PRESBYTERIAN HIGH SCHOOL

MATHEMATICS   **4045/01**
PAPER ONE

25 July 2022   **Monday**   **2 hours**

SECONDARY FOUR NORMAL (ACADEMIC)
PRELIMINARY EXAMINATIONS

MARK SCHEME

Answer **all** the questions.

1 **(a)** $\qquad 1.75^2 \qquad\qquad -1.3 \qquad\qquad \pi \qquad\qquad -1\frac{1}{3}$

Write these numbers in order of size, starting with the smallest.

Answer $\quad -1\frac{1}{3}, -1.3, 1.75^2, \pi$ [B1]   [1]

(b) Express 73.4% as a fraction.

$$\frac{73.4}{100} = \frac{367}{500}$$

Answer $\quad \dfrac{367}{500}$ [B1]   [1]

2

	Adult	Child (Ages 4 – 12)
One-Day Admission Ticket	$68	$58
Universal Express Pass	$50	$50

(Source: www.rwsentosa.com)

The table shows the cost for one-day admission ticket to Universal Studio.
The Universal Express Pass can be purchased to skip the queue at participating rides and shows.
Mr and Mrs Teo bought one-day admission tickets for themselves and their son (12 years old) and daughter (13 years old). They also bought Universal Express Passes for each of their two children.

Calculate the total cost of the tickets.

$$\text{cost} = (68 \times 3 + 58) + (50 \times 2) \quad \text{[M1 - cost of admission tickets/ cost of universal pass]}$$
$$= 362$$

Answer $ \underline{\quad 362 \quad \text{[A1]}\quad\quad\quad\quad\quad\quad}$ [2]

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3 Solve $\dfrac{4}{5x-2} = 3$.

$$\frac{4}{5x-2} = 3$$
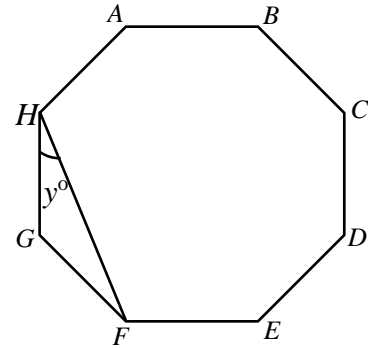$$3(5x-2) = 4 \qquad \text{[M1]}$$
$$15x - 6 = 4$$
$$15x = 10$$
$$x = \frac{2}{3}$$

Answer $x = \quad \dfrac{2}{3}$ [A1]   [2]

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4 The diagram shows a regular octagon *ABCDEFGH*.
Calculate *y*.

each int $\angle = \dfrac{(8-2)\times 180°}{8} = 135°$ } [M1 – either]

$y = \dfrac{180° - 135°}{2} = 22.5°$

Answer   *y* =   22.5   [A1]   [2]

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5 *p* is inversely proportional to q^2.
When *p* = 3 when *q* = 2.
Find the value of *p* when *q* = 10.

$p = \dfrac{k}{q^2}$

$3 = \dfrac{k}{2^2}$ [M1]

$k = 12$

$p = \dfrac{12}{10^2} = \dfrac{3}{25}$ or 0.12

Answer   *p* = $\dfrac{3}{25}$ or   0.12   [A1]   [2]

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6 **(a)** Given that $\dfrac{5^6}{5^{-3}\times 5^0} = 5^v$, find *v*.

$\dfrac{5^6}{5^{-3}\times 5^0} = 5^v$

$5^{6-(-3)} = 5^v$

$5^9 = 5^v$

Answer   *v* =   9   [B1]   [1]

(b) Given that $\dfrac{1}{64} = 4^w$, find *w*.
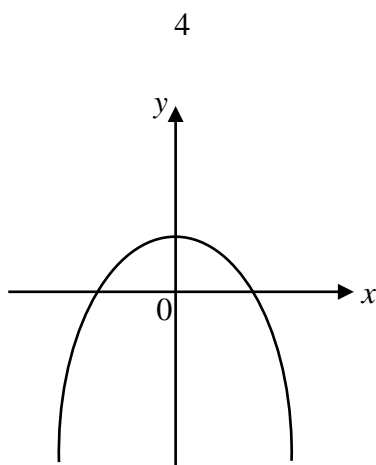
$\dfrac{1}{64} = 4^w$

$\dfrac{1}{4^3} = 4^w$

$4^{-3} = 4^w$

$w = -3$

Answer   *w* =   – 3   [B1]   [1]
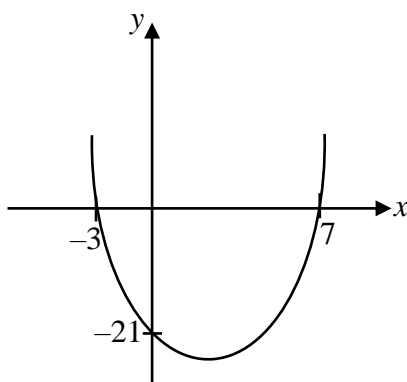
7 **(a)**

The diagram shows the sketch of $y = -x^2 + k$.
State a possible value of k.

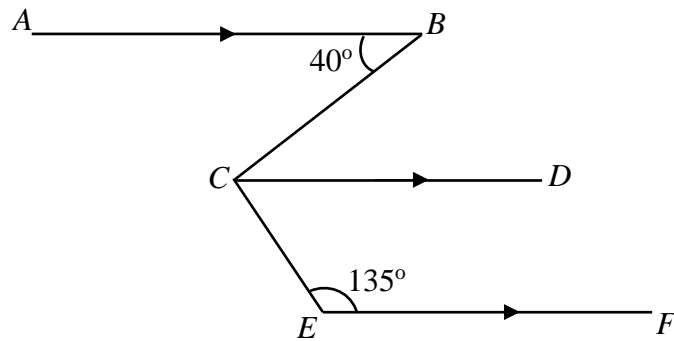*Answer* $k =$ ⎯1⎯ [B1] ⎯⎯⎯⎯⎯⎯⎯⎯ [1]
[Accept any value above 0]

(b)

State the equation of the line of symmetry of this graph of a quadratic function.

$x = \dfrac{-3+7}{2}$

$x = 2$

Answer ⎯$x = 2$⎯ [B1] ⎯⎯⎯⎯⎯⎯⎯⎯ [1]

8 In the diagram, *AB* is parallel to *CD* and *EF*. Angle $ABC = 40°$ and angle $CEF = 135°$.

Calculate angle *BCE*.

$\angle BCD = 40° \quad (\text{alt} \angle, AB//CD)$

$\angle DCE = 180° - 135° = 45° \quad (\text{int} \angle, CD//EF) \quad$ [M1 – int angle]

$\angle BCD = 40° + 45° = 85°$

Answer <u>85° [A1]</u>   [2]

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9 **(a)** $\sin m° = 0.6$

Given that *m* is an obtuse angle, find *m*.

$m = 180° - \sin^{-1}(0.6)$

$\quad = 143.13$

Answer $m =$ <u>143.1 [B1]</u>   [1]

(b) $\cos 133° = -\cos n°$

Given that *n* is an acute angle, find *n*.

$m = 180° - 133°$

$\quad = 47°$

Answer $n =$ <u>47° [B1]</u>   [1]

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10 On the grid below, draw a reduction of figure *P* using a scale factor of $\frac{1}{3}$.
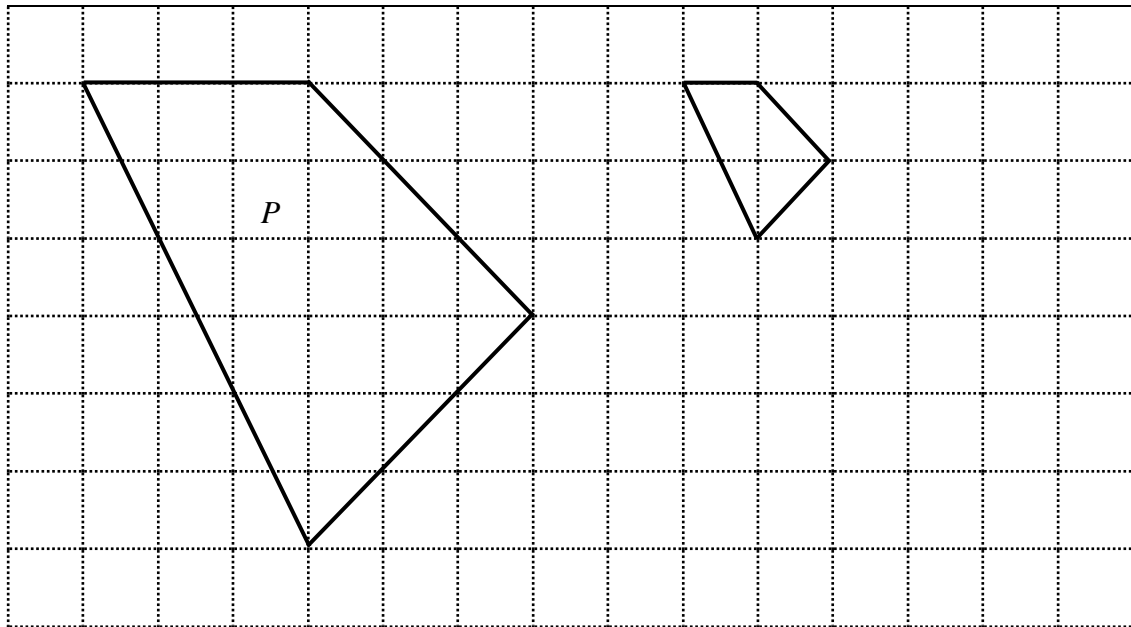
[B2 – 4 correct sides, B1 – 2 correct sides] [2]

11 A car is travelling at a constant speed of 90 km/h.

(a) How many minutes does the car take to travel 105 km?

$$\text{time} = \frac{105}{90} \quad \text{[M1]}$$
$$= \frac{7}{6}\text{h}$$
$$= 70 \text{ min}$$

Answer __70 minutes [A1]__ [2]

(b) Convert 90 km/h to m/s.

$$\frac{90 \times 1000}{1 \times 3600} = 25$$

Answer __25 m/s [B1]__ [1]

12 In 2021, Jane's annual salary was \$22 140.
This was an increase of 2.5% on her annual salary in 2020.

(a) Calculate her annual salary in 2020.

$$\text{annual salary} = \frac{22140}{102.5} \times 100\% \qquad \text{[M1]}$$
$$= 21600$$

Answer \$ <u> 21 600   [A1] </u> [2]

(b) The income tax rates for 2021 were:

Chargeable Income	Income Tax Rate (%)	Gross Tax Payable (\$)
First \$20 000	0	0
Next \$10 000	2	200

(Source: www.iras.gov.sg)

Calculate the amount of income tax she paid in 2021.

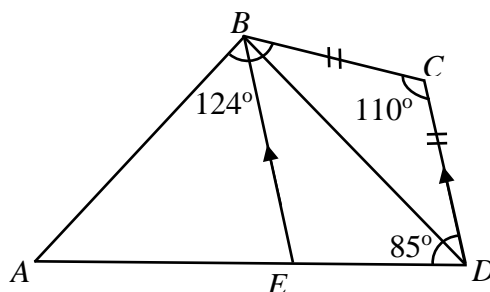$$\text{tax payable} = 2140 \times 2\%$$
$$= \$42.80$$

Answer \$ <u> 42.80   [B1] </u> [1]

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13 Express $\dfrac{5}{x^2-9} + \dfrac{1}{x+3}$ as a single fraction, in its simplest form.

$$\frac{5}{x^2-9} + \frac{1}{x+3} = \frac{5}{(x+3)(x-3)} + \frac{1}{x+3} \qquad \text{[M1 - } x^2-9 = (x+3)(x-3)\text{]}$$

$$= \frac{5}{(x+3)(x-3)} + \frac{1(x-3)}{(x+3)(x-3)} \qquad \text{[M1 - common denominator]}$$

$$= \frac{5+x-3}{(x+3)(x-3)}$$

$$= \frac{2+x}{(x+3)(x-3)}$$

Answer $\dfrac{2+x}{(x+3)(x-3)}$ [A1]   [3]

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14 *ABCD* is a quadrilateral and *E* is a point on *AD*. *BC = CD* and *BE* is parallel to *CD*.
Angle *ABC* = 124°, angle *BCD* = 110° and angle angle *CDA* = 85°.

(a) Find angle *BAD*.

$$\angle BAD = 360° - 110° - 85° - 124°$$
$$= 41°$$

Answer <u>41°   [B1]   </u> [1]

(b) Find angle *BDE*.

$$\angle CDB = \frac{180° - 110°}{2} = 35° \quad [M1]$$
$$\angle BDE = 85° - 35° = 50°$$

Answer <u>50° [A1]   </u> [2]

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15 (a) Jerry deposits $60 500 in a bank at 1.2% per annum interest for 3 years.
Calculate the simple interest earned for 3 years.

$$\text{interest} = \frac{60500 \times 1.2 \times 3}{100}$$
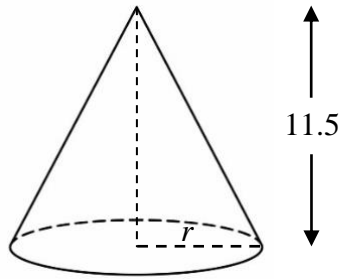$$= 2178$$

Answer $ <u>   2178   [B1]   </u> [1]

(b) Tom invests $12 000 for 4 years.
He receives compound interest at 1.15% per year.
How much is the investment worth at the end of the four years?
Give your answer to the nearest cent.

$$\text{Amt} = 12000\left(1 + \frac{1.15}{100}\right)^4 \quad [M1]$$
$$= 12561.595$$

Answer $ <u>   12561.60   [A1]   </u> [2]

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16 A cone has a vertical height of 11.5 cm, radius r cm and volume 434 cm^3.

11.5

Calculate r.

$$\frac{1}{3}\pi r^2 (11.5) = 434 \qquad \text{[M1]}$$

$$r^2 = \frac{434}{\frac{1}{3}\pi(11.5)} \qquad \text{[M1]}$$

$$r = \sqrt{\frac{434}{\frac{1}{3}\pi(11.5)}}$$

$$= 6.00318$$

Answer $r = $ ⟨6.00 [A1]⟩ [3]

17 The scale of the map is 1 : 40 000.

(a) The distance between two police stations on the map is 35 cm.
Find the actual distance. Give your answer in kilometres.

$$1\,\text{cm} : 40000\,\text{cm}$$
$$1\,\text{cm} : 0.4\,\text{km}$$
$$\text{actual distance} = 35 \times 0.4 \qquad \text{[M1]}$$
$$= 14$$

Answer <u>14 km [A1]</u> [2]

(b) The area of a forest has an area of 4.8 km^2.
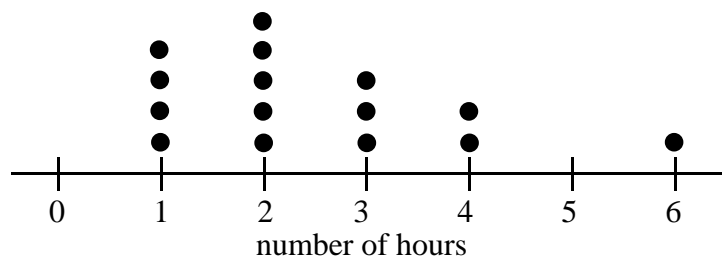Calculate the area of the forest on the map. Give your answer in cm^2.

$$1\,\text{cm} : 40000\,\text{cm}$$
$$(1\,\text{cm})^2 : (0.4\,\text{km})^2 \quad \text{[M1]}$$
$$\text{area on map} = \frac{4.8}{(0.4)^2}$$
$$= 30$$

Answer <u>30 cm^2 [A1]</u> [2]

18 This dot diagram shows the number of hours some boys spent on social media in a particular week.

number of hours

(a) How many boys are there?

Answer 15 boys   [B1] .... [1]

(b) Find the mode.

Answer 2 hours   [B1] .... [1]

(c) Find the number of boys who spent 2 hours on the social media in the week.

Answer 5 boys   [B1] .... [1]

(d) If a boy is selected at random, find the probability that the boy spent more than 3 hours on the social media in the week.

$$\frac{3}{15} = \frac{1}{5}$$

Answer $\frac{1}{5}$ [B1] .... [1]

19 (a) $x^2 - 6x - 11 = (x+h)^2 + k$

Find h and k.

$$x^2 - 6x - 11 = x^2 - 6x + \left(\frac{-6}{2}\right)^2 - \left(\frac{-6}{2}\right)^2 - 11$$

$$= (x-3)^2 - 20$$

Answer $h =$ ⸻ –3 [B1] ⸻ $k =$ ⸻ –20 [B1] ⸻ [2]

(b) Hence, solve $x^2 - 6x - 11 = 0$.

Give your answers correct to 2 decimal places.

$$x^2 - 6x - 11 = 0$$

$$(x-3)^2 - 20 = 0$$

$$(x-3)^2 = 20$$

$$x - 3 = \pm\sqrt{20} \qquad \text{[M1]}$$

$$x = \sqrt{20} + 3 \qquad \text{or} \qquad x = -\sqrt{20} + 3$$

$$x = 7.47213 \qquad \text{or} \qquad x = -1.47213$$

Answer $x =$ ⸻ 7.47 ⸻ or ⸻ –1.47 ⸻ [2]

[A1 – both answers]

20 A fruit stall sells mangoes at m each and nectarine at n each.

(a) 6 mangoes and 3 nectarines cost $7.50.
Show that $2m+n=2.5$.

Answer

$6m+3n=7.5$ [AG1]
$2m+n=2.5$ [1]

(b) 4 mangoes and 7 nectarines costs $8.50
Form another equation and hence solve the simultaneous equations algebraically.

$$2m+n=2.5 \quad \cdots(1)$$
$$4m+7n=8.5 \quad \cdots(2) \qquad [M1]$$
From (1), $n=2.5-2m \quad \cdots(3)$
subs eqn (3) into eqn (2)
$$4m+7(2.5-2m)=8.5$$
$$4m+17.5-14m=8.5$$
$$10m=9$$
$$m=0.9$$
subs $m=0.9$ into eqn (3)
$$n=2.5-2(0.9)$$
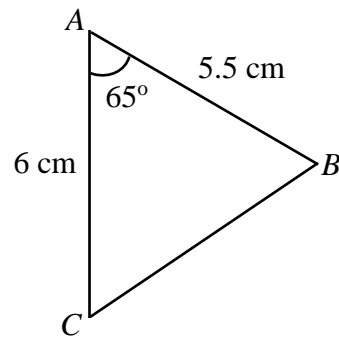$$=0.7$$

Answer $m =$ __0.90__ [A1]
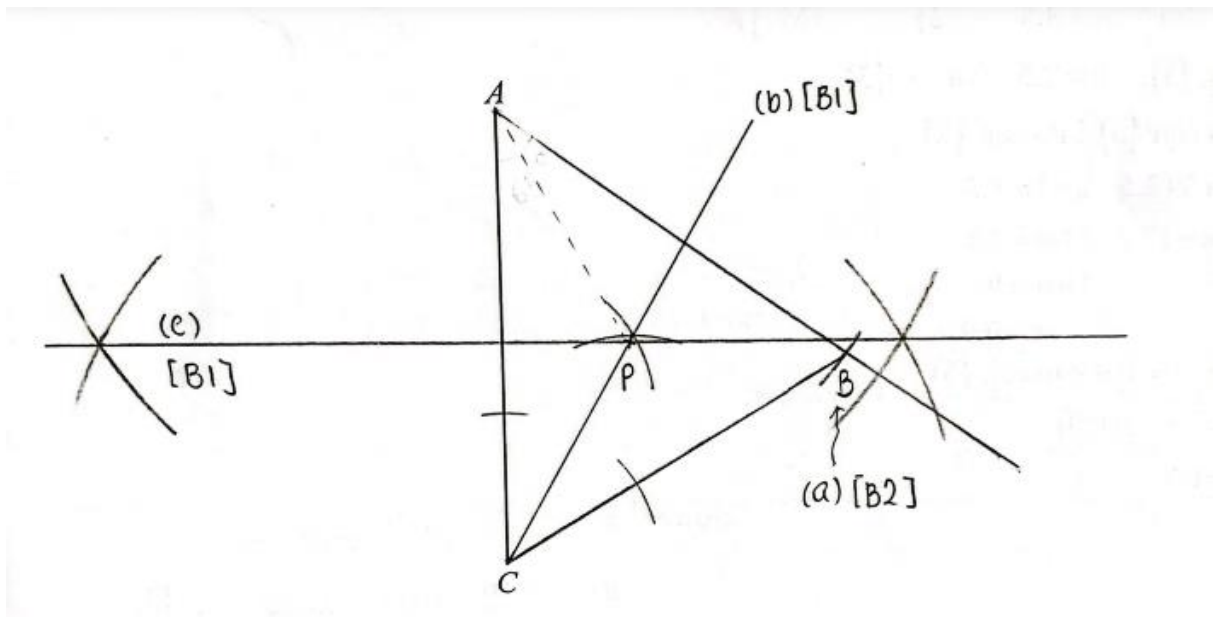
$n =$ __0.70__ [A1]   [3]

21 The diagram shows a sketch of triangle *ABC*.

(a) Construct an accurate full-sized drawing of triangle *ABC*.
The side *AC* has been drawn for you.

[2]

(b) Construct the bisector of angle *BCA*. [1]

(c) Construct the perpendicular bisector of *AC*. [1]

(d) Given that the two bisectors meet at *P*, measure *AP*.

Answer *AP* =   3.4 cm   [B1] .... [1]
(range = ±0.1)

22 The length of the rectangular playground is x metres.
The breadth of the playground is $(x+6)$ metres.
The area of the playground is 40 m^2.

(a) Write down an equation in x to represent this information and show that it reduces to $x^2 + 6x - 40 = 0$.

Answer
$$x(x+6) = 40 \qquad \text{[AG1]}$$
$$x^2 + 6x - 40 = 0 \qquad\qquad\qquad\qquad\qquad\qquad\qquad [1]$$

(b) Solve the equation $x^2 + 6x - 40 = 0$.

$$x^2 + 6x - 40 = 0$$
$$(x-4)(x+10) = 0 \qquad \text{[M1 - factorisation/ quad formula]}$$
$$x = 4 \quad \text{or} \quad x = -10$$

Answer $\underline{x = 4 \ \ \text{or} \ \ x = -10}$ _____ [2]
[A1 – both answers]

(c) Hence, find the perimeter of the playground.
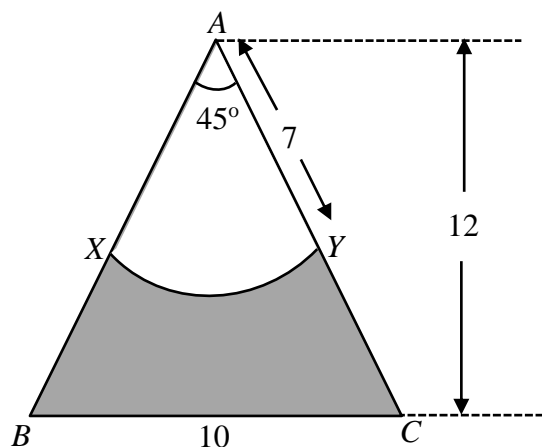
Choose $x = 4$.
perimeter = 4 + 4 + 10 + 10 = 28

Answer $\underline{\text{28 m} \quad \text{[B1]}}$ _____ [1]

(d) Explain why one of the values of x is rejected.

Answer
The positive value of x is chosen as the **length cannot be negative**.   [B1] _____ [1]

23 *AXY* is a sector of a circle, centre *O*, of radius 7 cm and angle *BAC* = 45°.
ABC is an isosceles triangle with a height of 12 cm and *BC* = 10 cm.

Calculate the percentage of the triangle that is shaded.

area of triangle = $\dfrac{1}{2}(10)(12)$ [M1]

$= 60$

area of sector = $\dfrac{45°}{360°} \times \pi(7)^2$ [M1]

$= 19.2422$

shaded area $= 60 - 19.2422$ [M1]

$= 40.7578$

% shaded $= \dfrac{40.7578}{60} \times 100\%$ [M1]

$= 67.929$

Answer   67.9%   [A1] ....   [5]

24 (a) The nth term of a sequence is given by $13 - 2n$.
Write down the first two terms of the sequence.

Answer <u> 11   [B1]   </u> , <u> 9 [B1] </u> [1]

(b) The first four terms of a different sequence are

15, 19, 23, 27.

(i) Find an expression for the nth term of this sequence.

$15 + (n-1)4 = 4n + 11$

Answer <u> $4n + 11$ [B1] </u> [1]

(ii) Find the 15th term.

Answer <u> 71   [B1] </u> [1]

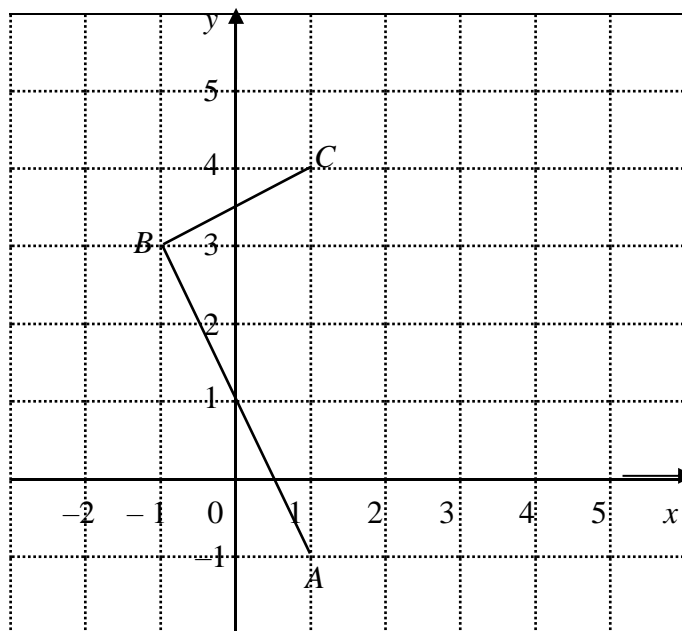**(iii)** The pth term in the sequence is 259.
Find p.

$$4p + 11 = 259 \qquad \text{[M1]}$$
$$4p = 248$$
$$p = 62$$

Answer $p =$ <u> 62 [A1] </u> [2]

25 The points $A(1, -1)$, $B(-1, 3)$, $C(1, 4)$ and D are the vertices of kite $ABCD$.

(a) Find the equation of line BC.

$$m = \frac{4-3}{1-(-1)} = \frac{1}{2} \qquad \text{[M1]}$$

$$\text{sub } C(1, 4) \text{ into } y = \frac{1}{2}x + c$$

$$4 = \frac{1}{2}(1) + c$$

$$c = \frac{7}{2} \qquad \text{[M1]}$$

$$y = \frac{1}{2}x + \frac{7}{2}$$

Answer $\quad y = \dfrac{1}{2}x + \dfrac{7}{2} \quad$ [A1]   [3]

(b) Find the coordinates of the point D.

Answer $\quad D\,(3,\ 3) \quad$ [B1]   [1]

(c) Tim claims that the length of line AB is 7.47 units.
Is he correct? Explain your answer.
Answer

$$\text{length} = \sqrt{(1-(-1))^2 + (-1-3)^2} \qquad \text{[M1]}$$

$$= 4.4721$$

He is incorrect as the length of line AB is 4.47.   [A1 – conclusion]   [2]

END OF MARK SCHEME