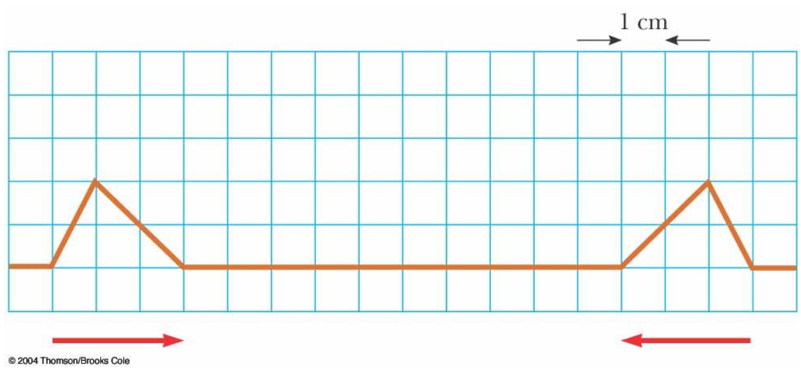
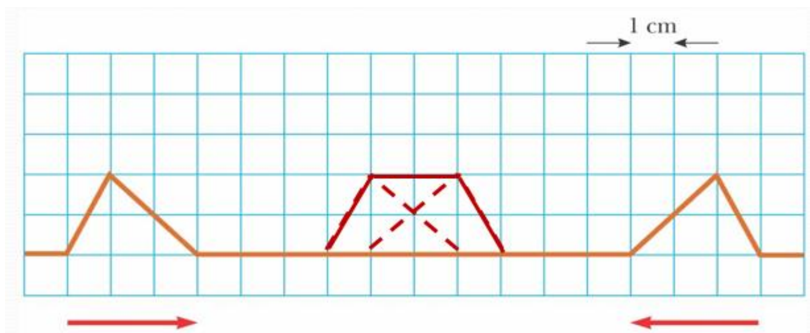
Superposition Lecture Examples Worked Solution

Example 10.3.1

Two pulses are travelling toward each other, at 10 cm s^{-1} on a long string, as shown in the figure below. Sketch the shape of the string after 0.6 s.



Solution:



Dotted lines show the shape of individual waves at t = 0.6 s.
Solid line shows the shape of the wave after interference using principle of superposition to add up the displacements of individual waves at a particular point.

Example 10.4.1: Two Source Interference

The figure (Fig. 10.4.5) shows two sources X and Y which emit identical sound waves of wavelength 2.0 m. The two sources emit in phase, and the waves emitted have equal amplitudes, each *A*.

What is the amplitude of the sound wave

(a) at R,

(b) at Q?

Suppose the source X is 180° out of phase with source Y. What does an observer hear

(c) at R,

(d) at Q?

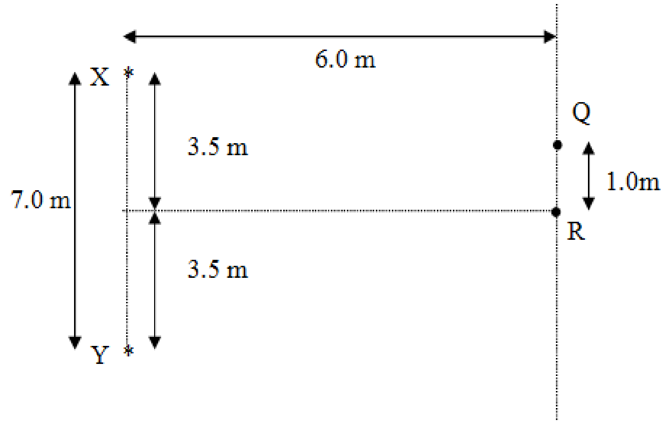


Fig. 10.4.5

Solutions:

<u>When two sources are **in Phase**:</u>

a) At R,

Path difference, δ = XR – YR = 0

Therefore, constructive interference occurs

$\Rightarrow A_{resultant} = A + A = 2A$

b) At Q,
using Pythagoras theorem,
XQ = $\sqrt{6.0^2 + (3.5 - 1.0)^2}$ = 6.5 m
YQ = $\sqrt{6.0^2 + (3.5 + 1.0)^2}$ = 7.5 m
Path difference, δ = YQ – XQ = 1.0 m = 0.5 λ
Therefore, the waves meet in antiphase
$\Rightarrow$ destructive interference occurs at Q
$\Rightarrow$ Resultant amplitude = A - A = 0

<u>If sources X & Y are **in Antiphase**:</u>

c) At R,
Path difference, δ = XR – YR = 0
Therefore, destructive interference occurs at R

$\Rightarrow A_{resultant} = A - A = 0$

$\Rightarrow$ An observer will hear <u>no sound.</u>

d) At Q,
The waves at Q will meet in phase instead,
Thus constructive interference occurs at Q.

$\Rightarrow A_{resultant} = A + A = 2A$

$\Rightarrow$ An observer will hear a <u>loud sound</u>.

Example 10.5.1 : Determination of wavelength of light using Young's Double Slit

A screen is separated from the double-slit source by 1.2 m. The distance between the two slits is 0.03 cm. The second-order bright fringe (m = 2) is measured to be 4.5 mm from the centre line

(i) What is the fringe separation between 2 neighbouring bright fringes formed on the screen?
(ii) Determine the wavelength of light.

Solution:

(i)

Since fringes are formed at small θ, the fringe separation is constant.

$$\Delta y = \frac{4.5}{2} = 2.25 \text{ mm}$$

(ii)

$$\Delta y = \frac{L\lambda}{d}$$

$$2.25 \times 10^{-3} = \frac{1.2\lambda}{0.03 \times 10^{-2}}$$

$$\lambda = 5.63 \times 10^{-7} \text{ m}$$

Example 10.6.1 : Diffraction Grating

White light from a source passes through a filter which transmits only wavelengths of 400 nm to 600 nm. When the filtered light falls normally on a diffraction grating, light of wavelength 600 nm in one order of the spectrum is diffracted at the same angle, 30°, as the 400 nm light in the adjacent order.

Find the number of lines per mm for the grating.

Solution:

Let 600 nm light be the nth order. Then (n+1)th order is the 400 nm light.

$$d\sin\theta = n\lambda_1$$
$$d\sin\theta = (n+1)\lambda_2$$

Since *d* and θ is the same,

$$n\lambda_1 = (n+1)\lambda_2$$
$$n = 2$$

Substitute n = 2 into $d\sin\theta = n\lambda_1$,

$$d = \frac{n\lambda_1}{\sin\theta} = \frac{2(600\times10^{-9})}{\sin(30°)} = 2.4\times10^{-6}m$$

$N = \frac{1}{d} = 4.17\times10^{5}$ lines per m = 417 lines per mm

Example 10.6.2 : Maximum no. of Fringes Observable.

A monochromatic source of 495 nm is incident normally on a diffraction grating which has 500 lines per mm. How many diffraction lines can be observed on the screen?

Strategy: The maximum angular deviation from the principle axis is 90^0. Using this condition, we can determine the maximum order and hence find the theoretical maximum number of fringes observable on the screen.

Solution:

Maximum theoretical angular displacement possible is $90°$

$$d \sin\theta = n\lambda$$

$$d \sin 90^o = n_{max}\lambda$$

Max. observable order,

$$n_{\max} = \frac{d}{\lambda} = \frac{1}{(500\times10^3)(495\times10^{-9})} = 4.04 = 4 \text{ (round down)}$$

Hence, max. no. of diffraction lines observable on the screen

= 2(4) + 1 = 9

Example 10.7.1: Single slit diffraction

The figure (Fig. 10.7.7a) shows light from a He-Ne laser of 633 nm is incident on a 2.0 x 10^{-4} m wide slit and the resulting diffraction pattern on the screen as shown in Fig 10.7.7b. What is the width of the central maxima (the distance between the dark fringes on either side of the central maxima) on a screen 2.0 m away?

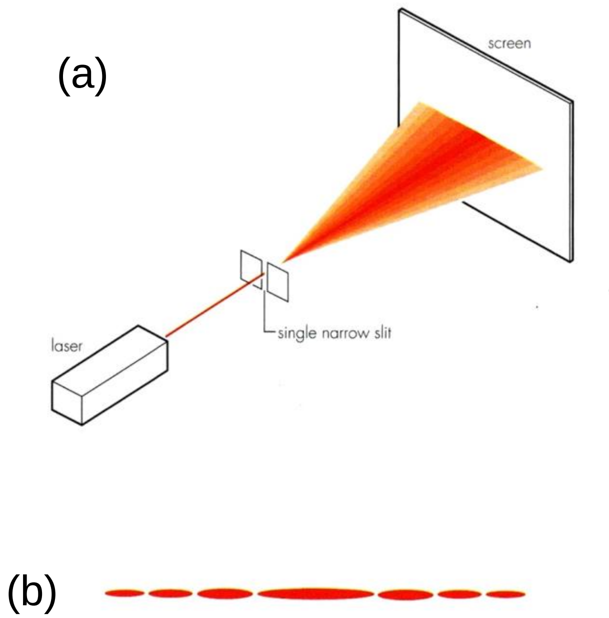


Fig. 10.7.7

Solution:

Using $\sin\theta = \frac{\lambda}{b}$

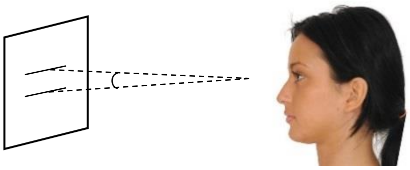
$$\theta = \frac{633\times10^{-9}}{2.0\times10^{-4}} = 0.003165 \text{ rad}$$

Note: We have used the small angle approximation $\sin\theta \approx \theta$ (in radians) since θ is very small.

Angular spread = 2θ
= 2(0.003165)
= 0.00633 rad

Width of the central maxima
= (2.0)(0.00633)
= 12.7 mm

Example 10.7.2



The resolving power of the eye may be determined by drawing two parallel lines at a distance of say 2 mm apart on a piece of card. The card is slowly moved away from the eye until the eye just cannot see the two lines as separate lines. The distance of the card from the eye is measured. Suppose for a particular person, the distance of the card from his eye is 5 m when his eye just fail to see the two lines distinctly.

(a) Determine the angle subtended at the eye by the two lines.

(b) By approximating the wavelength of the visible light to be 500 nm, estimate the aperture of the pupil of the eye.

Solution:

(a)

$(s = r\theta)$

Angular separation, $\alpha = 0.002 \div 5 = 0.0004$ rad

(b)

The first minima angle $\alpha = \theta_{\text{min}}$

$$\alpha = \frac{\lambda}{b}$$

$$0.0004 = \frac{500 \times 10^{-9}}{b}$$

$$b = 1.25 \text{ mm}$$

Example 10.8.1 : Stationary Waves

Fig 10.8.2 shows a car driven at a speed of 30 m s^{-1} along a straight road between 2 radio transmitters. The transmitters T_1 and T_2 are sending out the same programme, using a frequency of 1.50 MHz. The radio is heard to fade and strengthen regularly.

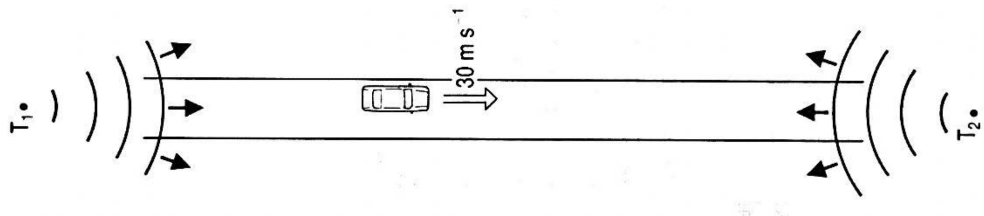


Fig. 10.8.2

What is the period of this regular fading?

Solution:

As two sources are identical and transmit radio waves in opposite direction, stationary wave will be formed between T_1 and T_2.

Wavelength of radio waves, $\lambda = \frac{c}{f} = \frac{3.00 \times 10^8}{1.5010^6} = 200$ m

Distance between 2 nodes $= \frac{\lambda}{2} = 200 \div 2 = 100$ m

Period of the regular fading equals the time taken to move between 2 nodes.

Period of regular fading $=$ distance $\div$ speed = $100 \div 30 = 3.3$ s

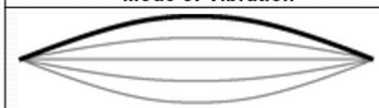
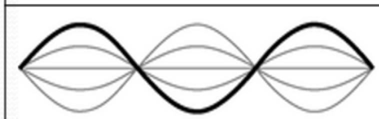
Example 10.8.2: Resonant Wavelengths of a Wire

A 1.0 m wire stretched between 2 points is plucked near one end. What are the three longest wavelengths present on the vibrating wire?

Solution:

Having the 3 longest (largest) wavelengths simply mean the 3 lowest(smallest) frequencies.
This is because assuming the **speed of the wave is a constant**, the frequency will be
inversely proportional to the wavelength

To solve:
Sketch the 3 modes of vibrations which produced the 3 lowest frequencies (corresponding to the 3 longest wavelengths) in string fixed at both ends.

Mode of Vibration	Wavelength
	$\lambda_1 = 2L = 2.0$ m
	$\lambda_2 = L = 1.0$ m
	$\lambda_3 = 2/3\ L = 0.67$ m

Ans: [2.0 m, 1.0 m, 0.67 m]

Example 10.8.3 **Serway p18.37.**

Calculate the length of a pipe that has a fundamental frequency of 240 Hz if the pipe is

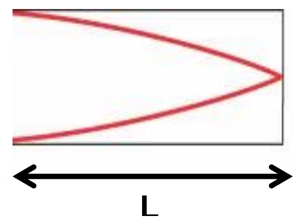
(a) closed at one end and
(b) open at both ends.

Take the speed of sound to be 343 m s^{-1}.

Solution:

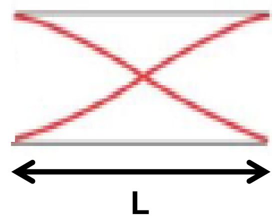
a) For pipe with one end closed and vibrating at fundamental frequency,

$$L = \frac{\lambda}{4} = \frac{v}{4f} = \frac{343}{4 \times 240} = 0.357m$$



b) For pipe open at both ends and vibrating at fundamental frequency,

$$L = \frac{\lambda}{2} = \frac{v}{2f} = \frac{343}{2 \times 240} = 0.715m$$

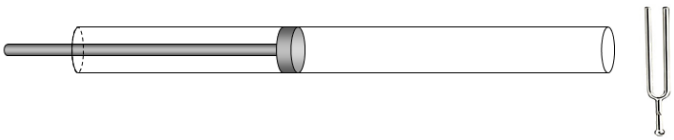


<u>Example 10.8.4</u> Serway P18.45.

An air column in a glass tube is open at one end and closed at the other by a movable piston. The air in the tube is warmed above room temperature, and a 384-Hz tuning fork is held at the open end. Resonance is first heard when the piston is 22.7 cm from the open end and again when it is 68.2 cm from the open end.

(a) What speed of sound is implied by these data?

(b) How far from the open end will the piston be when the next resonance is heard?

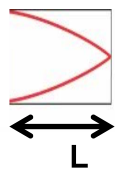


Solution:

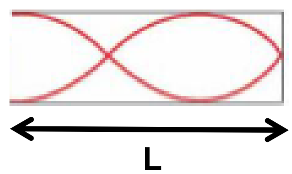
a) The difference between consecutive resonant lengths must correspond to 1 segment.

$$\frac{\lambda}{2} = (68.2 - 22.7)$$

$$\lambda = 91.0 \text{ cm}$$

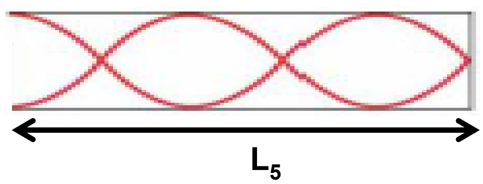


$$v = f\lambda = (384)(0.910) = 349 \text{ m s}^{-1}$$



b) The next resonant length must fit one additional segment.

$$68.2 + \frac{\lambda}{2} = 68.2 + \frac{91}{2} = 1.14 \text{ m}$$



Example 10.8.5: End Corrections

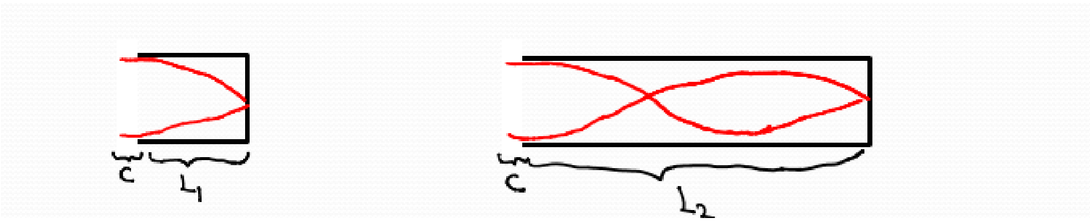
A source of sound of frequency 250 Hz is used with resonant tube, closed at one end, to measure the speed of sound in air. The two shortest resonant lengths are found to be 0.30 and 0.96 m. Calculate

(a) the speed of the sound, and

(b) the end correction of the tube.

Solution:

(a) Frequency is fixed (thus wavelength is also constant as velocity is constant) but length can vary.



$$\frac{\lambda}{4} = L_1 + c \quad \text{--- (1)}$$

$$\frac{3\lambda}{4} = L_2 + c \quad \text{--- (2)}$$

Taking (2) – (1):

$$\frac{\lambda}{2} = L_2 - L_1$$

Taking the difference here allows us to eliminate the unknown end correction.

Speed of sound is thus given by:

$$\begin{aligned} v &= f\lambda \\ &= 250(2)(0.96 - 0.30) \\ &= 330 \text{ m s}^{-1} \end{aligned}$$

(b) $\frac{\lambda}{4} = L_1 + c$

$$\begin{aligned} c &= \frac{\lambda}{4} - L_1 \\ &= \frac{v}{4f} - L_1 \\ &= \frac{330}{4(250)} - 0.30 \\ &= 0.030 \text{ m} \\ &= 3.0 \text{ cm} \end{aligned}$$