INDEX NUMBER

Anglo - Chinese School
(Independent)

FINAL EXAMINATION 2023

YEAR THREE EXPRESS

ADDITIONAL MATHEMATICS

PAPER 1 **4049/01**

Tuesday **3 October 2023** **1 hour 30 minutes**

Candidates answer on the Question Paper.
No additional materials are required.

READ THESE INSTRUCTIONS FIRST

Write your index number in the space at the top of this page.
Write in dark blue or black pen.
You may use an HD pencil for any diagrams or graphs.
Do not use staples, paper clips, glue or correction fluid.

Answer **all** questions.
Give non-exact numerical answers correct to 3 significant figures, or 1 decimal place in the case of angles in degrees, unless a different level of accuracy is specified in the question.
The use of an approved scientific calculator is expected, where appropriate.
You are reminded of the need for clear presentation in your answers.

The number of marks is given in brackets [ ] at the end of each question or part question.
The total number of marks for this paper is 60.

For Examiner's Use
60

This document consists of **13** printed pages and 1 blank page.

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Mathematical Formulae

1. ALGEBRA

Quadratic Equation

For the equation $ax^2 + bx + c = 0$,

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

Binomial expansion

$$(a+b)^n = a^n + \binom{n}{1} a^{n-1} b + \binom{n}{2} a^{n-2} b^2 + \ldots + \binom{n}{r} a^{n-r} b^r + \ldots + b^n,$$

where n is a positive integer and $\binom{n}{r} = \frac{n!}{r!(n-r)!} = \frac{n(n-1)\ldots(n-r+1)}{r!}$

2. TRIGONOMETRY

Identities

$$\sin^2 A + \cos^2 A = 1$$

$$\sec^2 A = 1 + \tan^2 A$$

$$\cos ec^2 A = 1 + \cot^2 A$$

Formulae for ΔABC

$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$

$$a^2 = b^2 + c^2 - 2bc \cos A$$

$$\Delta = \frac{1}{2} ab \sin C$$

Answer **all** the questions.

1 Solve the following pair of simultaneous equations, [4]

$$3x + y = 5,$$
$$2x^2 + xy - y^2 = 5.$$

2 A straight line graph, obtained by plotting $\frac{x}{y}$ against x^2, passes through the points $(-2, 5)$ and $(4, 8)$.

Prove that $y = \frac{2x}{x^2 + 12}$. [4]

3 Express $\dfrac{x^2+1}{x^2-1}$ in partial fractions. [4]

4 The population of bacteria, P, after t hours can be modelled by $P = P_0\left(2^t\right)$, where P_0 is a constant.

(i) Find the least number of hours (to the nearest hour) it will take for the population to reach 100 times its original size. [2]

(ii) Sketch the graph of P against t. [2]

5 Solve the equation $\log_3 x - 3\log_x 9 = 1$. [5]

6 ***Do not use a calculator in this question.***

Given that $\tan\theta = \frac{3}{5}$ and that θ is non-acute, find the exact value of

(i) $\cot\theta + \sec^2\theta$, [2]

(ii) $\sin\theta$, [1]

(iii) $\cos(180° - \theta)$, [1]

(iv) $\dfrac{\operatorname{cosec}(-\theta)}{\sec(\theta)}$. [2]

7 **(a)** Given that $5x^3 + x^2 + 27x + A \equiv Bx(x-1)^2 + C(x-1)(x+3)$ for all real values of *x*, find the constants *A*, *B* and *C*. [4]

(b) The polynomial f(*x*) leaves a remainder of 4 and -7 when divided by $2x-1$ and $x+2$ respectively. Find the remainder when *f*(*x*) is divided by $2x^2 + 3x - 2$. [4]

8 **(a)** Find the range of values of x which satisfies $\dfrac{x^2+2x+2}{x^2-5x+6} \geq 0$. [4]

(b) Show that the line $y = x + k$ always intersects the curve $xy = y + 2$ at two distinct points for all real values of k. [4]

9 In Singapore, Alternating Current (AC) power is provided by utility companies to households at a frequency of 50 Hz, or 50 cycles per second. The AC voltage oscillates between 230 and -230 volts during each cycle in a sinusoidal manner. The voltage supply, V, can be represented by the equation $V = 230\sin\left(100\pi t\right)$, where t is the time in seconds.

(i) Find p, the period of the graph. [1]

(ii) Find the time, in seconds, when it first reaches the maximum voltage. [2]

(iii) Sketch the graph of $V = 230\sin\left(100\pi t\right)$, for $0 \le t \le p$. [2]

(iv) Find the exact times within the first cycle when the voltage reaches 115 volts. [3]

10 ***Solutions obtained by accurate drawing will not be accepted.***

The coordinates of points *A*, *B* and *C* are $(-2,3)$, $(2,-4)$ and $(7,11)$ respectively.

(i) Find the area of triangle *ABC*. [2]

(ii) Given that the coordinates of point *D* are $(-1,-2)$, justify whether the points *A*, *B* and *D* are collinear. [2]

(iii) Find the coordinates of the foot of perpendicular from point *A* to the line *BC*. [5]

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