Name: _____( ) Class: _____

WOODLANDS SECONDARY SCHOOL
MID YEAR EXAMINATION 2021

Level:	Sec 4Exp	Marks:	90
Subject:	Additional Mathematics	Day:	Friday
Paper:	4049/01	Date:	7th May 2021
Duration:	2 hours 15 minutes	Time:	0945 – 1200

READ THESE INSTRUCTIONS FIRST

Answer on the Question Paper.

Write in dark blue or black pen.
You may use an HB pencil for any diagrams or graphs.
Do not use staples, paper clips, glue or correction fluid.

Answer **ALL** the questions.

Give non-exact numerical answers correct to 3 significant figures, or 1 decimal place in the case of angles in degrees, unless a different level of accuracy is specified in the question.
The use of an approved scientific calculator is expected, where appropriate.
You are reminded of the need for clear presentation in your answers.

At the end of the examination, fasten all your work securely together.
The number of marks is given in brackets [ ] at the end of each question or part question.

The total number of marks for this paper is **90**.

DO NOT TURN THE PAGE UNTIL YOU ARE TOLD TO DO SO.
This document consists of **15** printed pages and **1** blank page.

Mathematical Formulae

1. ALGEBRA

Quadratic Equation

For the equation $ax^2 + bx + c = 0$,

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

Binomial expansion

$$(a+b)^n = a^n + \binom{n}{1}a^{n-1}b + \binom{n}{2}a^{n-2}b^2 + \ldots + \binom{n}{r}a^{n-r}b^r + \ldots + b^n,$$

where n is a positive integer and $\binom{n}{r} = \frac{n!}{r!(n-r)!} = \frac{n(n-1)\ldots(n-r+1)}{r!}$.

2. TRIGONOMETRY

Identities

$$\sin^2 A + \cos^2 A = 1$$
$$\sec^2 A = 1 + \tan^2 A$$
$$\operatorname{cosec}^2 A = 1 + \cot^2 A$$
$$\sin(A \pm B) = \sin A\cos B \pm \cos A\sin B$$
$$\cos(A \pm B) = \cos A\cos B \mp \sin A\sin B$$
$$\tan(A \pm B) = \frac{\tan A \pm \tan B}{1 \mp \tan A\tan B}$$
$$\sin 2A = 2\sin A\cos A$$
$$\cos 2A = \cos^2 A - \sin^2 A = 2\cos^2 A - 1 = 1 - 2\sin^2 A$$
$$\tan 2A = \frac{2\tan A}{1 - \tan^2 A}$$

Formulae for ΔABC

$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$

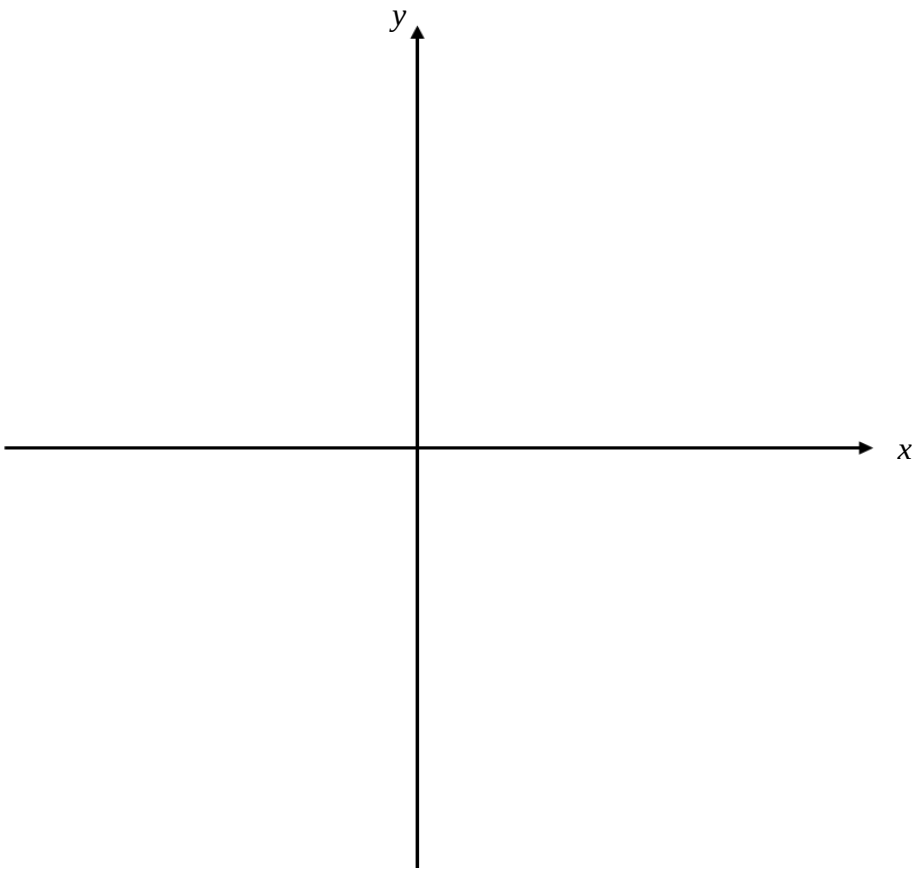
$$a^2 = b^2 + c^2 - 2bc\cos A$$

$$\Delta = \frac{1}{2}ab\sin C$$

1 Given that the coefficient of x and x^4 in the expansion of $(A-x)^4+(2-Bx)^5$ is – 272 and 811 respectively, find the value of A and B where both are positive integers. [5]

2 **(a)** Given that the curve $y = \log_a x$ passes through the point $\left(\frac{1}{27}, -3\right)$, find the value of a. [2]

(b) **(i)** Sketch the graph of $y = \log_a x$ for $0 < x \leq 27$. [1]



(b) (ii) Find the equation of a suitable straight line that can be inserted in part **(b)(i)** to solve the equation $\frac{27}{x} = 9^x$. [3]

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3 Without using a calculator, solve the equation $3x - x\sqrt{5} = 2\sqrt{5} + 4$, giving your answer in the form $m + n\sqrt{5}$. [3]

4 **(i)** Solve the equation $\lg(x) = 1 + \lg(x-2)$. [2]

(ii) Given that $\log_2 p = a$, $\log_8 q = b$ and $\frac{p}{q} = 2^c$, express c in terms of a and b. [3]

5 The equation of a curve is $y = x^2 - kx + 3 - k$, where k is a constant.

(i) Find the range of values of k for which the curve lies completely above the x-axis. [3]

(ii) Find the range of values of k for which the line $y + k + 1 = 0$ and the curve $y = x^2 - kx + 3 - k$ have two distinct roots. [4]

6 The value, V dollars, of Hi Watch in t months' time from its launch is modelled by the equation $V = V_0 e^{-kt}$, where k is a constant and V_0 is the original value of the watch. After 36 months from its launch, the value of the watch is half of its original value.

(i) Find the value of k. [3]

(ii) Twelve months after the launch, the price of Hi Watch is \$855.
Calculate the minimum number of months needed from the time of launch, for the watch value to fall below \$600. [5]

7 **(i)** Express $\frac{4x^2-4x-1}{2x^2-x}$ in partial fractions. [5]

(ii) Hence, find $\int \frac{4x^2-4x-1}{2x^2-x}\,\mathrm{d}x$. [2]

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8 It is given that $y = x^3 + px^2 + qx - 1$ where p and q are integers.

y is a decreasing function when $\frac{2}{3} < x < 5$. Find the value of p and q. [4]

9 **(i)** Prove that $\text{cosec}\, A - \cot A = \dfrac{\sin A}{1+\cos A}$. [3]

(ii) Hence, find all values of θ, where $0 \le \theta \le 2\pi$, which satisfy the equation

$$\frac{1}{2}(\text{cosec}\ \theta - \cot\theta) = 1 - \cos\theta. \quad [4]$$

10 The remainder when $2x^3 + ax^2 + bx + 3$, where a and b are constant, is divided by $x^2 + x - 2$ is $-6x + 9$.

(i) Show that $a = 5$ and $b = -7$. [5]

(ii) Solve the equation $2x^3 + ax^2 + bx + 3 = x + 3$, expressing non-integer roots in surd form. [3]

11 A circle C_1 has the equation $x^2 + y^2 - 6x + 2y + 1 = 0$.

(i) Find the coordinates of the centre of the circle and the radius of the circle. [3]

(ii) Determine if the origin lies inside or outside the circle.
Show your workings clearly. [2]

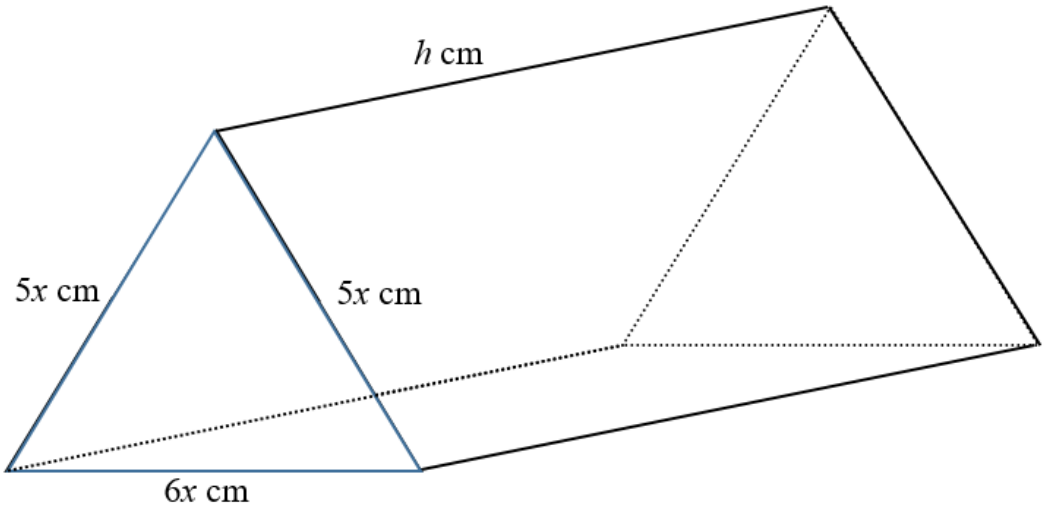
(iii) Another circle C_2, with centre (–9, 4) touches the circle C_1 such that the point of contact forms a straight line with the centres of C_1 and C_2.
Find the equation of circle C_2. [3]

12 A curve is such that $\frac{d^2y}{dx^2} = x + \frac{16}{x^3}$, and it has a stationary point at (2, 10).

(i) Find the equation of the curve. [5]

(ii) Find the coordinates of the other stationary point and determine its nature. [3]

13 A regular cross section of a triangular prism is an isosceles triangle whose sides are $5x$ cm, $5x$ cm and $6x$ cm. The length of the prism is h cm.
The volume of the prism is 600 cm^3.



(i) Show that the total surface area of the prism is $A = \left(24x^2 + \frac{800}{x}\right)$ cm^2. [4]

(ii) Hence, find the value of x for which A has a stationary value. [2]

(iii) Calculate the stationary value of A and determine whether it is a maximum or minimum. [2]

14 A particle moves in a straight line, such that, t seconds after leaving a fixed point O, its acceleration is given by $a = \frac{12}{(t+1)^2}$.

The particle comes to an instantaneous rest when $t = 3$.

Find

(i) an expression for the velocity of the particle in terms of t, [3]

(ii) the distance travelled in the next 4 seconds after the rest. [3]

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