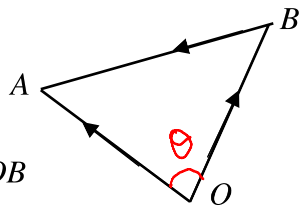
2020 ACJC H2 Math Prelim P2 Marker's Report

Marking Annotations used in scripts:
K:Knowledge Gap; C:Carelessness; R:Read/Interpret question wrongly; P:Presentation issue

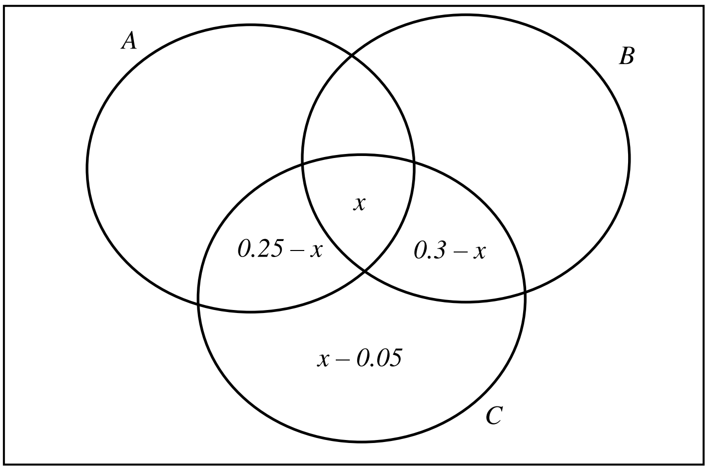
Qn	Solutions	Comments
1(a)	 By Cosine rule; $AB^2 = OA^2 + OB^2 - 2(OA)(OB)\cos AOB$ $\lvert\mathbf{a}\text{-}\mathbf{b}\rvert^2 = \lvert\mathbf{a}\rvert^2 + \lvert\mathbf{b}\rvert^2 - 2\lvert\mathbf{a}\rvert\lvert\mathbf{b}\rvert\cos AOB$ $3 = 4 + \lvert\mathbf{b}\rvert^2 - 2\left(\frac{5}{2}\right)$ $\lvert\mathbf{b}\rvert^2 = 4 \Rightarrow \lvert\mathbf{b}\rvert = 2$ **(since length |b| is postive)**	Many Students have forgotten the Cosine Rule formula $\mathbf{c}^2 = \mathbf{a}^2 + \mathbf{b}^2 - 2\mathbf{ab}\cos\theta$ $AB^2 = OA^2 + OB^2 - 2(OA)(OB)\cos AOB$ Some students did not realise that if angle AOB is used, **c** has to be the length AB. Many are able to start with $\mathbf{a}\bullet\mathbf{b} = \lvert\mathbf{a}\rvert\lvert\mathbf{b}\rvert\cos\theta = \frac{5}{2}$ but didn't know how to use it properly.
1(b)(i)	$\lvert\mathbf{r}\cdot\mathbf{k}\rvert$ represents the length of projection of **r** onto the *z*-axis. **<u>OR</u>** $\lvert\mathbf{r}\cdot\mathbf{k}\rvert$ represents the distance of point (a,b,c) from the *xy*-plane.	Some students did not portray $\underset{\sim}{r}$ or $\underset{\sim}{k}$ as vectors.
1(b)(ii)	$\lvert\mathbf{r}\times\mathbf{k}\rvert$ represents the area of a parallelogram with adjacent sides given by the vectors **r** and **k**. **<u>OR</u>** $\lvert\mathbf{r}\times\mathbf{k}\rvert$ represents the perpendicular distance of point (a,b,c) to the *z*-axis (or to the line with direction vector **k**).	Some students used 'adjacent sides OR and OK' without defining what is point R & K. Students who used 'perpendicular distance' didn't specify that it is from a point (a, b, c) towards the vector $\underset{\sim}{k}$
1(b)(iii)	From $\mathbf{k}\times\mathbf{r} = \mathbf{p}$ $\begin{pmatrix}0\\0\\1\end{pmatrix}\times\begin{pmatrix}a\\b\\c\end{pmatrix} = \begin{pmatrix}-b\\a\\0\end{pmatrix} = \mathbf{p}$ From $\mathbf{r}\times\mathbf{p} = \mathbf{k}$ $\begin{pmatrix}a\\b\\c\end{pmatrix}\times\begin{pmatrix}-b\\a\\0\end{pmatrix} = \begin{pmatrix}-ac\\-bc\\a^2+b^2\end{pmatrix} = \begin{pmatrix}0\\0\\1\end{pmatrix}$ $\therefore \quad ac = 0\cdots\cdots\cdots(1)$ and $bc = 0\cdots\cdots\cdots(2)$ and $a^2 + b^2 = 1$ (shown)$\cdots\cdots\cdots(3)$ From (1) : $a = 0$ or $c = 0$ From (2) : $b = 0$ or $c = 0$ From (3) : a and b cannot both be 0, $\therefore c = 0$	Many students were able to perform cross product correctly to get to $\begin{pmatrix}0\\0\\1\end{pmatrix}\times\begin{pmatrix}a\\b\\c\end{pmatrix} = \begin{pmatrix}-b\\a\\0\end{pmatrix} = \mathbf{p}$ but got the next cross product wrong eg. $\begin{pmatrix}-ac\\bc\\a^2+b^2\end{pmatrix}$ or $\begin{pmatrix}-ac\\-bc\\-a^2+b^2\end{pmatrix}$ For the last part getting to c = 0, many students didn't support with clear explanation.

	OR From $\mathbf{r}\times\mathbf{p}=\mathbf{k}$, $\mathbf{r}$ is perpendicular to **k.** $\mathbf{r}\cdot\mathbf{k}=0$ $\begin{pmatrix}a\\b\\c\end{pmatrix}\cdot\begin{pmatrix}0\\0\\1\end{pmatrix}=0\Rightarrow c=0$ From $\mathbf{k}\times\mathbf{r}=\mathbf{p}$ $\begin{pmatrix}0\\0\\1\end{pmatrix}\times\begin{pmatrix}a\\b\\c\end{pmatrix}=\begin{pmatrix}-b\\a\\0\end{pmatrix}=\mathbf{p}$ Substitute into $\mathbf{r}\times\mathbf{p}=\mathbf{k}$ $\begin{pmatrix}a\\b\\0\end{pmatrix}\times\begin{pmatrix}-b\\a\\0\end{pmatrix}=\begin{pmatrix}0\\0\\1\end{pmatrix}$ $\begin{pmatrix}0\\0\\a^2+b^2\end{pmatrix}=\begin{pmatrix}0\\0\\1\end{pmatrix}$ $\therefore a^2+b^2=1$	Some students who were able to see **r** and **k** being perpendicular to one another, didn't proceed to use the property of their scalar DOT product = 0
2(i)	$\frac{\mathrm{d}x}{\mathrm{d}t}=\frac{k}{\sqrt{x}}$ $\int\sqrt{x}\ \mathrm{d}x=\int k\ \mathrm{d}t$ $\frac{2}{3}x^{3/2}=kt+c$ When $t=0$, $x=1\Rightarrow c=\frac{2}{3}$ When $t=10$, $x=4\Rightarrow\frac{16}{3}=10k+\frac{2}{3}\Rightarrow k=\frac{7}{15}$ $\frac{2}{3}x^{3/2}=\frac{7}{15}t+\frac{2}{3}$ $x=\left(\frac{7}{10}t+1\right)^{2/3}$	Many students have the misconception that the "rate of… inversely proportional to square root of…" is $\frac{\mathrm{d}x}{\mathrm{d}t}=\frac{1}{k}\sqrt{x}$. A small number of students misread the question without "root", thus wrote $\frac{\mathrm{d}x}{\mathrm{d}t}=\frac{k}{x^2}$. Majority did not notice "*x*, **in hundreds**", and used $x=100$ and $x=400$ to calculate for the unknown constants. Many students did not make *x* the subject even though the question stated "*x* as a function of *t* ". Some expressed *t* in terms of *x* instead. Misconceptions of law of indices led to answers such as $x=\left(\frac{7}{10}t+1\right)^{-3/2}$, $x=\sqrt[\frac{2}{3}]{\frac{7}{10}t+1}$ and $x=\left(\frac{7}{10}t\right)^{2/3}+1$.
2(ii)	$\frac{\mathrm{d}x}{\mathrm{d}t}=m\left(100x-x^2\right)=mx\left(100-x\right)$	Majority are able to write

	$\int \frac{1}{x(100-x)}\,dx = \int m\,dt$ Using partial fraction, $\frac{1}{100}\int \frac{1}{x}+\frac{1}{100-x}\,dx = \int m\,dt$ $\int \frac{1}{x}+\frac{1}{100-x}\,dx = \int 100m\,dt$ $\ln\left\|\frac{x}{100-x}\right\| = 100mt + c$ $\frac{x}{100-x} = Ae^{100mt}$, where $A = \pm e^{c}$ $x = Ae^{100mt}(100-x)$ $x\left(1+Ae^{100mt}\right) = 100Ae^{100mt}$ $x = \frac{100Ae^{100mt}}{1+Ae^{100mt}}$ $x = \frac{100}{1+Ae^{-100mt}}$ $H = 100$	$\int \frac{1}{x(100-x)}\,dx = \int m\,dt$ and simplify the integral either using completing the square or by partial fraction. Some did the integration wrongly by writing $\int \frac{1}{100x-x^2}\,dx = \int \frac{1}{\left(\sqrt{100x}\right)^2 - x^2}\,dx$ or $\int \frac{1}{100-x}\,dx = \int \frac{1}{100-\left(\sqrt{x}\right)^2}\,dx$ and attempted to use the formula $\int \frac{1}{a^2-x^2}\,dx = \frac{1}{2a}\ln\left\|\frac{a-x}{a+x}\right\| + C.$ A few made the careless mistake of re-writing the integral as $\int \frac{1}{x(100-x)}\,dx = \int \frac{1}{100-x^2}\,dx \cdot$ Many students did not realise that $100-x<0$, where modulus is necessary. Some students stopped at this step and could not proceed on. $x = \frac{100Ae^{100mt}}{1+Ae^{100mt}}$.
2(iii)	The scientist's model. The scientist's model suggests that in the long term, the number of bacteria increases and tends to 10000. In a petri dish, there should be a limit to the maximum population of bacteria.	Many students were able to give the correct answer. There were many interesting answers given but did not fit the context of the question. Eg. The scientist since the scientist is smarter than the student.
3(a)(i)	Using similar triangle, $\frac{y}{2a} = \frac{x}{x-a^2} \quad \Rightarrow \quad y = \frac{2ax}{x-a^2}$ Area of S = $\frac{1}{2}x\,y$ = $\frac{1}{2}x\left(\frac{2ax}{x-a^2}\right)$ = $\frac{ax^2}{x-a^2}$ (shown)	Most students attempted to express y in terms of x. A number of students used similar figures, $\frac{A_1}{A_2} = \left(\frac{l_1}{l_2}\right)^2$, and were successful in showing. Some students attempted to use the equation of a straight line $Y-2a = -\frac{y}{x}\left(X-a^2\right)$, where x and X got cancelled out when simplifying, which is incorrect.
3(a)(ii)	$\frac{dS}{dx} = \frac{\left(x-a^2\right)2ax - ax^2}{\left(x-a^2\right)^2} = \frac{ax\left(x-2a^2\right)}{\left(x-a^2\right)^2}$	Many students did long division to simplify S, which is not necessary.

	For maximum or minimum, $\dfrac{dS}{dx} = 0$ $\dfrac{(x-a^2)2ax - ax^2}{(x-a^2)^2} = 0$ $ax^2 - 2a^3x = 0$ $x\,(x-2a^2) = 0$ $x = 0$ (reject since $x > 0$) or $x = 2a^2$ $\dfrac{d^2S}{dx^2} = \dfrac{(x-a^2)^2(2ax-2a^3)-(ax^2-2a^3x)2(x-a^2)}{(x-a^2)^4}$ $= \dfrac{2a(x-a^2)^2-(x-2a^2)2ax}{(x-a^2)^3} = \dfrac{2a^5}{(x-a^2)^3}$ When $x = 2a^2$, $\dfrac{d^2S}{dx^2} = \dfrac{2a^5}{(2a^2-a^2)^3} = \dfrac{2}{a} > 0$ Hence by the second derivative test, S is minimum at $x = 2a^2$. **<u>Alternative for checking minimum:</u>** Using first derivative: <table><tr><td>x</td><td>$(2a^2)^-$</td><td>$2a^2$</td><td>$(2a^2)^+$</td></tr><tr><td>$\frac{dS}{dx}$</td><td>- ve</td><td>0</td><td>+ ve</td></tr></table>When $x < 2a^2$, $x-2a^2<0$, and since $ax>0$ and $(x-a^2)^2>0$, $\therefore \dfrac{dS}{dx} = \dfrac{ax(x-2a^2)}{(x-a^2)^2} < 0$ Similarly, when $x > 2a^2$, $\dfrac{dS}{dx} = \dfrac{ax(x-2a^2)}{(x-a^2)^2} > 0$ Hence by the first derivative test, S is minimum at $x = 2a^2$. Minimum value of $S = \dfrac{ax^2}{x-a^2} = \dfrac{a(2a^2)^2}{(2a^2)-a^2} = \dfrac{4a^5}{a^2} = 4a^3$	Majority found $\frac{dS}{dx}$ correctly and equated to zero. However, when simplifying, there were many algebraic errors, resulting in the wrong value of x found. Almost all students did the "max. or min." check using either 1st or 2nd derivative, but very few successful ones. Students who used 2nd derivative test, majority did not know that they have to find the value of $\frac{d^2S}{dx^2}$ **when $x = 2a^2$**. Most of them left the expression as $\frac{2a(x-a^2)^2-(x-2a^2)2ax}{(x-a^2)^3}$ and tried to explain that it is > 0. Students who did 1st derivative test, very few explained why $\frac{dS}{dx}$ < 0 when $x=(2a^2)^-$ etc, thus get the credit. Students who simply drew the table without explaining why $\frac{dS}{dx}$ < 0 or $\frac{dS}{dx}$ > 0, got penalized. **<u>Note:</u>** **It is not encouraged to use 1st derivative test when the value of x contains unknown constants.** Most of the students did not realise that they are required to find the value of S at $x = 2a^2$, as stated in the question.
3(b)	$\dfrac{1}{R_P} = \sum_{i=1}^{3}\dfrac{1}{R_i} = \dfrac{1}{R_1}+\dfrac{1}{R_2}+\dfrac{1}{R_3}$	This question was badly done. A common mistake is

	$\frac{d}{dt}\left(\frac{1}{R_P}\right) = \frac{d}{dt}\left[\frac{1}{R_1}+\frac{1}{R_2}+\frac{1}{R_3}\right]$ $\frac{-1}{(R_P)^2}\left(\frac{dR_P}{dt}\right)=\frac{-1}{(R_1)^2}\left(\frac{dR_1}{dt}\right)+\frac{-1}{(R_2)^2}\left(\frac{dR_2}{dt}\right)$ $\frac{1}{(R_P)^2}\left(\frac{dR_P}{dt}\right)=\frac{r}{(R_1)^2}+\frac{2r}{(R_2)^2}$ $\frac{dR_P}{dt} = \left[\frac{r}{(R_1)^2}+\frac{2r}{(R_2)^2}\right] \times (R_P)^2$ $= \left[\frac{r}{(R_1)^2}+\frac{2r}{(R_1)^2}\right] \times \left(\frac{2R_1}{5}\right)^2 = \frac{12r}{25}$.	$\frac{1}{R_P}=\frac{1}{R_1}+\frac{1}{R_2}+\frac{1}{R_3}$ $\Rightarrow\ R_P = R_1+R_2+R_3$. Majority carried out the differentiation after substituting $R_2 = R_1$ and $R_3 = 2R_1$, without realising that these are constant values at that point of time. ie $\frac{d}{dt}\left(\frac{1}{R_P}\right)=\frac{d}{dt}\left[\frac{1}{R_1}+\frac{1}{R_1}+\frac{1}{2R_1}\right]$, which is incorrect.
4(a)(i)	$f(x)=x^6-ax^4-x^2-b$ $f(-x) = (-x)^6-a(-x)^4-(-x)^2-b$ $= x^6-ax^4-x^2-b = f(x)$ (shown)	Well done.
4(a)(ii)	Since $f(x)$ is of degree 6, there is a total of 6 roots. As there is only one real root for $x\geq 0$, i.e $x=\beta$, and $f(x)=f(-x)$, thus $x=-\beta$ is the other real root. With only 2 real roots, the equation $f(x)=0$ has 4 non-real roots.	Many students assumed that the question is asking how many roots for $x\geq 0$ and hence gave the answer as 2 non real roots. Since the curve is symmetrical about the y-axis from (i), there should are 2 real roots (see the graph) and 4 non real roots.
4(a)(iii)	As $z_1 = re^{i\theta}$ is a root, its conjugate $z_2 = re^{-i\theta}$, is also a root, since $f(x)$ has real coefficients. With $f(x)=f(-x)$, $f(z_1)=f(-z_1)=f(-re^{i\theta})=0$. Similarly, $f(z_2)=f(-z_2)=f(-re^{-i\theta})=0$. Thus $z_3=-re^{i\theta}$ and $z_4=-re^{-i\theta}$ are also the roots of $f(x)=0$. The roots in modulus-argument form are: $z_1 = re^{i\theta}$, $z_2=re^{-i\theta}$, $z_3=-re^{i\theta}=re^{i(\theta-\pi)}$, $z_4=-re^{-i\theta}=re^{i(\pi-\theta)}$, $z_5=\beta e^{i\pi}$, $z_6=\beta e^{i0}$ are the remaining roots of $f(x)=0$.	Most students knew the six roots but fail to present in the required form. This answer $z_3=-re^{i\theta}$ is unacceptable as question wanted **modulus**-argument form. Hence students should note that $-1=e^{i\pi}$ Hence, $z_3=re^{i(\theta-\pi)}$
4(a)(iv)	$f(x) = (x-\beta)(x+\beta)(x-re^{i\theta})(x-re^{-i\theta})(x-re^{i(\theta-\pi)})(x-re^{i(\pi-\theta)})$ $= (x^2-\beta^2)\ (x^2-rxe^{i\theta}-rxe^{-i\theta}+r^2)\ (x^2-rxe^{i(\pi-\theta)}-rxe^{i(\theta-\pi)}+r^2)$	Most students are able to list the 6 linear factors correctly but they have forgotten to group the factors with conjugate pairs together. There were unsuccessful attempts such as

	$= \left(x^2-\beta^2\right)\left(x^2-rx\left[\mathrm{e}^{\mathrm{i}\theta}+\ \mathrm{e}^{-\mathrm{i}\theta}\right]+r^2\right)\left(x^2-rx\left[\mathrm{e}^{\mathrm{i}(\pi-\theta)}+\ \mathrm{e}^{\mathrm{i}(\theta-\pi)}\right]+r^2\right)$ $= \ \left(x^2-\beta^2\right)\ \left(x^2-rx[2\cos\theta]+r^2\right)\ \left(x^2-rx\left[2\cos\left(\pi-\theta\right)\right]+r^2\right)$ $= \ \left(x^2-\beta^2\right)\ \left(x^2-2rx\cos\theta+r^2\right)\ \left(x^2+2rx\cos\theta+r^2\right)$ $A=\beta^2,\ B=2,\ C=-2$	grouping in this way: $\left(x-r\mathrm{e}^{\mathrm{i}\theta}\right)\left(x+r\mathrm{e}^{\mathrm{i}\theta}\right)$
4(b)	$\left\|\dfrac{\mathrm{i}\left(2\mathrm{i}z+z^2\right)^*}{z\left(2z^*-4\mathrm{i}\right)}\right\| \ = \ \left\|\dfrac{\mathrm{i}}{2z}\right\| \ \left\|\dfrac{z^*\left(2\mathrm{i}+z\right)^*}{\left(z^*-2\mathrm{i}\right)}\right\|$ $= \ \left\|\dfrac{\mathrm{i}}{z}\right\| \ \left\|\dfrac{z^*}{2}\right\| \ \left\|\dfrac{z^*-2\mathrm{i}}{z^*-2\mathrm{i}}\right\|$ $= \ \dfrac{1}{2}$	Majority found this question challenging. Many started working in Cartesian form but couldn't get far. Students should be familiar with the properties of complex nos to do this question efficiently.
5(i)	Since A and B are independent, $\mathrm{P}\left(A\|B\right)=\mathrm{P}\left(A\right)=0.5$ $\mathrm{P}\left(A\cup B\right)=\mathrm{P}\left(A\right)+\mathrm{P}\left(B\right)-\mathrm{P}\left(A\cap B\right)$ $0.8=0.5+\mathrm{P}\left(B\right)-0.5\mathrm{P}\left(B\right)$ (since A and B are independent) $0.5\mathrm{P}\left(B\right)=0.3 \qquad \therefore \mathrm{P}\left(B\right)=0.6$	Students who didn't get this well probably forgot that 'independent events' for $\mathrm{P}(A\cap B)=\mathrm{P}(A)\times\mathrm{P}(B)$
5(ii)	Since A and C are independent and B and C are independent, $\mathrm{P}\left(A\cap C\right)=0.5\times0.5=0.25$ $\mathrm{P}\left(B\cap C\right)=0.6\times0.5=0.3$ Let $\mathrm{P}\left(A\cap B\cap C\right)=x$. Then $\mathrm{P}\left(A\cap B'\cap C\right)=0.25-x$, $\mathrm{P}\left(A'\cap B\cap C\right)=0.3-x$, $\mathrm{P}\left(A'\cap B'\cap C\right)=0.5-x-\left(0.25-x\right)-\left(0.3-x\right)$ $=x-0.05$.  Hence, $0.05\le\mathrm{P}\left(A\cap B\cap C\right)\le0.25$.	Many students were able to use $\mathrm{P}\left(A\cap C\right)=0.5\times0.5=0.25$ $\mathrm{P}\left(B\cap C\right)=0.6\times0.5=0.3$ But some made a mistake to assume that to find $\mathrm{P}\left(A\cap B\cap C\right)$

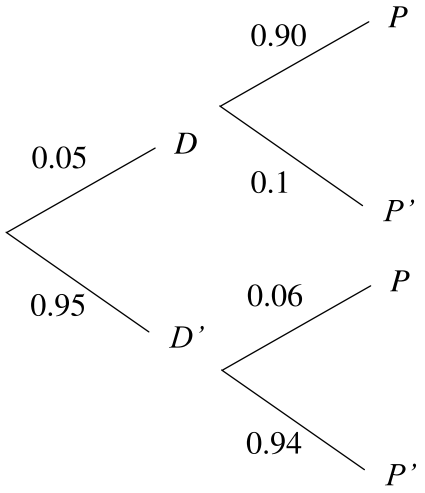
6(i)	Total number of socks: $2(3+2+1)=12$ s: 2, 3, 4 $P(S=s)$: $\frac{1}{3}$, $\frac{74}{165}$, $\frac{12}{55}$ $P(S=2)=P(RR)+P(WW)+P(BB)$ $=\frac{6}{12}\cdot\frac{5}{11}+\frac{4}{12}\cdot\frac{3}{11}+\frac{2}{12}\cdot\frac{1}{11}=\frac{1}{3}$ $P(S=3)=2\left[P(RXR)+P(WXW)+P(BXB)\right]$ $=2\left[\frac{6}{12}\cdot\frac{6}{11}\cdot\frac{5}{10}+\frac{4}{12}\cdot\frac{8}{11}\cdot\frac{3}{10}+\frac{2}{12}\cdot\frac{10}{11}\cdot\frac{1}{10}\right]=\frac{74}{165}$ $P(S=4)=1-\frac{1}{3}-\frac{74}{165}=\frac{12}{55}$ If calculated: $P(S=4)=3!\left[P(RXYR)+P(WXYW)+P(BXYB)\right]$ $=6\left[\frac{6}{12}\cdot\frac{4}{11}\cdot\frac{2}{10}\cdot\frac{5}{9}+\frac{4}{12}\cdot\frac{6}{11}\cdot\frac{2}{10}\cdot\frac{3}{9}+\frac{2}{12}\cdot\frac{6}{11}\cdot\frac{4}{10}\cdot\frac{1}{9}\right]$ $=\frac{12}{55}$	Surprisingly, this question was very poorly answered. This should be a routine question, but quite a number of scripts even fail to see that *S* can only take on values 2,3 and 4. When calculating the probabilities, the number of permutations were also not identified correctly. Out of the correct responses, many of them also split each outcome into too many small cases, leading to tedious computation.
6(ii)	$E(S)=2\left(\frac{1}{3}\right)+3\left(\frac{74}{165}\right)+4\left(\frac{12}{55}\right)=\frac{476}{165}$ or 2.88 $E(S^2)=2^2\left(\frac{1}{3}\right)+3^2\left(\frac{74}{165}\right)+4^2\left(\frac{12}{55}\right)=\frac{1462}{165}$ or 8.86 $\text{Var}(S)=\frac{1462}{165}-\left(\frac{476}{165}\right)^2=0.538$	Generally okay, for students who managed to do (i). Quite a number of scripts left this part blank though, students should continue to try to work out the expectation and variance as they may get method marks.
6(iii)	$\left(1-\frac{1}{3}\right)^7=\frac{128}{2187}=0.0585$	Generally quite well done, if they get (i) right, or even just the probability of *S* = 2.
7(i)	$(8-1)!-(7-1)!\times 2!=3600$ **OR** $(6-1)!\times {}^6C_2\times 2!=3600$	This question is generally well done. Some students who did the 2nd method didn't separate the 2 person i.e. ${}^6C_2$
7(ii)	$6P4=360$ **OR** $6C2\times 2!\times 4C2\times 2!=360$ **OR** $6C4\times 4C2\times 2!\times 2C2\times 2!=360$	This part is generally well done.
7(iii)	For there to be ties, there are 4 cases to consider for the voting: $(4,4,0,0),(3,3,1,1),(3,3,2,0)$ and $(2,2,2,2)$. $(4,4,0,0)$: $8C4\times 4C4\times 4C2=420$ $(3,3,1,1)$: $8C3\times 5C3\times 2C1\times 1C1\times 4C2=6720$	Many students were able to list at least 2 of the 4 cases correctly, but proceeded to calculate the ways wrongly. For cases with 3 movies highest (i.e. 3,3,1,1, and 3,3,2,0), some

	$(3,3,2,0)$: $8C3\times5C3\times2C2\times\frac{4!}{2!}=6720$ $(2,2,2,2)$: $8C2\times6C2\times4C2\times2C2=2520$ Total number of ways the votes could have happened: $420+6720+6720+2520=16380$	student missed out one type. Some students do not understand what the question requires of them.
8(i)	Unbiased estimate of population mean, $\bar{x} = \frac{\sum(x-200)}{36} + 200$ $= \frac{662.4}{36} + 200 = 218.4$ Unbiased estimate of population variance, $s^2 = \frac{1}{36-1}\left(\Sigma(x-200)^2 - \frac{\left[\Sigma(x-200)\right]^2}{36}\right)$ $= \frac{1}{35}\left(307141.56 - \frac{(662.4)^2}{36}\right) = 8427.24$ To test $H_0: \mu = 200$ against $H_1: \mu > 200$ at 10% level of significance. μ represents the (population) mean monthly income returns of the investment plan. Under H_0, $\bar{X} \sim N\left(200, \frac{8427.24}{36}\right)$ approximately by Central Limit Theorem since $n = 36$ is large. Using GC, Value of test statistic: $z = \frac{218.4-200}{\sqrt{\frac{8427.24}{36}}} = 1.202614379$ p-value $= 0.114562852$ Since p-value $= 0.115 > 0.1$ ($\therefore$ Do not reject H_0) There is insufficient evidence at 10% level of significance that the investment plan generate more than \$200 monthly income. Hence Wally should not invest in the plan.	Many rounded the values of $\bar{x}$ and s^2 although they can be written exactly. Students can also express these answers in fraction wherever possible. If can't, it means that they need to round to 3 sf unless otherwise stated in the question. Many students missed out the word 'population' in the definition of μ. Some went on the define Ho and H1 and $\bar{x}$ which are not required. Some students wrote CLT instead of spelling it out in full. Instead of 200, some wrote 218.4 in the distribution: $\bar{X} \sim N\left(218.4, \frac{8427.24}{36}\right)$ Students must conclude with a final statement that answers the question if it is different from the H1 statement. Prior to this, they will conclude by writing the H1 statement in context of the

		question.
8(ii)	'10% significance level' means that there is a 10% probability of concluding that the investment plan's promised mean monthly income is more than \$200 when it is \$200.	Many did not write in context of the question. Many gave many different variations for this question. Students are strongly encouraged to follow the format in the solution given instead. Students must note that the meaning of 1) significance level 2) *p* value are different.
9(i)	Let random variable *A* be the BFE of Brand *A* mask. Since $\text{P}(A<95.7)=\text{P}(A>95.78)$, $\mu=\frac{95.7+95.78}{2}=95.74$ $\text{P}(A<95.7)=0.0912$ $\text{P}(Z>\frac{95.7-95.74}{\sigma})=0.0912$ $\frac{-0.04}{\sigma}=-1.333401746$ $\sigma=0.0299984608=0.03$ (2 d.p)	Many students solve the question by forming 2 equations in terms of using μ and σ using $\text{P}(A<95.7)=0.0912$ and $\text{P}(A>95.78)=0.0912$. This could be students not realizing that as the two tails have the same area, hence μ can be simply obtained by taking the average of 95.7 and 95.78. Quite a number of students wrote 0.957 instead of 95.7 and 0.9578 instead of 95.78. Hence their final answers are affected. Some students could not recall the standardization formulae correctly. A handful did not leave their answer in 2 d.p.
9(ii)	Let random variable *B* be the BFE of Brand *B* mask. $B\sim\text{N}\left(92.19,0.03^2\right)$ $A\sim\text{N}\left(91.09,0.08^2\right)$ $\bar{A}\sim\text{N}\left(91.09,\frac{0.08^2}{n}\right)$ $B-\bar{A}\sim\text{N}\left(1.1,0.03^2+\frac{0.08^2}{n}\right)$ $\text{P}(\lvert B-\bar{A}\rvert\le 1.15)\ge 0.9405$ $\text{P}(-1.15\le B-\bar{A}\le 1.15)\ge 0.9405$ Note: It is not necessary to carry out standardisation (as shown below) to solve this question.	Many students could not interpret the questions correctly. Among these students, either the modulus sign (since difference in value is needed for this question) was missing or students misunderstood/misread "sample mean" as $A_1+A_2+\ldots+A_n$ or nA. Another common mistake made is when finding $\text{Var}\left(B-\bar{A}\right)$. Note that $\text{Var}\left(B-\bar{A}\right)\neq\text{Var}\left(B\right)-\text{Var}\left(\bar{A}\right)$. Not many students successfully uses the GC method to solve. The "table method" is recommended as *n* is an integer value. For those who attempted to solve

	$\mathrm{P}\left(\frac{-1.15-1.1}{\sqrt{0.03^2+\frac{0.08^2}{n}}}\le Z\le\frac{1.15-1.1}{\sqrt{0.03^2+\frac{0.08^2}{n}}}\right)\ge 0.9405$ Using GC, <table><tr><td>n</td><td>$\mathrm{P}(\lvert B-\overline{A}\rvert\le 1.15)$</td></tr><tr><td>49</td><td>0.9403</td></tr><tr><td>**50**</td><td>**0.9406**</td></tr><tr><td>51</td><td>0.9408</td></tr></table>$\therefore$ Least $n=50$.	algebraically, many made the mistake of assuming that $\frac{-1.15-1.1}{\sqrt{0.03^2+\frac{0.08^2}{n}}}$ and $\frac{1.15-1.1}{\sqrt{0.03^2+\frac{0.08^2}{n}}}$ are symmetrical about the mean.
9(iii)	$X\sim\mathrm{N}\left(203,\sigma_1^{\ 2}\right)$ $\quad Y\sim\mathrm{N}\left(203,\sigma_2^{\ 2}\right)$ $3X-\left[Y+Y_2+Y_3\right]\sim\mathrm{N}\left(0,\ 9\sigma_1^{\ 2}+3\sigma_2^{\ 2}\right)$ $\mathrm{P}\left(3X>Y_1+Y_2+Y_3\right)=\mathrm{P}\left(3X-\left[Y_1+Y_2+Y_3\right]>0\right)$ $=0.5$	Many students are able to state the parameters of the distribution of $3X-\left[Y_1+Y_2+Y_3\right]$. Common mistake is failing to interpret what "total mass of 3" means by writing $3Y$ instead of $Y_1+Y_2+Y_3$. Few manage to realize that since the <u>mean</u> of $3X-\left[Y+Y_2+Y_3\right]$ is **zero**, hence $\mathrm{P}\left(3X-\left[Y_1+Y_2+Y_3\right]>0\right)=0.5$.
10(i)	There is no need to make any assumption about the distribution of the dividend yield earned by the manager's clients. This is because the sample size is large enough for Central Limit Theorem to be applicable.	The many students did not get this mark even though they have stated there is no need for any assumption to be made as Central Limit Theorem(CLT) is applicable due to the large sample size. This is because in their answer they **incorrectly** wrote that by applying CLT, the "mean dividend yield" or "dividend yield" or the "sample" will be normally distributed which is incorrect. It is the <u>sampling distribution</u> that will approximately follow a normal distribution by CLT.
10(ii)	To test $\mathrm{H_0}:\mu=12$ against $\mathrm{H_1}:\mu<12$ at 5% level of significance. $s^2=\frac{n}{n-1}\left(\text{sample variance}\right)=\left(\frac{40}{39}\right)\left(2.5^2\right)$ Under H_0, since n = 40 is large, by Central Limit theorem $\overline{X}\sim\mathrm{N}\left(12\ ,\frac{\left(\frac{40}{39}\right)\left(2.5^2\right)}{40}\right)$ approximately.	Most student are able to state correctly the null and alternative hypothesis. However, most students did not realize that 2.5% is the sample standard deviation and hence did not find s^2 resulting in the wrong p-value. Instead of using population mean of 12, a significant number of students uses the sample mean, 10.9 and wrote $\overline{X}\sim\mathrm{N}\left(10.9\ ,\frac{\left(\frac{40}{39}\right)\left(2.5^2\right)}{40}\right)$

	Test Statistic, $Z=\dfrac{\overline{X}-12}{\sqrt{\dfrac{\left(2.5^2\right)}{39}}}\sim \mathrm{N}(0,1)$ approximately. Using GC, Value of test statistic: $z=\dfrac{10.9-12}{\sqrt{\dfrac{\left(2.5^2\right)}{39}}}=-2.747799$ p-value $=0.00299989$ Since p-value$=0.003<0.05$ ($\therefore$ Reject H_0) There is sufficient evidence at 5% level of significance that the manager overstated his claim.	There were also a number of instances where students had problem concluding their hypothesis test appropriately. Some even have results that contradicts their conclusion. Generally, most students know how to carry out a hypothesis test but they need to work on their presentation and use of appropriate notation.
10(iii)	• The sample of the manager's clients obtained is assumed to be randomly chosen. • There is a need to assume that the distribution of the dividend yield earned by the manager's clients follows a normal distribution.	Not many students get full marks for this part. Quite a number of student state either one of the two required assumptions. It is interesting to note that quite a significant number of students wrote the conditions for a distribution to be well modelled by a binomial distribution which is not relevant to this question at all. Quite a number of student assume the need to use Central Limit Theorem even though the question states that the sample size is small.
10(iv)	Let random variable X be the dividend yield of the manager's clients. To test $H_0: \mu=12$ against $H_1: \mu\neq 12$ at 5% level of significance. Under H_0, $\overline{X}\sim \mathrm{N}\left(12,\dfrac{2.3^2}{n}\right)$. Test Statistic, $Z=\dfrac{\overline{X}-12}{\sqrt{\dfrac{2.3^2}{n}}}\sim \mathrm{N}(0,1)$ Value of test statistic: $z=\dfrac{10.2-12}{\sqrt{\dfrac{2.3^2}{n}}}$ To reject H_0,	Quite a number of students stated the wrong alternative hypothesis. They did not realized that in this part of the question, what is needed is a two-tail test as we are testing against the manger's claim that the mean dividend yield of the client is 12%. There are also some students who did not realize that the given value of 2.3% is the population standard deviation and tried incorrectly find s^2. Among those who realize that it is a two tail test, quite a number took the non-critical region as the critical region. There is also a group who uses only one tail of the distribution curve as the critical region and did state why

	$\dfrac{10.2-12}{\sqrt{\dfrac{2.3^2}{n}}} < -1.95996$ or $\dfrac{10.2-12}{\sqrt{\dfrac{2.3^2}{n}}} > 1.95996$ $\dfrac{-1.8\sqrt{n}}{2.3} < -1.95996$ or $\dfrac{-1.8\sqrt{n}}{2.3} > 1.95996$ $\sqrt{n} > 2.504398$ or $\sqrt{n} < -2.504398$ $n > 6.2719$ or (rejected, since $\sqrt{n}>0$) $\therefore$ Least $n = 7$.	they did not consider the other tail when solving for *n*. Generally, not many students manage to solve for *n* successfully.
11(i)	Let D denote the event that a person is infected. Let P denote the event that the quick test shows a positive result.  $\text{P}(\text{accurate test}) = \text{P}(D \cap P) + \text{P}(D' \cap P')$ $= (0.05)(0.9) + (0.95)(0.94)$ $= 0.938$	Quite well done. Some students forgot how to draw a proper tree diagram (where to label the events, probabilities etc.). Some students did not define the events, leading to some confusion when 'N' was used for both not infected and negative. Quite a few scripts with the correct tree diagram left out this part, but it was generally well-answered.
11(ii)	$\text{P}(D \mid P) = \dfrac{\text{P}(D \cap P)}{\text{P}(P)}$ $= \dfrac{(0.05)(0.9)}{(0.05)(0.9) + (0.95)(0.06)}$ $= \dfrac{45}{102}$ or $\dfrac{15}{34}$ or 0.441	Generally well-answered, with only a few students not being able to interpret the question as a conditional probability or getting the formula wrong.
11(iii)	From (ii), $\text{P}(D \mid P) = 0.441$, hence 55.9% of the people who tested positive are actually not infected, which is quite a high probability of inaccuracy when the quick test result is positive. Thus, people who tested positively should be sent for the laboratory confirmation test.	This part was poorly interpreted, with a lot of semi-coherent responses that did not actually address the question of why the recommendation is to send people who tested positively to the laboratory confirmation test.

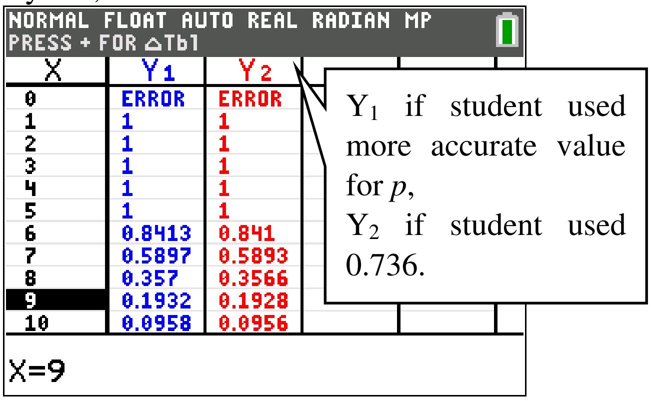
	OR From (ii), $\mathrm{P}(P)=(0.05)(0.9)+(0.95)(0.06)=0.102$, or 10.2% of the total population. Hence a smaller proportion of the population will need to be sent for the laboratory confirmation test, saving money. **OR** $\mathrm{P}(D'\cap P)=(0.95)(0.06)=0.057$; $\mathrm{P}(D\cap P')=(0.05)(0.1)=0.005$ Since $\mathrm{P}(D'\cap P)>\mathrm{P}(D\cap P')$, there is likely to be a larger increase in the proportion of the population with the correct diagnosis if the people who tested positively were sent for the laboratory confirmation test. The one drawback is that the infected people who tested negative on the quick test will be omitted and may not be provided with adequate treatment.	A properly structured answer should include (i) stated probability (or probabilities), and (ii) comment(s) about the value(s) or a comparison between values to justify the recommendation. Students' responses also showed poor consideration for the probabilities they are trying to describe, leading to a lot of confusion in their responses. The drawback was generally identified correctly, though some responses were really poor phrased.
11(iv)	Whether a randomly chosen person is infected is independent of whether other selected people are infected. The probability that a randomly chosen person is infected is constant at 0.05. Let the random variable *X* denote the number of infected people out of 20 people. $X\sim\mathrm{B}(20,0.05)$ $\mathrm{P}(X<2)=\mathrm{P}(X\le 1)$ $=0.736$	The assumption was generally given correctly. Do note that students need to know BOTH the assumptions. Some phrasing issues, which should also be fixed. The conceptual error of stating that probabilities/chances are independent is still quite prevalent among wrong responses. Mostly well answered, with some students finding $\mathrm{P}(X\le 2)$ instead, either through a misinterpretation of the question or worse still, not dealing with the inequality properly.
11(v)	Let the random variable Y_n denote the number of infected people out of *n* people. $Y_n\sim\mathrm{B}(n,0.05)$ $\mathrm{P}(>2\text{ infected and 1st infected was the tenth chosen})$ $=\mathrm{P}(\text{1st 9, 0 infected})\mathrm{P}(\text{10th infected})\mathrm{P}(\text{last 10, }>1\text{ infected})$ $=\mathrm{P}(Y_9=0)(0.05)\mathrm{P}(Y_{10}>1)$ $=\mathrm{P}(Y_9=0)(0.05)\left[1-\mathrm{P}(Y_{10}\le 1)\right]$ $=0.00271$	This part was poorly answered, with only a minority getting it fully correct. Most responses recognise the need to split the event up, but few dealt with it properly. A few almost correct responses took $\mathrm{P}(Y_{10}\ge 1)$ instead. Quite a number of responses took $\mathrm{P}(X>2)$ and multiplied it with other probabilities involving a subset of the 20 people, these students should note that the events they used are not independent.
11(vi)	Let the random variable *W* denote the number of samples with	Generally well-answered, with

fewer than 2 infected people out of n samples.

$W \sim \mathrm{B}(n, \mathrm{P}(X < 2)) = \mathrm{B}(n, 0.7358395)$

$\mathrm{P}(W \leq 5) > 0.1$

By GC,



Largest value of n is 9.

most scripts presenting clear working with an accompanying table.

Some more prominent careless mistakes include comparing the table values to 0.01 instead of 0.1, or using binompdf instead of binomcdf.