Year 4 Mathematics 2
Applications of Integration - Kinematics
Supplementary Worksheet

Name : _____ ( ) Class : _____ Date : _____

1 A particle starts from point *O* and moves in a straight line so that its displacement, *s* cm, from *O*, *t* seconds after leaving *O*, is given by $s = t(t-6)^2$.

Obtain an expression for the velocity of the particle in terms of *t*.
Hence, determine the value of *t* when the particle first comes to instantaneous rest and find the acceleration at this instant. The particle is next at *O* when $t = T$. Find

(i) the value of *T*,

(ii) the distance travelled from $t = 0$ s to $t = T$.

2 The height, *h* m, of a stone *t* seconds after it has been thrown vertically upwards from ground level is given by $h = 24t - 3t^2$. Find

(i) its velocity after 3 seconds,

(ii) the maximum height reached,

(iii) the time of the flight.

3 A particle *P* starts at a point 3m away from *O* and travels in a straight line so that its velocity *v* ms^{-1} is given by $v = 9t - 3t^2$ where *t* is the time in seconds measured from the start of the motion. Calculate

(i) its acceleration when $t = 1$,

(ii) the maximum velocity attained by the particle,

(iii) the distance of *P* from *O* when it comes to instantaneous rest,

(iv) the total distance travelled by *P* in the first 4 seconds.

4 A particle moves in a straight line so that, at time *t* seconds after leaving a fixed point *O*, its velocity , *v* m s^{-1}, is given by $v = \frac{1}{2} - 2\text{e}^{-\frac{1}{2}t}$.

(a) Find

(i) the initial acceleration of the particle,

(ii) the value of *t* when the particle is instantaneously at rest,

(iii) the distance of the particle from *O* when $t = 2$.

(b) **(i)** Sketch the velocity-time curve for $t \geq 0$, indicating the coordinates of the points of intersection with the axes.

(ii) Find the distance travelled during the third second.

5 A particle starts from a point *O* and moves in a straight line with a velocity *v* m/s given by $v = t + \sin 2t$ where *t* seconds is the time after leaving *O*.

(i) Find an expression for the displacement of the particle from *O* in terms of *t*,

(ii) Calculate the distance travelled by the particle when $t = \frac{\pi}{2}$ and its acceleration at this instant.

6 A particle X moves along a horizontal straight line so that its displacement, s m, from a fixed point O, t seconds after motion has begun, is given by $s = 28 + 4t - 5t^2 - t^3$. Obtain expressions, in terms of t, for the velocity and acceleration of X, and state the initial velocity and the initial acceleration of X.

A second particle Y moves along the same horizontal straight line as X, and starts from O at the same instant that X begins to move. The initial velocity of Y is 2 m s^{-1} and its acceleration, a m s^{-2}, t seconds after motion has begun, is given by $a = 2 - 6t$. Find the value of t at the instant when X and Y collide and determine whether or not X and Y are travelling in the same direction at this instant.

(*Pass GCE 'O' Level Examination Additional Mathematics, Shinglee*)

7 A particle travels in a straight line with velocity, v m/s, given by $v = t - \frac{1}{2}t^2$ where t is the time in seconds after passing a fixed point O. Calculate

(i) the distance from O when the acceleration is zero,

(ii) the distance travelled by the particle during the 2nd second,

(iii) the total distance travelled by the particle after four seconds.

8 The velocity, v m/s, of a particle, t seconds after passing a fixed point O, is given by $v = 3t^2 - \frac{48}{t^2}$, where $t \geq 1$. Calculate

(i) the acceleration of the particle when $t = 2$, [2]

(ii) the time when it is momentarily at rest, [1]

(iii) the total distance moved by the particle for $1 \leq t \leq 3$. [5]

[2006 / DHS EOY Y4 P1 / Q17]

9 A particle moves in a straight line so that t seconds after leaving a fixed point O, its velocity, v ms^{-1}, is given by $v = 2\sin t - 1$. Find

(i) the time at which the particle first comes to instantaneous rest, [2]

(ii) the distance travelled by the particle in the first 2 seconds. [4]

[2008 / DHS EOY Y4 P2 / Q13]

Answer

1 $v = 3(t-6)(t-2)$ m/s, $t = 2, -12$ m/s^2 **(i)** 6 **(ii)** 64 m

2(i) 6 m/s **(ii)** 48 m **(iii)** 8 s

3(i) 3 m/s^2 **(ii)** 6.75 m/s **(iii)** 16.5 m **(iv)** 19 m

4(a)(i) 1 m/s^2 **(ii)** 2.77 s **(iii)** 1.53 m **(b)(ii)** 0.0914 m

5(i) $s = t^2 - \frac{1}{2}\cos 2t + \frac{1}{2}$ **(ii)** 2.23 m

6 Initial velocity = 4 m/s; Initial acceleration = –10 m/s; $v_Y = 2t - 3t^2 + 2$; $S_Y = t^2 - t^3 + 2t$

$t = \frac{7}{3}$ for collision to occur. Both X and Y are travelling in the same direction because both velocities are negative at this instant.

7(i) 1/3 m **(ii)** 1/3 m **(iii)** 4 m

8(i) 24 m/s^2 **(ii)** 2 s **(iii)** 28 m

9(i) $\frac{\pi}{6}$ s **(ii)** 1.09 m