NAME: _____ **( )** **CLASS:** _____

FAIRFIELD METHODIST SCHOOL (SECONDARY)

PRELIMINARY EXAMINATION 2023
SECONDARY 4 EXPRESS

ADDITIONAL MATHEMATICS **4049/02**
Paper 2

Date: 25 August 2023 **Duration: 2 hours 15 minutes**

READ THESE INSTRUCTIONS FIRST

Write your name, index number and class on all the work you hand in.
Write in dark blue or black pen.
You may use a pencil for any diagrams or graphs.
Do not use staples, paper clips, highlighters, glue or correction fluid.

Answer **all** questions.
Write your answers on the space provided.
Give non-exact numerical answers correct to 3 significant figures, or 1 decimal place in the case of angles in degrees, unless a different level of accuracy is specified in the question.
The use of a scientific calculator is expected, where appropriate.
You are reminded of the need for clear presentation in your answers.

At the end of the examination, fasten all your work securely together.
The number of marks is given in brackets [ ] at the end of each question or part question.
The total number of marks for this paper is 90.

For Examiner's Use

Table of Penalties		Question Number		
Presentation	□ 1 □ 2			
Rounding off	□ 1		Parent's/Guardian's Signature	**90**

Setter : Mr Wilson Ho

This paper consists of <u>19</u> printed pages including this cover page.

Name: _____ ( ) Class: _____

Mathematical Formulae

1. ALGEBRA

Quadratic Equation

For the equation $ax^2 + bx + c = 0$,

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

Binomial expansion

$$(a+b)^n = a^n + \binom{n}{1} a^{n-1}b + \binom{n}{2} a^{n-2}b^2 + \ldots + \binom{n}{r} a^{n-r}b^r + \ldots + b^n,$$

where n is a positive integer and $\binom{n}{r} = \frac{n!}{r!(n-r)!} = \frac{n(n-1)\ldots(n-r+1)}{r!}$

2. TRIGONOMETRY

Identities

$$\sin^2 A + \cos^2 A = 1$$

$$\sec^2 A = 1 + \tan^2 A$$

$$\operatorname{cosec}^2 A = 1 + \cot^2 A$$

$$\sin(A \pm B) = \sin A \cos B \pm \cos A \sin B$$

$$\cos(A \pm B) = \cos A \cos B \mp \sin A \sin B$$

$$\tan(A \pm B) = \frac{\tan A \pm \tan B}{1 \mp \tan A \tan B}$$

$$\sin 2A = 2 \sin A \cos A$$

$$\cos 2A = \cos^2 A - \sin^2 A = 2\cos^2 A - 1 = 1 - 2\sin^2 A$$

$$\tan 2A = \frac{2 \tan A}{1 - \tan^2 A}$$

Formulae for ΔABC

$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$

$$a^2 = b^2 + c^2 - 2bc \cos A$$

$$\Delta = \frac{1}{2} ab \sin C$$

Name: _____ ( ) Class: _____

1 **(a)** By using long division, divide $2x^3 + 6x^2 + x + 3$ by $x + 3$. [1]

(b) Express $\dfrac{9x^2 - 10x - 16}{2x^3 + 6x^2 + x + 3}$ in partial fractions. [5]

Name: _____ ( ) Class: _____

2 **(a)** Given that $y = e^{2x}(5x - 4)$, show that $\frac{dy}{dx} = e^{2x}(10x - 3)$. [3]

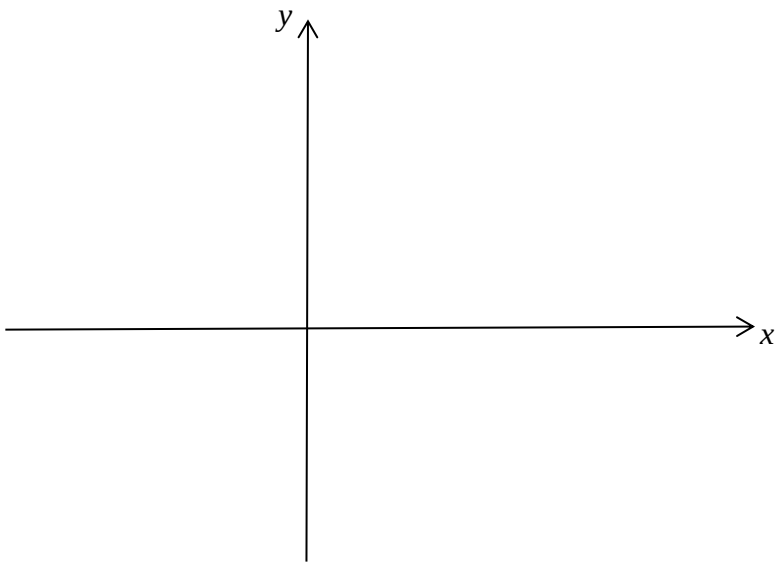
(b) Hence find $\int 4xe^{2x}dx$ and evaluate $\int_0^3 4xe^{2x}\,dx$. [5]

Name: _____ ( ) Class: _____

3 The graph of $y = \log_p x$ passes through the points (27, 3) and $(q, -2)$.

(a) Find the value of p and of q. [2]

(b) Sketch, on the same diagram, the graphs of $y = \log_p x$ and $y = 3^{-x}$. [3]



(c) State the number of solutions for $\log_p x = 3^{-x}$. [1]

Name: _____ ( ) Class: _____

4 **(a)** Solve the equation $\log_6\left(2^y+1\right)-\log_6\left(2^y-4\right)=1$. [4]

(b) Given that $\left(\log_x xy\right)\left(\log_y x^6\right)-8=0$, express y in terms of x. [4]

Name: _____ ( ) Class: _____

5 It is given that $f(x)$ is such that $f''(x) = 4\cos 4x + 2\sin 2x$. Given also that $f(0) = 0$ and $f(\frac{\pi}{4}) = \frac{3}{4}$.

(a) Find $f(x)$. [5]

5 **(b)** Show that $f(\frac{\pi}{6}) = \frac{7-2\sqrt{3}}{8}$. [3]

6 A circle, C_1, has equation $x^2 + y^2 - 4x + 6y = 12$. The equation of the normal to this circle at a point is $3y - 4x = k$.

(a) Find the value of the constant *k* and the radius of C_1. [4]

A second circle, C_2, centre (14, 2), just touches C_1.

(b) Find the equation of C_2. [3]

Name: _____ ( ) Class: _____

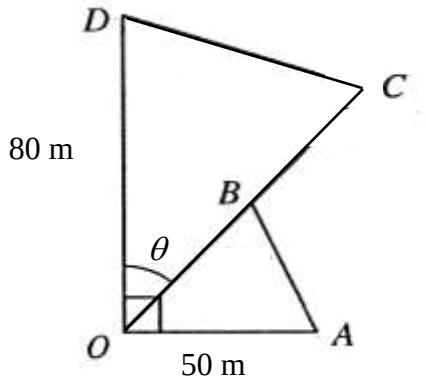
7 **(a)** Prove that $\dfrac{2\tan x + \sec^2 x}{1 - \tan^2 x} = \dfrac{\cos x + \sin x}{\cos x - \sin x}$. [4]

Name: _____ ( ) Class: _____

7 **(b)** Find all the values of x between $0°$ and $360°$ for which $\operatorname{cosec}^2 x - 5\cot x = -5$. [4]

Name: _____ ( ) Class: _____

8



The diagram shows two triangular plots of land *OAB* and *OCD*. It is given that triangle *OAB* and triangle *OCD* are isosceles triangles with *OA* = *OB* = 50 m and *OC* = *OD* = 80 m.

Angle *AOD* = 90° and angle *COD* = θ.

The sum of the areas of the two plots of land is *S* m^2.

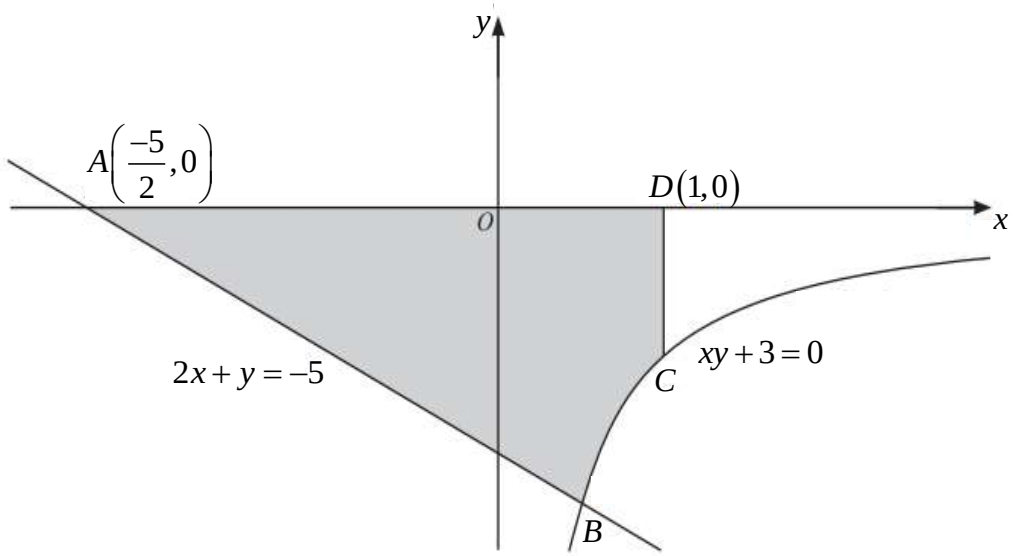
(a) Show that $S = 3200\sin\theta + 1250\cos\theta$. [2]

8 **(b)** Express S in the form $R\sin(\theta+\alpha)$ where $R>0$ and α is an acute angle. [4]

(c) Given that θ can vary, find the value of θ for which S will be a maximum. [2]

Name: _____ ( ) Class: _____

9



The diagram shows the straight line $2x + y = -5$ and part of the curve $xy + 3 = 0$. The straight line intersects the *x*-axis at the point $A\left(\frac{-5}{2}, 0\right)$ and intersects the curve at the point *B*. The point *C* lies on the curve. The point *D* has coordinates (1, 0). The line *CD* is parallel to the *y*-axis.

(a) Find the coordinates of point *B*. [3]

9 **(b)** Find the area of the shaded region, giving your answer in the form $p + \ln q$ where p and q are positive integers. [5]

(c) Find the equation of the normal at point C. [4]

Name: _____ ( ) Class: _____

10 Given that $x^2 + 2x - 3$ is a factor of the function f(x) = $x^4 + 6x^3 + 2ax^2 + bx - 3a$,

(a) find the value of a and of b, [6]

(b) find the other quadratic factor of f(x), [2]

10 **(c)** show that the equation f(x) = 0 has two real distinct roots. [3]

Name: _____ ( ) Class: _____

11 The variables x and y are related by the equation $y = 10^{-M} n^x$, where M and n are constants. The table shows values of x and y.

x	15	20	25	30	35	40
y	0.15	0.38	0.95	2.32	5.90	14.80

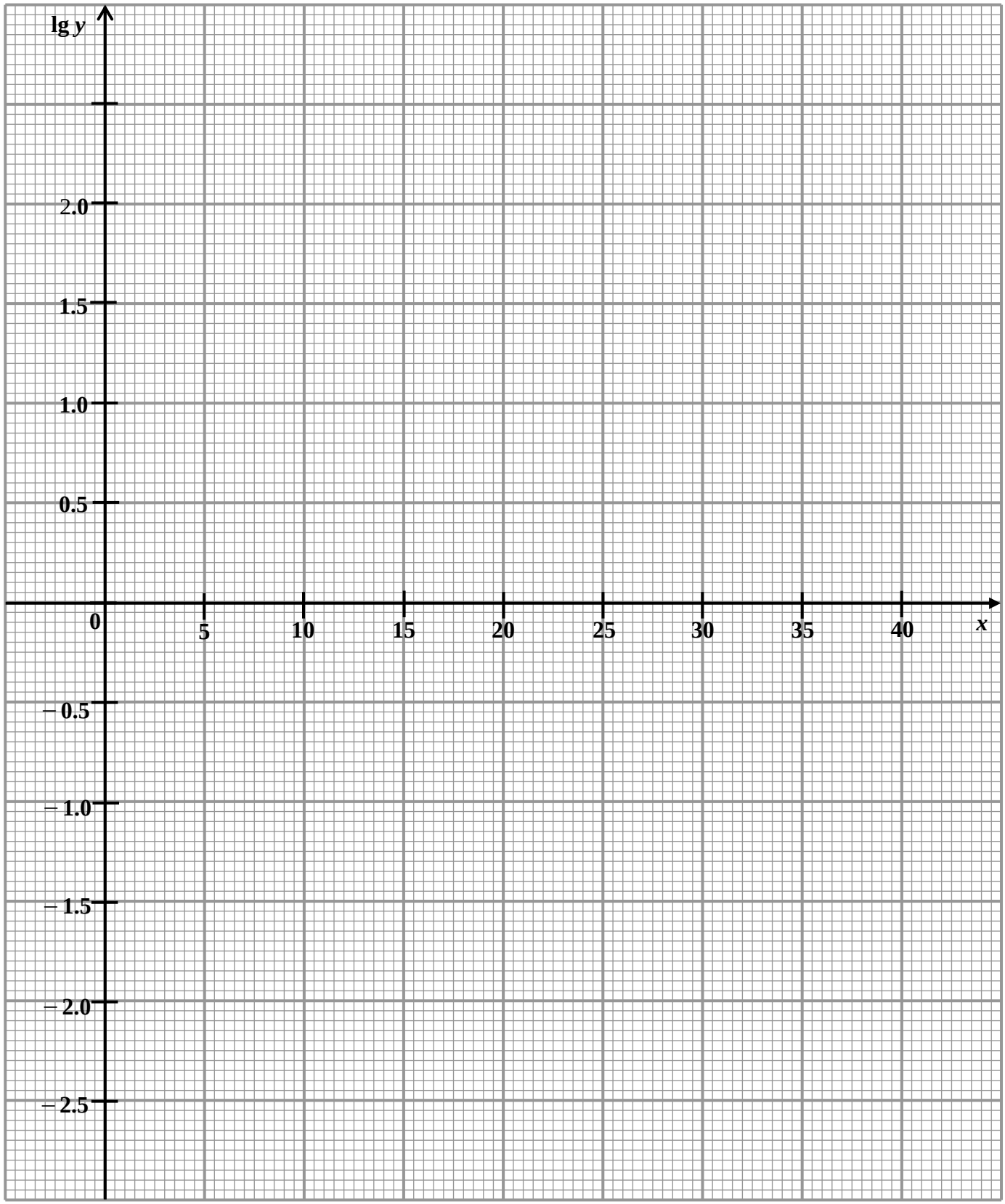
(a) Show your working clearly and draw a straight line graph of lg y against x on the grid provided. [2]

(b) Use your graph to estimate

(i) the value of M and of n, [4]

(ii) the value of x when $y = 10$. [2]

Name: _____ ( ) Class: _____



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