Chapter 3: Binomial Distribution

1.	JPJC JC2 Prelim 8865/2019/Q6
	Mandarin oranges are supplied to the supermarkets in boxes of 12. It is found that on average, 7% of the mandarin oranges are bad when the boxes are opened. (i) For a randomly chosen box of mandarin oranges, find the probability that (a) all the mandarin oranges are good, [1] (b) at least 25% of the mandarin oranges are bad. [2] (ii) A customer bought 20 boxes of mandarin oranges. Find the probability that fewer than 4 boxes have at least 25% bad mandarin oranges. [2] Answer: (ia) 0.419, (ib) 0.0468, (ii) 0.987

1.	JPJC JC2 Prelim 8865/2019/Q6 (Solutions)
(ia)	Let X be the random variable denoting the number of bad mandarin oranges out of 12 $X \sim \mathrm{B}(12, 0.07)$ $\mathrm{P}(X = 0) = 0.418596 \approx 0.419$ (3sf)
	Alternatively, Let X' be the random variable denoting the number of good mandarin oranges out of 12 $X' \sim \mathrm{B}(12, 0.93)$ $\mathrm{P}(X' = 12) = 0.418596 \approx 0.419$ (3sf)
(ib)	$\mathrm{P}(X \geq 0.25 \times 12) = \mathrm{P}(X \geq 3)$ $= 1 - \mathrm{P}(X \leq 2)$ $= 1 - 0.95320$ $= 0.046797 \approx 0.0468$ (3sf)
(ii)	Let Y be the random variable denoting the number of cartons out of 20 with at least 25% bad mandarin oranges. $Y \sim \mathrm{B}(20, 0.0467968)$ $\mathrm{P}(Y < 4) = \mathrm{P}(Y \leq 3) = 0.98728 \approx 0.987$ (3sf)

2.	MI PU2 Prelim 8865/2019/Q10
	Millennia Tour operates a half-day city tour on an open double-decked bus. A bus has 22 seats and Millennia Tour will accept a maximum of 22 passenger bookings per bus. Past records show that on average 30% of the passengers do not turn up for the tour after booking. Tickets sold are non-refundable and non-exchangeable. The tour is always fully booked on weekends. Find the probability that, in a bus on a randomly chosen Saturday, (i) there are exactly 7 passengers who do not turn up for the tour after booking, [1] (ii) there are at least 17 passengers who turn up for the tour after booking. [2]

(iii) 8 open double-decked buses are fully booked on a randomly chosen Saturday. Find the probability that at least 3 of these buses have at least 17 passengers who turn up for the tour after booking. [3]

During holiday peak seasons, Millennia Tour decides to maximise profit by accepting n bookings per bus on weekends, where $n \geq 22$.

(iv) The probability that sufficient seats are available for all passengers who turn up for the tour on a randomly chosen Saturday during a holiday peak season is more than 0.9. Find the largest value of n. [3]

Answer: (i) 0.177 (ii) 0.313 (iii) 0.482 (iv) 27

2. MI PU2 Prelim 8865/2019/Q10 (Solutions)

(i) Let W be the random variable denoting the number of passengers who do not turn up for the tour after booking, out of 22. Then $W \sim \mathrm{B}(22, 0.3)$.

$\mathrm{P}(W = 7) = 0.17707 = 0.177$

(ii) Method 1

Let Q be the random variable denoting the number of passengers who turn up out of 22.
$Q \sim \mathrm{B}(22, 0.7)$.

$\mathrm{P}(Q \geq 17) = 1 - \mathrm{P}(Q \leq 16) = 0.31341 = 0.313$

Method 2

$\mathrm{P}(W \leq 5) = 0.31341 = 0.313$

(iii) Let X be the random variable denoting the number of buses with at least 17 passengers who turn up for the tour, out of 8.
$X \sim \mathrm{B}(8, 0.31341)$

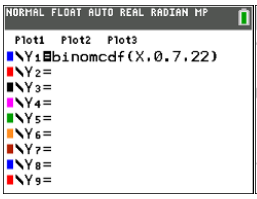
$\mathrm{P}(X \geq 3) = 1 - \mathrm{P}(X \leq 2) = 0.48216 = 0.482$

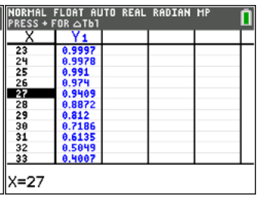
(iv) Let Y be the random variable denoting the number of passengers who turn up, out of n passengers (bookings).
Then $Y \sim \mathrm{B}(n, 0.7)$.

$\mathrm{P}(Y \leq 22) \geq 0.9$

Using GC,

n	$\mathrm{P}(Y \leq 22)$
26	0.974
27	0.9409
28	0.8872





Hence the maximum bookings should be 27.

3.	HCI JC2 Prelim 8865/2019/Q10

At the coffee and tea stall in the canteen of Hwa Chong Institution, on average, 70% of the students will order ice lemon tea. A random sample of n students are interviewed on their preference.

(i) State in context, two assumptions needed for the number of students who will order ice lemon tea to be well modelled by a binomial distribution. [2]

Assume now that the number of students who will order ice lemon tea follows a binomial distribution.

It is given that $n = 15$ for the rest of the question.

(ii) Find the probability that exactly 10 students will order ice lemon tea. [1]

(iii) Find the probability that more than 9 students will order ice lemon tea. [2]

(iv) If the probability of at least r students ordering ice lemon tea is not less than 0.8, find the greatest value of r. [3]

Answer: (ii) 0.206 (iii) 0.722 (iv) 9

3.	HCI JC2 Prelim 8865/2019/Q10 (Solutions)
(i)	Assumptions: 1. The probability of students ordering ice lemon tea is constant at 0.7 for every student. 2. The order of a student is independent of the order of another student.
(ii)	Let X be the random variable denoting the number of students ordering ice lemon tea (out of 15 students). $X \sim \mathrm{B}(15, 0.7)$ $\mathrm{P}(X = 10) = 0.20613 \approx 0.206$ (3 s.f.)
(iii)	$\mathrm{P}(X > 9)$ $= 1 - \mathrm{P}(X \le 9)$ $= 0.721621$ ≈ 0.722
(iv)	$\mathrm{P}(X \ge r) \ge 0.8$ $1 - \mathrm{P}(X \le r-1) \ge 0.8$ $\mathrm{P}(X \le r-1) \le 0.2$ From G.C., $\mathrm{P}(X \le 9-1) = 0.1311 < 0.2$ $\mathrm{P}(X \le 10-1) = 0.2784 > 0.2$ $\therefore r - 1 = 9 - 1 \Rightarrow r = 9$ Hence the greatest value of r is 9.

4. EJC JC2 Prelim 8865/2019/Q7

In a large number of apples, 2% of the apples are rotten. Uncle Ee buys cartons of apples from the fruit wholesaler every day. He randomly selects 6 apples from a carton and accepts the carton if all 6 are not rotten.

(i) Explain the significance of the phrase ‘large number’ in the first sentence of the question. [1]

(ii) Find the probability that Uncle Ee accepts a carton of apples. [1]

(iii) Uncle Ee first picks 3 cartons of apples, of which at least 2 cartons are accepted. He then decides to buy another 5 cartons. Find the probability that exactly 6 of the 8 cartons are accepted. [4]

Answer: (ii) 0.886 (iii) 0.163

4. EJC JC2 Prelim 8865/2019/Q7 (Solutions)

(i)

This is to assume that the conditional probability of the event that each apple (among the ones in each carton) is rotten is approximately the same and hence the trials (selection of apples) may be considered to be independent.

(ii)

Let X be the random variable denoting the number of apples, out of 6, that are rotten.

$X \sim \mathrm{B}(6, 0.02)$

$\mathrm{P}(\text{carton is accepted}) = \mathrm{P}(X = 0)$

$= 0.88584 = 0.886$ (3sf)

Alternatively,

Let Y be the random variable denoting the number of apples, out of 6, that are not rotten

$Y \sim \mathrm{B}(6, 0.98)$

$\mathrm{P}(\text{carton is accepted}) = \mathrm{P}(Y = 6)$

$= 0.88584 = 0.886$ (3sf)

(iii)

Let Q be the random variable denoting the number of cartons, out of first 3, that are accepted.

$Q \sim \mathrm{B}(3, 0.88584)$

Let R be the random variable denoting the number of cartons, out of next 5, that are accepted.

$R \sim \mathrm{B}(5, 0.88584)$

|
|---|
| $\mathrm{P}(\text{exactly 6 out of 8 accepted}\mid\text{at least 2 out of first 3 cartons accepted})$
$=\dfrac{\mathrm{P}(Q=2,R=4)+\mathrm{P}(Q=3,R=3)}{\mathrm{P}(Q\geq 2)}$
$=\dfrac{\mathrm{P}(Q=2)\mathrm{P}(R=4)+\mathrm{P}(Q=3)\mathrm{P}(R=3)}{1-\mathrm{P}(Q\leq 1)}$
$=0.163\ (3\text{sf})$ |

5.	NJC JC2 Prelim 8865/2019/Q7
	A survey was conducted with a large number of young employees who were asked to select one SkillsFuture programme that they would like to enroll in to further develop their skills. The survey showed that 53% would like to enroll in infocomm technology programme, 38% would like to enroll in financial management programme and 9% would like to enroll in human resource programme. Fifteen young employees who took part in the survey were randomly selected. Find the probability that (i) exactly twelve young employees said that they would like to enroll in either infocomm technology programme or financial programme, [2] (ii) no less than five young employees said that they would like to enroll in human resource programme. [2] Suppose now that there are *n* young employees who took part in the survey were randomly selected, write down an inequality in terms of *n* such that the probability of thirty or thirty-one young employees said that they would not like to enroll in financial management programme is at least 0.22. Hence, find the least value of *n*. [3] Answer: (i) 0.107 (ii) 0.0082 (iii) 47

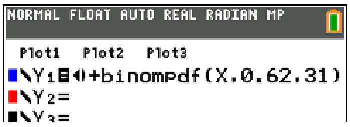
5.	NJC JC2 Prelim 8865/2019/Q7 (Solutions)
(i)	Let A be the random variable denoting the number of young employees, out of 15, who would like to enroll in either infocomm technology programme or financial programme. $A\sim \mathrm{B}(15, 0.91)$ $\mathrm{P}(A=12)=0.1069635\approx 0.107$
(ii)	Let H be the random variable denoting the number of young employees, out of 15, who would like to enroll in human resource programme. $H\sim \mathrm{B}(15, 0.09)$ $\mathrm{P}(H\geq 5)=1-\mathrm{P}(H\leq 4)$ $=0.0082037$ ≈ 0.00820
	Let M be the random variable denoting the number of young employees, out of n, who would not like to enroll in financial management programme. $M\sim \mathrm{B}(n, 0.62)$

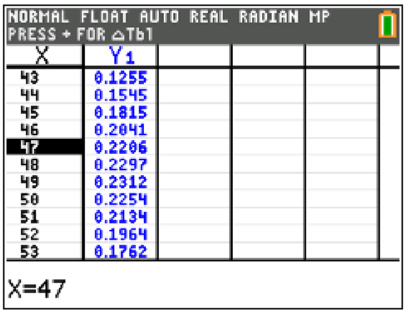
$\mathrm{P}(M = 30 \text{ or } 31) \geq 0.22$

$\Rightarrow \mathrm{P}(M = 30) + \mathrm{P}(M = 31) \geq 0.22$

$\Rightarrow \binom{n}{30}(0.62)^{30}(0.38)^{n-30} + \binom{n}{31}(0.62)^{31}(0.38)^{n-31} \geq 0.22$

Using GC,





Therefore, least value of n is 47.

6. RI JC2 Prelim 8865/2019/Q7

At a certain restaurant, customers can pay for their food using either cash, NETS or credit card. There are no other mode of payments and the choice of payment for each customer is independent of another customer. It is given that the probability of a customer paying using cash, NETS or credit card is p, $2p$ and $3p$ respectively.

During a particular lunch hour, there are 6 customers in the queue at the cashier.

(i) Find the probability that at least 2 of them pay with a credit card. [3]

(ii) The 3rd customer in the queue pays by NETS. Find the probability that fewer than 3 customers in the queue pay by NETS. [3]

Answer: (i) 0.891 (ii) 0.461

6. RI JC2 Prelim 8865/2019/Q7 (Solutions)

(i) Total probability = $p + 2p + 3p = 1 \Rightarrow p = \frac{1}{6}$

Let X be the random variable denoting the number of customers (out of 6) who pay with a credit card, $X \sim \mathrm{B}\left(6, \frac{1}{2}\right)$

$\mathrm{P}(X \geq 2) = 1 - \mathrm{P}(X \leq 1) = 0.891$ (3 sf)

(ii) Let Y be the random variable denoting the number of customers (out of 5) who pay by NETS,

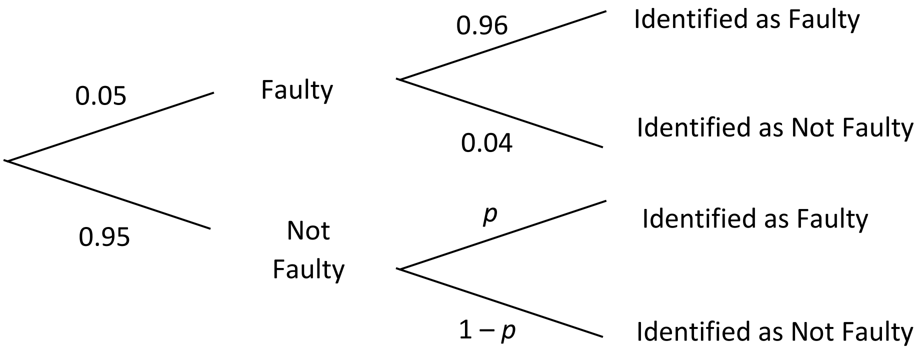
$Y \sim \mathrm{B}\left(5, \frac{1}{3}\right)$

$\mathrm{P}(Y \leq 1) = 0.461$ (3 sf)

7. DHS JC2 Prelim 8865/2019/Q8
At a funfair, Alice plays a game by tossing a biased coin 80 times using a computer simulation. Let Y be the random variable denoting the number of heads obtained and p be the probability of obtaining a head. It is given that $80+\mathrm{E}(Y)=6\mathrm{Var}(Y)$.   (i) Show that $p=\frac{1}{3}$. [3]   (ii) Find the probability of Alice obtaining at least 30 heads in the game, given that she had obtained heads in her first 5 tosses. [3]   Answer: (i) $\dfrac{1}{3}$ (ii) 0.543

7.	DHS JC2 Prelim 8865/2019/Q8 (Solutions)
(i)	$Y \sim \mathrm{B}(80,\ p)$ $80+80p=480p(1-p)$ $1+p=6p-6p^2$ $6p^2-5p+1=0$ $p=\dfrac{1}{3}$ or $p=\dfrac{1}{2}$ (rejected as coin is not fair)
(ii)	Let W be the random variable denoting the number of heads obtained in the last 75 tosses $W \sim \mathrm{B}(75,\ \frac{1}{3})$ Required probability $=\mathrm{P}(W \geq 25)$ $=1-\mathrm{P}(W \leq 24)$ $=0.543$

8. TMJC JC2 Prelim 8865/2019/Q10
A company produces a certain type of batteries. The batteries are packed in boxes of 24. On average, 5% of the batteries produced by the company are faulty.   (i) The number of batteries that are faulty in a box is denoted by X. State, in context, one assumption needed for X to be well modelled by a binomial distribution. [1]   Assume now that X follows a binomial distribution.   (ii) Find $\mathrm{P}(X=3 \text{ or } 4)$. [2]   (iii) Find the probability that a box of 24 batteries contains more than two batteries that are faulty. [2]   The boxes are packed into cartons. Each carton contains four boxes.

	(iv) Find the probability that a carton contains at most one box with more than two batteries that are faulty. [2] The company conducts a test to check if the batteries are faulty. Batteries identified to be faulty are discarded. If a battery is faulty, the probability that the test correctly identifies the battery as faulty is 0.96. If a battery is not faulty, the probability that the test incorrectly identifies the battery as faulty is p. The probability that the test correctly identifies a battery as faulty or not faulty is 0.9315. (v) Show that $p = 0.07$. [1] (vi) Find the probability that a battery identified as faulty by the test is not faulty. [3] (vii) Discuss briefly if the test is worthwhile. [1] Answer: (ii) 0.110 (iii) 0.116 (iv) 0.931 (v) 0.07 (vi) 0.581
8.	TMJC JC2 Prelim 8865/2019/Q10 (Solutions)
(i)	A battery being faulty is independent of any other battery being faulty.
(ii)	Let X be the random variable denoting the number of batteries out of 24 that are faulty $X \sim \mathrm{B}(24, 0.05)$ $\mathrm{P}(X = 3 \text{ or } 4)$ $= \mathrm{P}(X = 3) + \mathrm{P}(X = 4)$ $= 0.110$ (to 3 s.f)
(iii)	$\mathrm{P}(X > 2)$ $= 1 - \mathrm{P}(X \le 2)$ $= 0.11594$ $= 0.116$ (to 3 s.f)
(iv)	Let Y be the random variable denoting the number of boxes, out of 4, that contain more than two batteries that are faulty $Y \sim \mathrm{B}(4, 0.11594)$ $\mathrm{P}(Y \le 1)$ $= 0.931$ (to 3 s.f)
(v)	

	P(test correctly identifies batteries as faulty or not faulty) = 0.9315 $(0.05)(0.96)+(0.95)(1-p)=0.9315$ $p=1-\frac{0.8835}{0.95}=0.07$ (shown)
(vi)	P(battery is not faulty | identified as faulty) $=\frac{\text{P(battery is not faulty and identified as faulty)}}{\text{P(identified as faulty)}}$ $=\frac{(0.95)(0.07)}{(0.05)(0.96)+(0.95)(0.07)}$ $=0.581(\text{to 3 s.f})$
(vii)	Even though it is given that the test correctly identifies batteries as faulty or not faulty with a high probability of 0.9315, from part (vi), it is known that 58.1% of the batteries that are identified as faulty by the test are not faulty but end up being discarded. Therefore, the test is not worthwhile. The test is worthwhile since the probability that the test correctly identifies batteries as faulty or not faulty is high at 0.9315