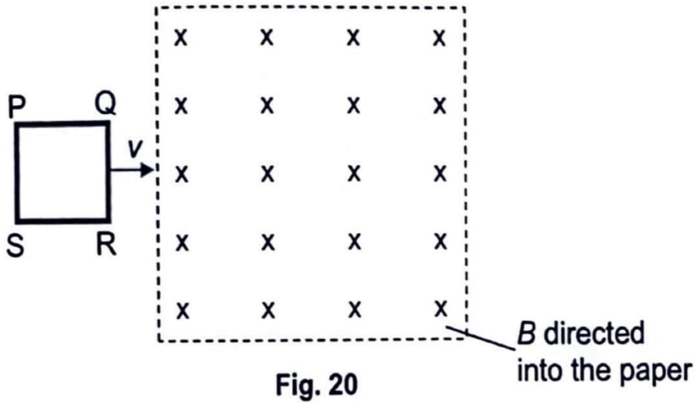
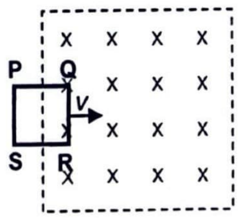


Fig. 20

Deduce the magnitude and direction of the current flowing in the frame when
(a) QR has just entered the field,
(b) the whole frame is entirely within the field and
(c) QR just moves out of the field.

Solution



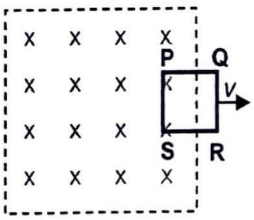
(a) When side QR just enters the field, there is an induced e.m.f. E along that side of the frame given by BLV.

Hence, magnitude of the induced current is

$$I = \frac{E}{R} = \frac{BLV}{R} = \frac{(1.0\times10^{-2})(0.10)(0.20)}{5.0} = 4.0\times10^{-5}\text{ A}$$

Using Fleming's Right-Hand Rule, the direction of the induced current is from R to Q. Hence, current flows anti-clockwise around the frame.

(b) With the whole frame PQRS moving through the magnetic field, the flux linkage through PQRS is constant. No e.m.f. (and hence no current) is induced.



(c) When QR just moves out of the field, there is an e.m.f. induced along the left side of the frame given by $E = BLv$.

The magnitude of the induced current is the same as that in part (a), i.e.

$$I = 4.0\times10^{-5}\text{ A}$$

Using Fleming's Right-Hand Rule, the direction of the induced current is from S to P. Hence, current flows clockwise around the frame.

Example 10
[J93/II/4(part)]

A current-carrying solenoid is placed near to a search coil as shown in Fig. 21.

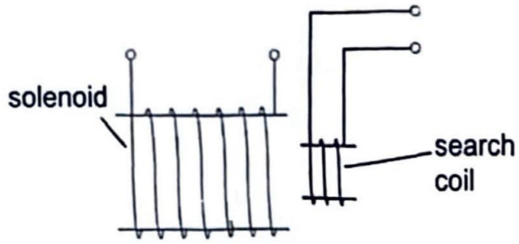
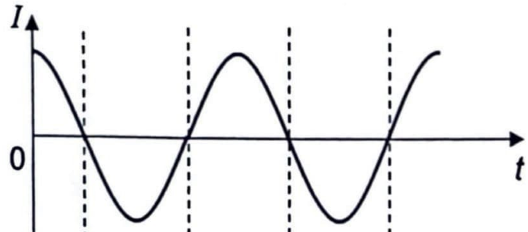


Fig. 21

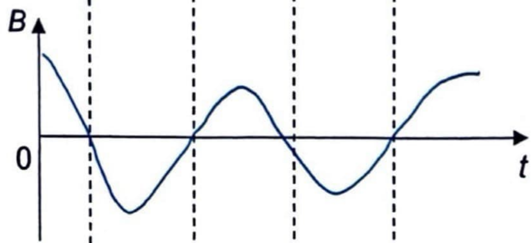
The variation with time t of the current I in the solenoid is shown in Fig. 22a.

Fig. 22a



$I = I_{max} \cos \omega t$

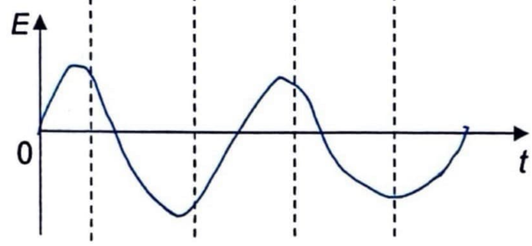
Fig. 22b



$B \propto I$

$B = B_{max} \cos \omega t$

Fig. 22c



$E = -\frac{dB}{dt}$

$E = E_{max} \sin \omega t$

(a) Sketch on the given axes in Fig. 22b and Fig. 22c, the variation with t of
  (i) the magnetic flux density B in the solenoid
  (ii) the e.m.f. E induced in the search coil.

(b) Describe and explain the effect on the amplitude and frequency of E if, separately,
  (i) a ferrous core is slowly introduced into the solenoid
  (ii) the frequency of the current in the solenoid is increased, whilst maintaining the same amplitude.

Solution

$$E = -\frac{d\Phi}{dt} \propto \frac{dB}{dt} \propto \frac{dI}{dt}$$

Since magnetic flux density *B* at the centre of a solenoid carrying current *I* is given by $B\ \mu_o n I$, where *n* is the number of turns per unit length.

(b)(i) With the introduction of a ferrous core, the magnetic flux density *B* in the solenoid would increase in magnitude but not frequency. Hence *E* will have a larger magnitude but no change in frequency.

(ii) An increased frequency of current leads to an increased rate of change of magnetic flux linkage. *E* will thus have both larger frequency and amplitude.

Example 11
[J94 P3/Q3]

The north pole of a magnet is placed inside a coil of wire, as shown in the diagram. The terminals of the coil are connected to the Y-plates of a cathode ray oscilloscope (c.r.o.) which may be assumed to have infinite input resistance.

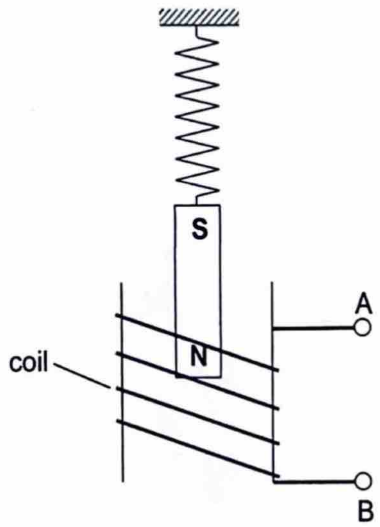


Fig. 23

(a) Sketch a graph to show how the induced e.m.f. in the coil will vary with time *t* when the magnet oscillates in the coil. Mark relevant times (for example, t_1, t_2, t_3) on the *t*-axis of your graph.

(b) Use the laws of electromagnetic induction to explain the shape of your graph.

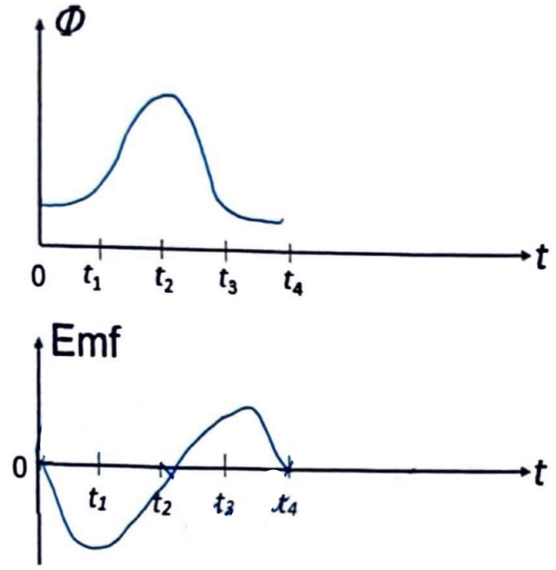
(c) A high resistance resistor is now connected in parallel with the c.r.o. between the points A and B. Draw a second graph to show how the e.m.f. will vary with time *t*.

(d) Explain, in terms of the principle of conservation of energy, why this graph is different from your first graph.

(e) Describe, with the aid of a sketch graph, the changes which would occur in the shape of the graph drawn in **(c)** if the resistance of the resistor has been reduced to a very low value.

Solution

(a) Assume that the magnet is moved vertically up a certain displacement and released.



***t* = 0 s** : The magnet is at the highest point of its oscillations, minimum overlap with the coil and hence, minimum flux linkage.

$\frac{d\Phi}{dt} = 0$

$\Rightarrow$ induced e.m.f $= 0$

t_1 : A point somewhere in between the points of maximum overlapping and minimum overlapping of the magnet and the coil.

$\frac{d\Phi}{dt} =$ maximum

$\Rightarrow$ maximum induced e.m.f

t_2 : The magnet is at the lowest point of its oscillation, maximum overlap with the coil and hence maximum flux linkage.

$\frac{d\Phi}{dt} = 0$

$\Rightarrow$ induced e.m.f $= 0$

t_3 : Same point as t_1 , but the magnet is moving out of the coil. E.m.f. is also maximum, but of opposite sign to that at t_1.

$\frac{d\Phi}{dt} =$ maximum $\Rightarrow$ maximum induced e.m.f

t_4 : The magnet is back at the highest point of its oscillations

$\frac{d\Phi}{dt} = 0$

$\Rightarrow$ induced emf $= 0$

(b) From $t = 0$ to t_2

The flux linkage Φ of the coil is increasing as it moves into the coil. Hence there is an induced e.m.f. according to Faraday's Law.

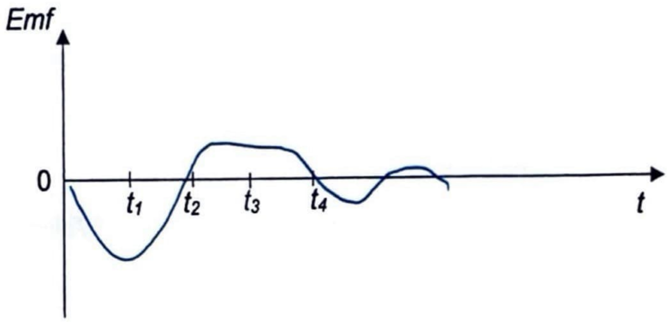
The e.m.f. peaks at t_1, where $\frac{d\Phi}{dt}$ is maximum.

From t_2 to t_4

The flux linkage of the coil is decreasing as the magnet moves out of the coil. Again, e.m.f. is induced, but in the opposite direction.

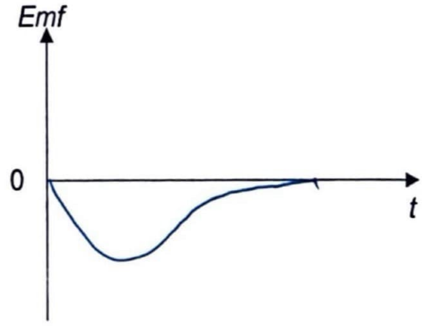
The e.m.f. peaks at t_3, where $\frac{d\Phi}{dt}$ is maximum.

(c)



(d) A small current now flows through the resistor and hence heat is dissipated. Heat energy is produced at the expense of electrical energy. The oscillation dies off (due to light damping) and the magnitude of the induced e.m.f. becomes smaller with time.

(e)



If the resistance is very low, the induced current is very large and this leads to a higher rate of heat dissipation. Hence, there is a larger degree of damping and the oscillations die off faster or there may not be any oscillations due to heavy damping.

Summary

1. Magnetic flux is defined as the product of an area and the component of the magnetic flux density perpendicular to that area.
2. The magnetic flux linkage of a coil is the product of the magnetic flux through the coil and the number of turns of the coil.
3. Magnetic flux and magnetic flux linkage are measured in weber (Wb).
4. Faraday's law of electromagnetic induction states that the induced e.m.f. is proportional to the rate of change of magnetic flux linkage.
5. Lenz's law states that the direction of induced e.m.f. is such as to cause effects to oppose the change producing it.
6. Mathematically, induced e.m.f. is given by:

$$\text{Induced e.m.f.} = -\frac{d\Phi}{dt}$$

When this is applied to different situations, different expressions are obtained:

	Situation	Formulae for induced e.m.f.
1	Moving rod	e.m.f. = BLv
2	Generator	e.m.f. = $NBA\omega \sin\omega t$
3	Rotating Disc	e.m.f. = $BAf = \frac{1}{2} Br^2\omega$

7. To find the *direction* of the induced e.m.f. (or current), use either Fleming's Right-Hand Rule, Lenz's Law or consider the direction of the magnetic force experienced by the charge carriers in the object.

APPENDIX

Biography of Michael Faraday

Michael Faraday (1791 – 1867)

Michael Faraday was born in Newington Butts, near present-day Elephant and Castle in South London, England. His family was extremely poor; his father, James Faraday, was a Yorkshire blacksmith who suffered ill-health throughout his life. After the most basic of school educations, Faraday had to educate himself. At fourteen he became apprenticed to a local bookbinder and book seller George Riebau and, during his seven-year apprenticeship, he read many books, including Isaac Watts' *The Improvement of the Mind*, the principles and suggestions contained therein he enthusiastically implemented. He developed an interest in science and specifically electricity. In particular, he was inspired by the book *Conversations in Chemistry* by Jane Marcet.

At the age of twenty, in 1812, at the end of his apprenticeship, Faraday attended lectures by the eminent English chemist and physicist Humphry Davy of the Royal Institution and Royal Society, and John Tatum, founder of the City Philosophical Society. Many tickets for these lectures were given to Faraday by William Dance (one of the founders of the Royal Philharmonic Society). Afterwards, Faraday sent Davy a three hundred page book based on notes taken during the lectures. Davy's reply was immediate, kind and favorable. When Davy damaged his eyesight in an accident with nitrogen trichloride, he decided to employ Faraday as a secretary. When John Payne, one of the Royal Institution's assistants, was sacked, the now Sir Humphry Davy was asked to find a replacement. He appointed Faraday as Chemical Assistant at the Royal Institution on 1 March 1813.

Faraday's earliest chemical work was as an assistant to Davy. He made a special study of chlorine, and discovered two new chlorides of carbon. He also made the first rough experiments on the diffusion of gases, a phenomenon first pointed out by John Dalton, the physical importance of which was more fully brought to light by Thomas Graham and Joseph Loschmidt. He succeeded in liquefying several gases; he investigated the alloys of steel, and produced several new kinds of glass intended for optical purposes.

Faraday also discovered chemical substances such as benzene (which he called bicarburet of hydrogen), inventing the system of oxidation numbers, and liquefying gases such as chlorine. He prepared the first clathrate hydrate. Faraday also discovered the laws of electrolysis and popularized terminology such as anode, cathode, electrode, and ion, terms largely created by William Whewell. For these accomplishments, many modern chemists regard Faraday as one of the finest experimental scientists in history.

Faraday's greatest work was with electricity. The first experiment which he recorded was the construction of a voltaic pile with seven halfpence pieces, stacked together with seven disks of sheet zinc, and six pieces of paper moistened with salt water. With this pile, he decomposed sulphate of magnesia.

At this stage, there is also evidence to suggest that Davy may have been trying to slow Faraday's rise as a scientist (or natural philosopher as it was known then). In 1825, for instance, Davy set him onto optical glass experiments, which progressed for six years with no great results. It was not until Davy's death, in 1829, that Faraday stopped these fruitless tasks and moved on to endeavors that were more worthwhile. Two years later, in 1831, he began his great series of experiments in which he discovered electromagnetic induction, though the discovery may have been anticipated by the work of Francesco Zantedeschi. His breakthrough came when he wrapped two insulated coils of wire around a massive iron ring, bolted to a chair, and found that upon passing a current through one coil, a momentary current was induced in the other coil. The iron ring-coil apparatus is still on display at the Royal Institution. In subsequent experiments he found that if he moved a magnet through a loop of wire, an electric current flowed in the wire. The current also flowed if the loop was moved over a stationary magnet.

His demonstrations established that a changing magnetic field produces an electric field. This relation was mathematically modelled by Faraday's law, which subsequently went on to become one of the four Maxwell equations.

In 1848, as a result of representations by the Prince Consort, Michael Faraday was awarded a Grace and favour house in Hampton Court, Surrey free of all expenses or upkeep. This was the Master Mason's House, later called Faraday House, and now No.37 Hampton Court Road. In 1858 he retired to live there.

During his lifetime, Faraday rejected a knighthood and twice refused to become President of the Royal Society.

He died at his house at Hampton Court on August 25, 1867. He has a memorial plaque in Westminster Abbey, near Isaac Newton's tomb, but he turned down burial there and is interred in the Sandemanian plot in Highgate Cemetery.

Measurement of the Magnetic Flux Density Using a Search Coil

A search coil can be used to measure the magnetic flux density of a magnetic field. It is placed in the magnetic field to be measured so that the flux density is at right angles to the plane of the coil.

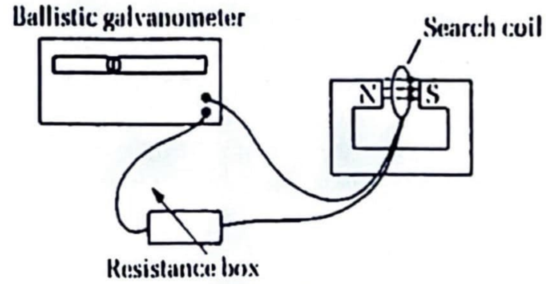


Fig. 24

If N = number of turns in the search coil and A = area of the coil, then the initial magnetic flux linkage is $\Phi_0 = NBA$

The search coil is then quickly removed from the magnetic field and the magnetic flux linkage is now $\Phi_t = 0$

This sudden change in the magnetic flux linkage causes a current to be induced in the search coil. When charges flow through the ballistic galvanometer, the first 'throw' θ produced by the change in flux is noted.

The charge Q driven through the coil is proportional to θ. That is, $Q = k\theta$, where k is found by calibrating the galvanometer.

From $I = \frac{dQ}{dt}$, $Q = \int_0^t I dt$.

From $E = IR$, $I = \frac{E}{R}$, where E is the induced e.m.f. and R the resistance of the search coil.

Therefore,

$$Q = \int_0^t I dt = \int_0^t \frac{E}{R} dt = -\frac{1}{R}\int_0^t \frac{d\Phi}{dt} dt = -\frac{1}{R}\int_0^t d\Phi = \frac{1}{R}[\Phi_0 - \Phi_t] = \frac{NBA}{R}$$

Hence, $\frac{NBA}{R} = k\theta$ and $B = \frac{k\theta R}{AN}$

Application of EMI in Protection for Moving Coil Meters

Suspension-type meters are very sensitive but easily damaged by careless handling. To protect the meter when not in use, the meter terminals should be connected together. When the meter is moved carelessly, the motion of its coil in the magnetic field creates induced current in the coil. The induced current causes magnetic forces to act on the coil to oppose its motion relative to the field. So movement of the sensitive coil relative to the magnet is greatly reduced by connecting the terminals together.

If the coil is wound on a metal frame, current is induced in the frame too, so that also helps to prevent the coil from swinging too freely. Also, when in use, induced current in the frame serves to slow the frame down, which stops the meter pointer from over-shooting.

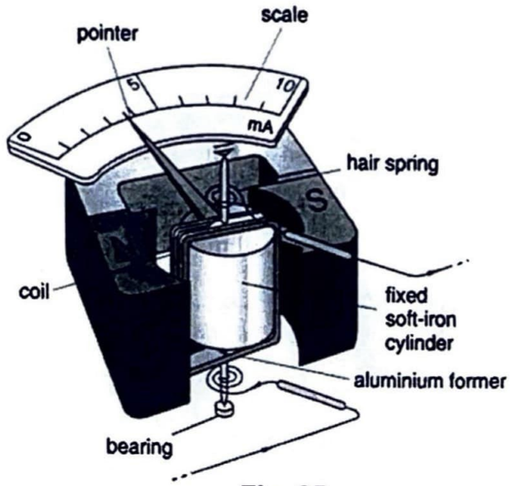


Fig. 25

RAFFLES INSTITUTION
PHYSICS DEPARTMENT

Tutorial 17 ELECTROMAGNETIC INDUCTION

Self-check Questions

S1 Define magnetic flux and give a mathematical expression to determine its value.
S2 Define magnetic flux linkage and give a mathematical expression to determine its value.
S3 State Faraday's law of electromagnetic induction.
S4 State the factors that would affect the magnitude of induced e.m.f.
S5 State Lenz's law and explain how it is consistent with the principle of conservation of energy.
S6 Describe the ways which can be used to determine direction of induced current.
S7 Describe and explain some practical applications of electromagnetic induction in our daily life.

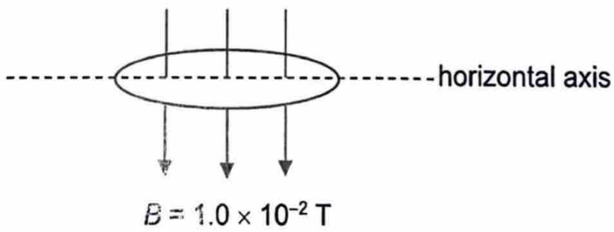
Self-Practice Questions

SP1 A flat, circular loop of wire of radius r is placed with its plane at right angles to a uniform magnetic field. The magnetic flux density is B and the magnetic flux through the loop is Φ.

What is the magnetic flux density and the magnetic flux through a new loop of radius ½ r in the same plane?

	magnetic flux density	magnetic flux through new loop
A	B	¼ Φ
B	B	½ Φ
C	¼B	Φ
D	½B	Φ

SP2 A narrow coil of 10 turns and area 4.0×10^{-2} m^2 is placed in a uniform magnetic field of flux density 1.0×10^{-2} T such that the magnetic field lines pass through the coil perpendicularly.

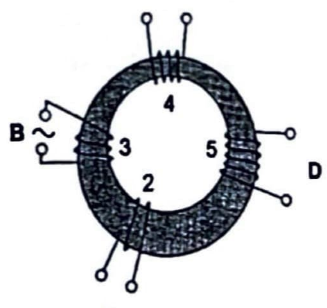


Calculate the average induced e.m.f. in the coil if

(a) it is removed from the field in 0.50 s
(b) it is turned over the horizontal axis once in 0.50 s
(c) it is turned over the horizontal axis twice in 0.50 s.

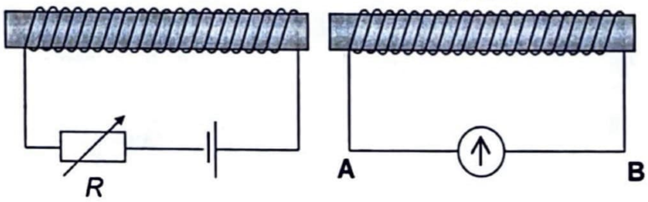
SP3 A soft-iron ring of variable cross-section has four coils wound round it at the positions shown. The coils have 2, 3, 4 and 5 turns. The 3-turn coil is connected to an a.c. supply.

In which coil does the magnetic flux density have the greatest variation?



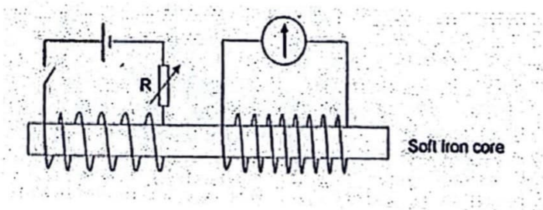
(2010 P1 Q32)

SP4 The diagram shows a coil X connected in series with a variable resistor *R* and a battery while another coil Y is connected in series with a galvanometer. The two coils are not in contact.



Determine the direction of the induced current flowing through the galvanometer at the instant when
(a) coil Y is moved towards coil X,
(b) the resistance *R* is increased.

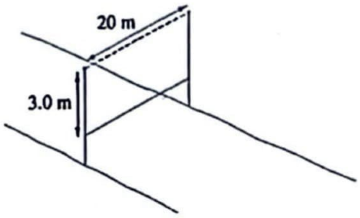
SP5 Two coils of wire are wound around a soft iron core as shown.



The following sequence of events happened.
(i) The circuit shown was arranged with the switch open and the value of *R* of the variable resistor set at its highest.
(ii) The switch was then closed.
(iii) *R* was steadily reduced until its lowest setting.
(iv) The switch was then opened.

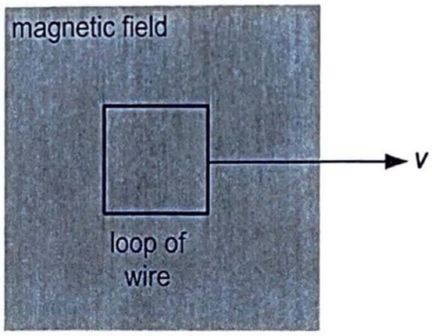
When did the galvanometer give the highest reading?
A At the end of (i)
B During (ii)
C During (iii)
D During (iv)

SP6 At the beginning of a horse race, a horizontal straight wire of length 20 m is raised vertically through a height of 3.0 m in 0.20 s.



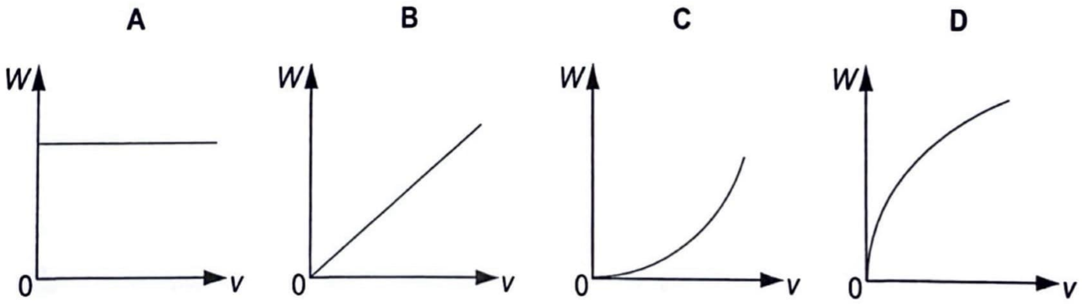
The horizontal component of the Earth's magnetic field strength perpendicular to the wire is 2.0×10^{-5} T. What is the average e.m.f. induced across the ends of the wire? **[J01/I/19]**

SP7 A square loop of wire is placed in a region of uniform magnetic field. The direction of the field is perpendicular to the plane of the loop of wire.



The loop is pulled out of the field at a uniform speed v. During this operation, the loops remains in the same plane.

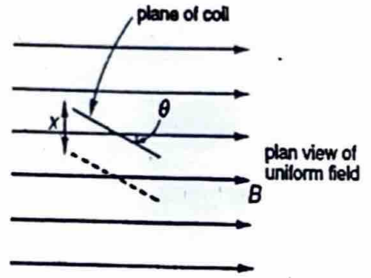
Which graph shows how the total work done W in pulling the loop out of the field depends on the speed v?



(2008 P1 Q10)

SP8 A plane coil of wire containing N turns each of area A is placed so that the plane of the coil makes an angle θ with the direction of a uniform magnetic field of flux density B. The coil is now moved through a distance x in time t to the position shown dotted in Fig. 3.

Fig. 3

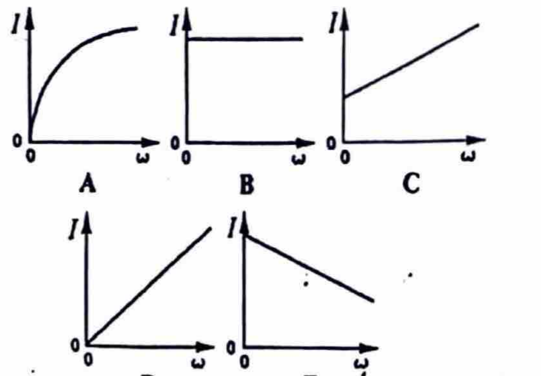


What is the e.m.f. induced in the coil?

A zero
B NAB/t
C $NAB\,x/t$
D $(NB\,x\cos\theta)/t$
E $(NAB\,x\cos\theta)/t$

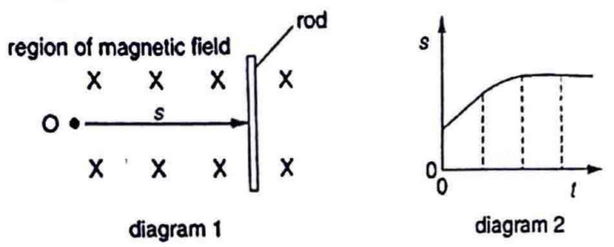
J82/II/20

SP9 A d.c. electric motor that has a permanent magnet as its field magnet is joined to a battery of constant e.m.f. and negligible internal resistance. When the motor is used to drive various loads, the corresponding values of its speed of rotation ω and the current I passing through it are measured. Which one of the following graphs most nearly shows how I varies with ω?

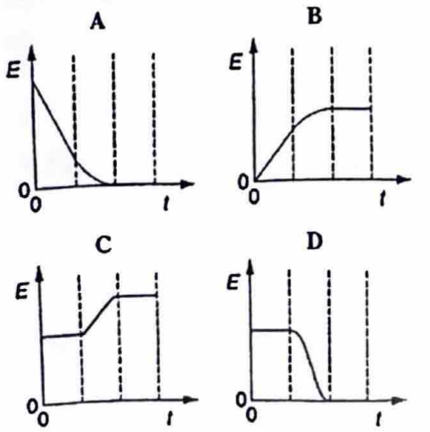


N82/II/23

SP10 Diagram 1 shows an aluminium rod, moving at right angles to a uniform magnetic field. Diagram 2 shows the variation with time t of the distance s from O.

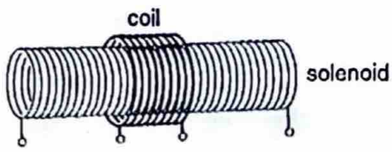


Which graph best shows the variation with t of the e.m.f. E induced in the rod?

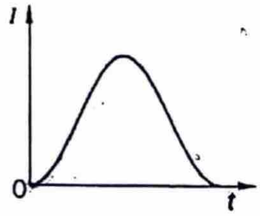


J99/I/20

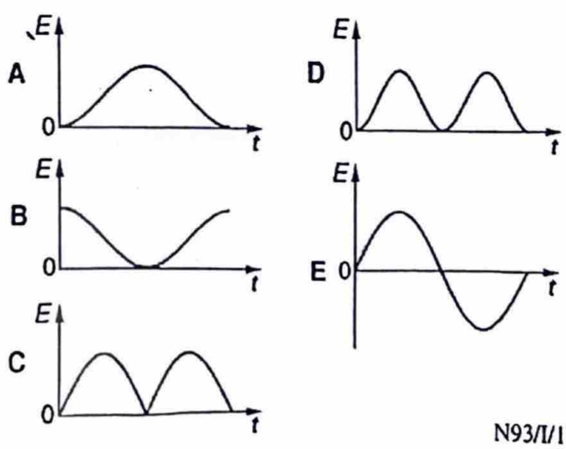
SP11 The diagram shows a short coil wound over the middle part of a long solenoid.



The solenoid current I is varied with time t as shown in the sketch graph. As a consequence, the flux density of the magnetic field due to the solenoid varies with time. The relation between B and I is $B = \mu_0 nI$.

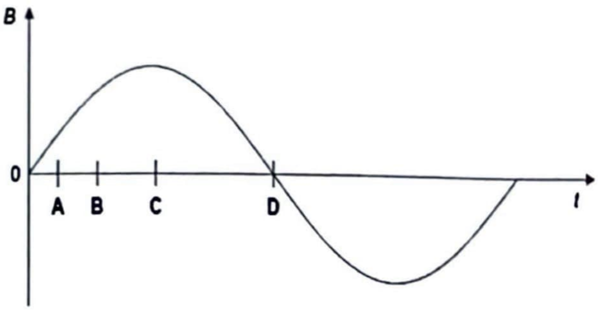


Which graph shows how the e.m.f. E induced in the short coil varies with time t?



N93/I/17

SP12 An e.m.f. is induced in a coil subjected to a changing magnetic field. The flux density *B* of this field varies sinusoidally with time *t* as shown.

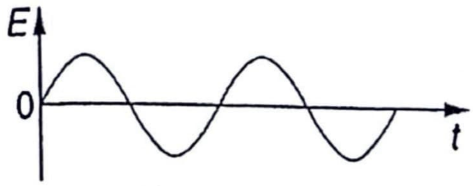


At which time, A, B, C or D, is the magnitude of the e.m.f. induced in the coil a maximum?

SP13 When a coil of wire is rotating in a magnetic field, the e.m.f. induced in the coil does not depend on

A the angular velocity of the coil
B the resistance of the coil
C the number of turns of the coil
D the magnetic flux density

SP14 When a coil is rotated in a magnetic field, the variation with time *t* of the induced e.m.f. *E* is as shown below.



On the same axes above, sketch the variation with *t* of *E* if the speed of rotation of the coil is doubled. **[J98/I/19]**

SP15 A rectangular coil of wire, initially placed as shown in Fig. a, is rotated with constant angular velocity in a magnetic field which acts in the direction of XX'. The sinusoidal e.m.f. represented in Fig. b is produced across the ends of the coils.

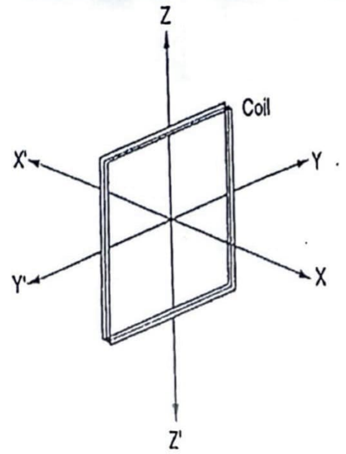


Fig. a

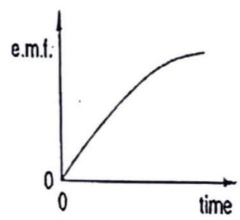


Fig. b

Which one of the following movements would have given this result?

A Rotation of a quarter revolution about axis YY'
B Rotation of a half revolution about axis XX'
C Rotation of a half revolution about axis ZZ'
D Rotation of a quarter revolution about axis XX'
E Rotation of a half revolution about axis YY'

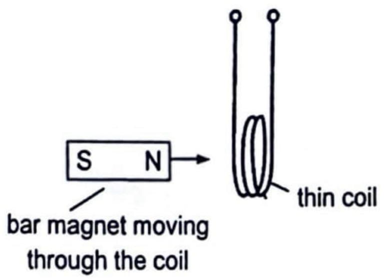
[N83/II/21]

Discussion Questions

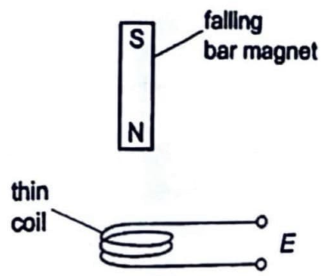
D1 For each of the following situations, sketch two graphs to represent the variation with time *t* of
- the magnetic flux linkage Φ through the coil
- the induced e.m.f. *E* in the coil

(a) A square coil of wire moving at a constant velocity across a uniform magnetic field (see diagram in lecture notes Example 9).

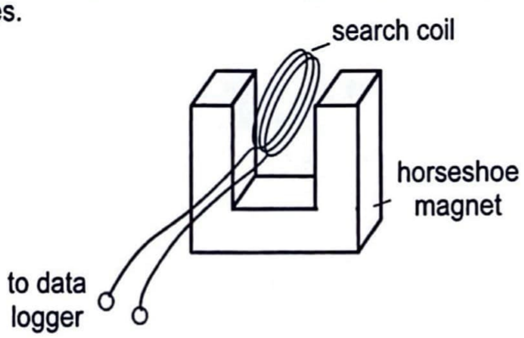
(b) A bar magnet moving at constant velocity through a thin coil of wire.



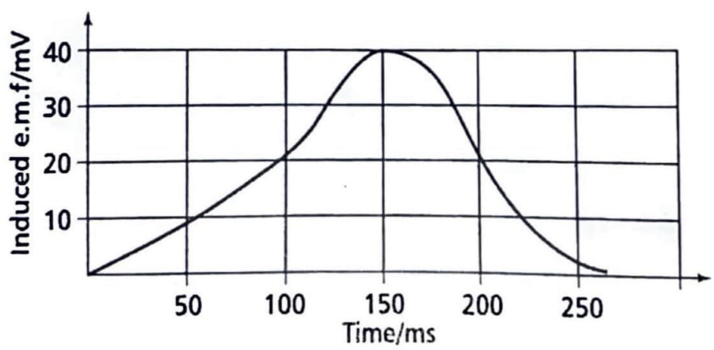
(c) A bar magnet released vertically from rest at a height above the coil of wire.



D2 A circular coil of diameter 34 mm with 100 turns is connected to a data logger. The coil is then positioned at rest between the poles of a horseshoe magnet with the plane of the coil parallel to the magnet's pole faces.



The coil is then rapidly removed from the magnet and the data logger is used to measure the induced e.m.f. at intervals of 10 ms. A graph of the measurements is shown below.

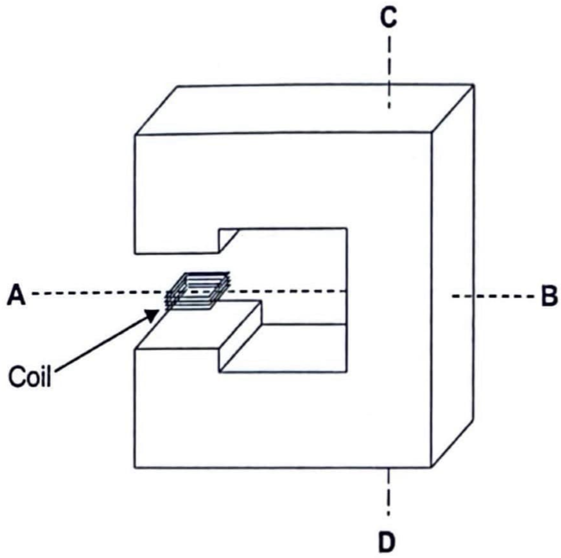


(a) Use Faraday's Law to explain why the area under the curve of the graph represents the change in flux linkage through the coil.

(b) From the graph, estimate the change of flux linkage when coil is removed from the magnetic field.

(c) Hence, calculate the magnetic flux density between the pole pieces.

D3 A small square coil has its plane set at right angles to the uniform magnetic field between the pole pieces of a horseshoe magnet as shown.



(a) The magnet is now rotated at constant angular velocity about the axis AB. Draw sketch graphs, on the same time axis, to show the variation of

(i) the magnetic flux through the coil,
(ii) the emf induced in the coil.

(b) Draw a second set of graphs for the case where the magnet rotates at constant angular velocity about the axis CD. (You may assume that the magnetic flux density in the region outside the pole pieces is zero.)

[N95/III/4 part]

D4 A revolving aluminium disc has small magnets equally spaced around its rim as shown in Fig. 8. The magnets are all aligned in the same direction with the north poles on the same side of the disc.

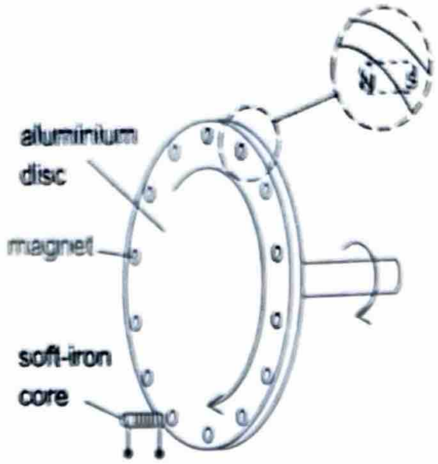


Fig. 8

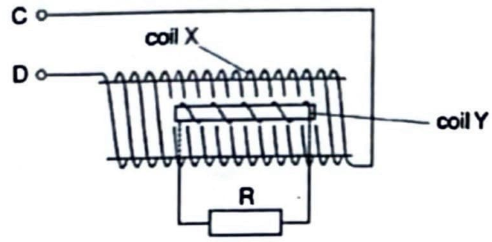
A small coil, wound on a soft-iron core, is fixed so that the north poles of the magnets pass close by the end of the coil without touching it. The terminals of the coil are connected to a detector which monitors the e.m.f. induced in the coil.

(a) Draw a sketch graph to show the possible variation with time of the e.m.f. induced in the coil as the magnets pass the coil. [3]

(b) Explain, on the basis of the laws of electromagnetic induction, the shape of your graph. [4]

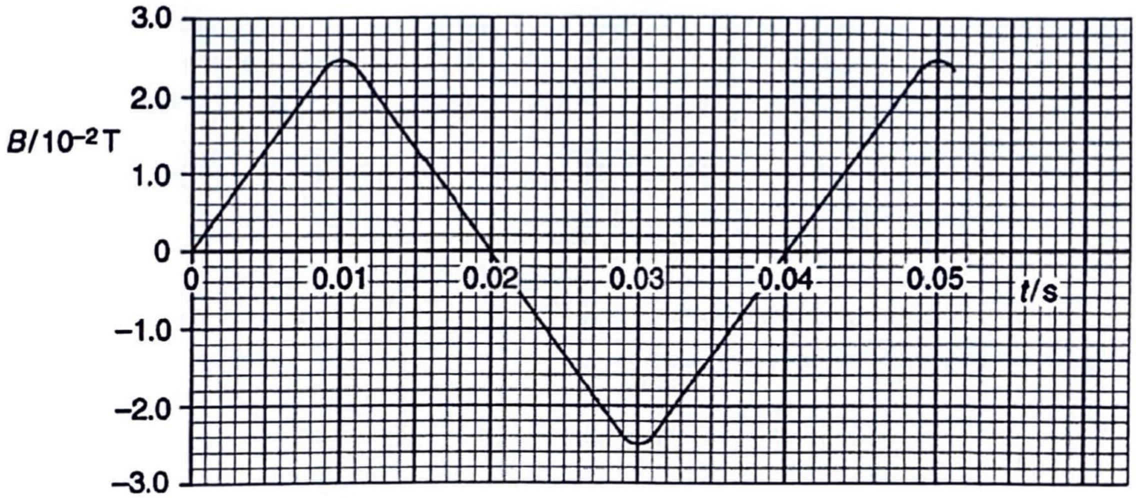
[N95/III/4 part]

D5 A pair of concentric coils is shown in the following figure.



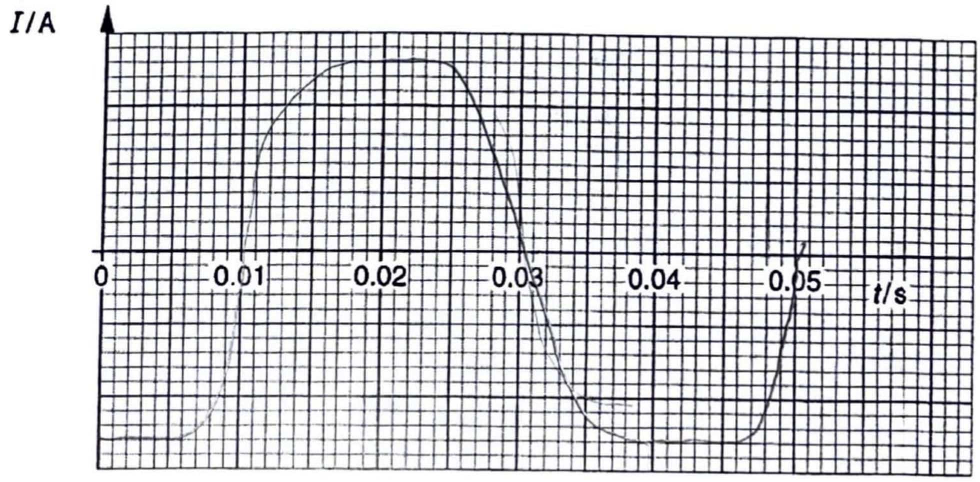
The outer coil X has 2500 turns and is connected to a variable power supply by the terminals CD. The inner coil Y has 500 turns, a cross-sectional area of 7.25×10^{-4} m^2 and a resistance of 5.00 Ω. Coil Y is connected to a resistor R of resistance 10.0 Ω.

The variation with time *t* of the magnetic flux density *B* in coil Y is shown.



(a) Calculate the maximum current in R. [3]

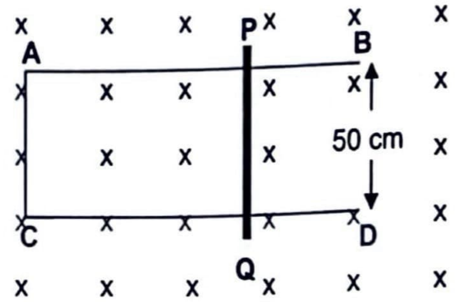
(b) On the axis given, sketch the variation with time *t* of current *I* in R.



[2]

[A-Level/2012/P2/Q5 part]

D6 **(a)** A conducting rod PQ of length 60 cm makes contact with the metal rails AB and CD that are 50 cm apart in a uniform magnetic field of flux density 0.50 T perpendicular to the plane of the paper as shown below. The resistance of PQ is 0.30 Ω and that of the rails is negligible.



(i) Determine the magnitude of the current induced in the rod when it is moved to the right with a velocity of 5.0 m s^{-1}.

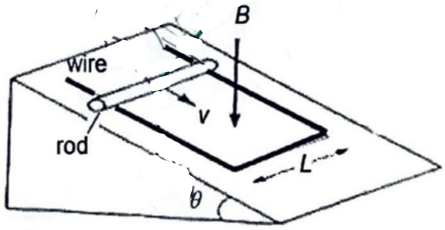
(ii) State the direction of the induced current in the rod PQ.

(iii) Determine the magnitude of the applied force required to keep the rod in motion.

(iv) Compare the rate at which mechanical work is done by the applied force with the rate of thermal energy dissipated in the circuit.

(b) The rod PQ now slides down a frictionless slope (inclined at θ to the horizontal) where a magnetic field B acts vertically downwards as shown. PQ makes contact with a rectangular circuit composed of wire of negligible resistance

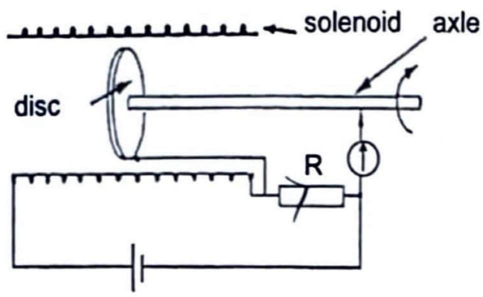
The mass of the rod is m and the portion of the rod that is in contact with the wire is of length L and resistance R.



(i) Sketch and label the forces acting on the rod as it slides down.

(ii) Find an expression, in terms of the quantities listed in this part of the question, for the terminal velocity v of the rod.

D7 The figure below shows a long solenoid which is connected in series with a d.c. supply and a resistor of resistance R.



A small copper disc mounted at the centre of the solenoid spins on an axle which lies along the axis of the solenoid.

By means of brushes, one terminal of the resistor is connected to the edge of the disc and the other is connected to the axle via a galvanometer. An e.m.f. is generated between the axle and the edge of the disc when the disc rotates.

(a) Label, with an arrow, the direction of the current I flowing through R before the disc spins

(b) When the disc rotates, the magnitude of the deflection on the galvanometer starts to decrease until there is zero deflection. Give an explanation for this.

(c) If the disc has an area A and is rotating at f revolutions per unit time when the galvanometer registers no deflection, show that the resistance is given by the expression:

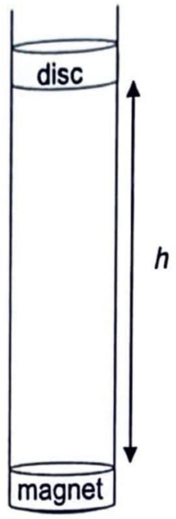
$$R = \mu_o nAf$$

where n is the number of turns per unit length of the solenoid.

(The magnetic flux density B at the centre of the solenoid is given by the expression $B = \mu_o nI$ where I is the current in the solenoid.)

[N87/III/12modified]

D8 A conducting disc of mass m, placed within a vertical smooth cylinder, is released from height h above a strong magnet as shown.

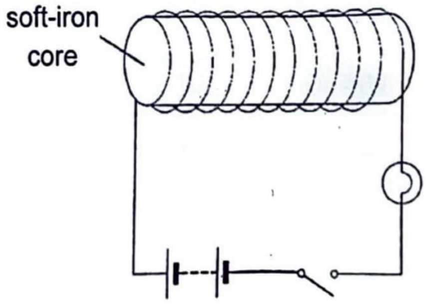


Using the laws of electromagnetic induction, explain the following observations as the disc falls along the cylinder :

(a) the temperature of the disc increases

(b) the acceleration of the disc is smaller than the acceleration of free fall.

D9 A coil, consisting of many turns of insulated metal wire wrapped around a soft-iron core, is connected in series with a battery, a switch and a lamp, as shown in the figure.



(a) State what happens to the magnitude of the magnetic flux in the coil as the current increases from zero when the switch is closed. [1]

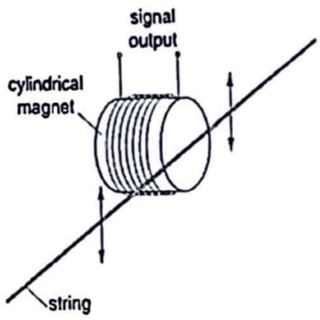
(b) Hence, explain why an e.m.f. is induced in the coil as the current increases. [1]

(c) Hence, explain why there is a noticeable delay before the lamps light up after the switch is closed. [2]

(d) State and explain what will happen to the length of the delay if the soft-iron core is replaced by one made of wood. [2]

[J95/II/4(part)]

D10 The pick-up on an electric guitar produces an electrical signal from the vibrations of the guitar strings made of steel. The pick-up consists of a small coil of insulated wire wound round a small cylindrical bar magnet as illustrated in the figure below. When the string vibrates, an electrical signal is generated between the terminals of the coil.



Explain why an electrical signal is generated. [3]

[N01/II/6]

Challenging Question

C1. A rigid wire, firmly supported, is in contact with another wire that is bent into a semi-circle of radius 0.20 m. The semi-circular portion may be rotated with a handle, as shown in Fig.10.

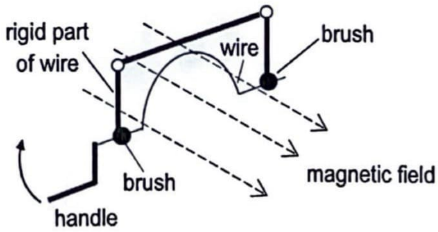


Fig.10

Conducting brushes allow the rigid part to make contact with the semi-circular wire. The setup is placed in a uniform magnetic field of flux density 0.80 T, and the semi-circular wire is rotated generating an alternating e.m.f. with a peak value of 0.31 V.

At what angular speed is the wire rotating?

Suggested Solutions

Self-check Questions

S1 **Magnetic flux** defined as the product of an area and the component of the magnetic flux density perpendicular to that area.

The expression for magnetic flux ϕ is $\phi = BA\cos\theta$

where θ is the angle that the magnetic flux density vector B makes with the normal to the area A.

S2 The **magnetic flux linkage** of a coil is the product of the magnetic flux through the coil and the number of turns of the coil.

The expression for magnetic flux linkage is $\Phi = N\phi = NBA\cos\theta$,

where N is the number of turns in the coil.

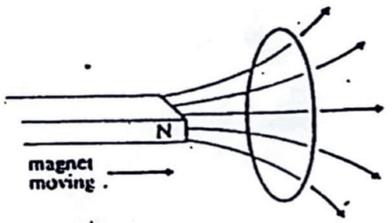
S3 **Faraday's law of electromagnetic induction** states that the induced e.m.f. is proportional to the rate of change of magnetic flux linkage.

Induced e.m.f. = $-\dfrac{d\Phi}{dt}$

S4
- Rate of change of magnetic flux density through the coil/circuit
- Rate of change of area of the coil/circuit
- Number of turns in the coil

S5 **Lenz's law** states that the direction of induced e.m.f. is such as to cause effects to oppose the change producing it.

Consider a bar magnet being pushed into a circular coil as shown.



As the magnet moves towards the coil, the flux linkage through the coil increases. A current will be induced in an anticlockwise direction in the coil (as viewed from the left side of the coil).

By inducing a current that flows anti-clockwise, the coil sets up a north pole facing the incoming north pole of the magnet. This means that the mechanical work done by the force pushing the magnet will have to do work to overcome the repulsive force between the north poles of the magnet and the coil. This gives rise to the electrical energy to drive the current. In other words, mechanical energy is converted to electrical energy. This is a consequence of the principle of conservation of energy

S6 Using either Fleming's right-hand rule, Lenz's Law or first principles. (refer to lecture notes Section 17.3)

S7 Refer to lecture notes Section 17.7

Self-Practice Questions

SP1 **Ans: A**

As the magnetic field is uniform, the magnetic flux density remains unchanged.

$$\Phi = BA = B\pi r^2$$

As Φ is proportional to r^2, the magnetic flux through the new loop would be ¼ Φ.

SP2 (a)

$$|E| = \left|-\frac{d\Phi}{dt}\right| = \left|\frac{\Phi_{final} - \Phi_{initial}}{\Delta t}\right| = \left|NA\left(\frac{B_{final} - B_{initial}}{\Delta t}\right)\right|$$

$$= \left|(10)(4.0\times10^{-2})\left(\frac{0-(1.0\times10^{-2})}{0.50}\right)\right|$$

$$= 8.0\times10^{-3}\ \text{V}$$

(b)

$$|E| = \left|-\frac{d\Phi}{dt}\right| = \left|\frac{\Phi_{final} - \Phi_{initial}}{\Delta t}\right|$$

$$= \left|(10)(4.0\times10^{-2})\left(\frac{(-1.0\times10^{-2})-(1.0\times10^{-2})}{0.50}\right)\right|$$

$$= 1.6\times10^{-2}\ \text{V}$$

(c)

$$|E| = \left|-\frac{d\Phi}{dt}\right| = \left|\frac{\Phi_{final} - \Phi_{initial}}{\Delta t}\right|$$

$$= \left|(10)(4.0\times10^{-2})\left(\frac{(1.0\times10^{-2})-(1.0\times10^{-2})}{0.50}\right)\right|$$

$$= 0\ \text{V}$$

SP3 **Ans: C**

Magnetic flux ϕ through all coils is the same. Since magnetic flux density $B = \phi / A$, the coil which has the smallest cross sectional area A would have the largest variation of B.

SP4 (a) Coil X has a north pole facing coil Y (using right-hand grip rule). By moving the coils closer to each other, there is an increase in flux linkage through Y. Hence, by Lenz's Law, the induced current in Y flows in a way so as to create a north pole facing coil X (to oppose the increase in flux linkage). Using right-hand grip rule, we conclude that the current in Y is from B to A through the galvanometer.

(b) Current in X decreases (when resistance increases), hence the magnetic flux linkage in Y is decreasing. The magnetic field generated by the induced current must oppose this decrease in flux linkage (by producing a magnetic field in the same direction as that in X). Using right-hand grip rule, we conclude that the current in Y is from A to B through the galvanometer.

SP5 **Ans: D**

$$E = -\frac{d\Phi}{dt} \propto \frac{dB}{dt} \propto \frac{dI}{dt}$$

The rate of change of flux linkage of the secondary depends directly on the rate of change of current through the primary coil. When the switch was opened, the current in the primary coil is reduced from a maximum value to zero in a very short time.

SP6 $E = BLv$

$$= \left(2.0 \times 10^{-5}\right)(20)\left(\frac{3.0}{0.2}\right)$$

$$= 6.0 \times 10^{-3} \text{ V}$$

SP7 **Ans: B**

Let L be the length of one side of the square loop.
There is no net force on the loop while the entire loop is within the magnetic field.

As the rightmost vertical section of the loop starts to leave the magnetic field, there is a current induced on the leftmost vertical section, giving rise to an induced magnetic force F.

$$F = BIL = B\left(\frac{BLv}{R}\right)L = \frac{B^2L^2}{R}v$$

Work W must be done to overcome this force as the loop moves a distance L to completely exit the field.

$$W = FL = \frac{B^2L^3}{R}v$$

Hence $W \propto v$.

SP8 **Ans: A**

The is no change in magnetic flux linkage through the coil as the coil moves downwards. This is because the angle θ between the magnetic flux and the plane of the coil remains unchanged.

SP9 **Ans: E**

As the rate of rotation increases, the magnitude of the back e.m.f. (and hence opposing current) increases. This leads to a decrease in the net current that flows through the coil.

SP10 **Ans: E**

$$E = BLv = BL\left(\frac{ds}{dt}\right)$$

As seen from the *s-t* graph, the speed of the rod is initially constant, then decreases to zero.

SP11 **Ans: E**

$$E = -\frac{d\Phi}{dt} \propto -\frac{dB}{dt} \propto -\frac{dI}{dt}$$

The gradient of the graph of I vs t will give an indication of how E varies with t. There are 3 turning points on the graph of I vs t where E is zero. The best option is option E.

SP12 **Ans: D**

$$E = -\frac{d\Phi}{dt} = -NA\frac{dB}{dt}$$

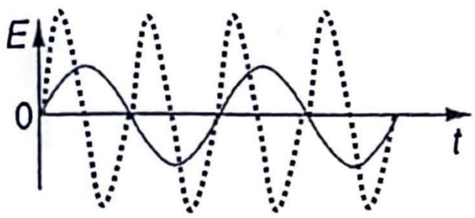
For e.m.f induced to be a maximum, the rate of change of B has to be maximum. Gradient at D is the largest.

SP13 **Ans: B**

$E = NBA\omega \sin(\omega t)$

E is dependent on N, B, A and ω. It is independent of R (but note that induced current will depend on R).

SP14
$E = NBA\omega \sin(\omega t)$ where $\omega = \dfrac{2\pi}{T}$



Since ω is doubled, the amplitude would be doubled and the period would be halved.

SP15 **Ans: A**

Rotating about the axis XX' will not change the flux linkage. Hence there won't be any e.m.f. induced. (option B & D eliminated)

On the other hand, turning the coil about the axes YY' and ZZ' results in sinusoidal change in the flux linkage, which in turn will induce a sinusoidal e.m.f.

The graph shows ¼ of a sinusoidal variation, leaving us with only option A as the possible answer.

Answers for Discussion Questions

D2 **(b)** 4.8×10^{-3} Wb, **(c)** 0.052 T

D5 **(a)** 0.0644 A

D3 **(a)(i)** 5.0 A, **(a)(ii)** Q to P, **(a)(iii)** 1.25 N, **(a)(iv)** 6.25 W; **(b)(ii)** $v = \dfrac{mgR\sin\theta}{B^2L^2\cos^2\theta}$

Answer for Challenging Question

C1. 6.17 rad s^{-1}