



**ST JOSEPH'S INSTITUTION
PRELIMINARY EXAMINATION 2024
(YEAR 4)**

CANDIDATE
NAME

--

CLASS

--	--	--	--	--

INDEX
NUMBER

--	--

ADDITIONAL MATHEMATICS

4049/02

Paper 2

20 August 2024

Candidates answer on the Question Paper.

**2 hours 15 minutes
(0805 - 1020)**

READ THESE INSTRUCTIONS FIRST

Write your class, index number and name on all the work you hand in.

Write in dark blue or black pen in the space provided.

You may use an HB pencil for any diagrams or graphs.

Do not use staples, paper clips, glue or correction fluid.

Answer all questions.

If working is needed for any question it must be shown with the answer.

Omission of essential working will result in loss of marks.

The use of an approved scientific calculator is expected, where appropriate.

If the degree of accuracy is not specified in the question, and if the answer is not exact, give the answer to three significant figures. Give answers in degrees to one decimal place.

For π , use either your calculator value or 3.142, unless the question requires the answer in terms of π .

The number of marks is given in brackets [] at the end of each question or part question.

The total number of marks for this paper is 90.

This document consists of **19** printed pages and **1** blank page.

1. ALGEBRA

Quadratic Equation

For the equation $ax^2 + bx + c = 0$,

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

Binomial expansion

$$(a + b)^n = a^n + \binom{n}{1}a^{n-1}b + \binom{n}{2}a^{n-2}b^2 + \dots + \binom{n}{r}a^{n-r}b^r + \dots + b^n,$$

where n is a positive integer and $\binom{n}{r} = \frac{n!}{r!(n-r)!} = \frac{n(n-1)\dots(n-r+1)}{r!}$

2. TRIGONOMETRY

Identities

$$\sin^2 A + \cos^2 A = 1$$

$$\sec^2 A = 1 + \tan^2 A$$

$$\operatorname{cosec}^2 A = 1 + \cot^2 A$$

$$\sin(A \pm B) = \sin A \cos B \pm \cos A \sin B$$

$$\cos(A \pm B) = \cos A \cos B \mp \sin A \sin B$$

$$\tan(A \pm B) = \frac{\tan A \pm \tan B}{1 \mp \tan A \tan B}$$

$$\sin 2A = 2 \sin A \cos A$$

$$\cos 2A = \cos^2 A - \sin^2 A = 2 \cos^2 A - 1 = 1 - 2 \sin^2 A$$

$$\tan 2A = \frac{2 \tan A}{1 - \tan^2 A}$$

Formulae for $\triangle ABC$

$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$

$$a^2 = b^2 + c^2 - 2bc \cos A$$

$$\Delta = \frac{1}{2}ab \sin C$$

[Turn over

- 1 Solve the inequality $8 \leq x^2 - 2x < 24$ and represent your solution on a number line. [6]



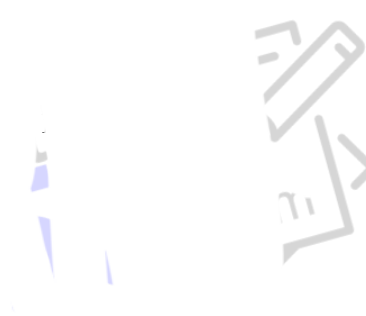
[Turn over

2 The expression $3 \cos 2\theta + 6 \sin 2\theta$ is defined for $0 < \theta < \pi$.

(a) Using $R \cos(2\theta - \alpha)$, where $R > 0$ and $0 < \alpha < \frac{\pi}{2}$, solve the equation

$$3 \cos 2\theta + 6 \sin 2\theta = 5.$$

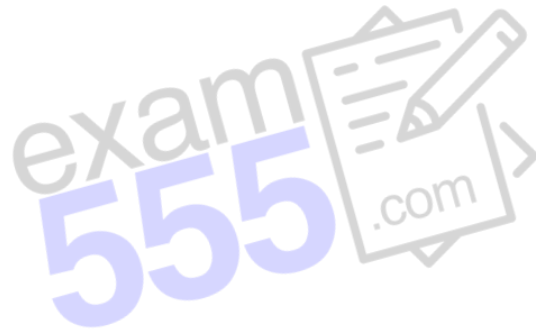
[6]



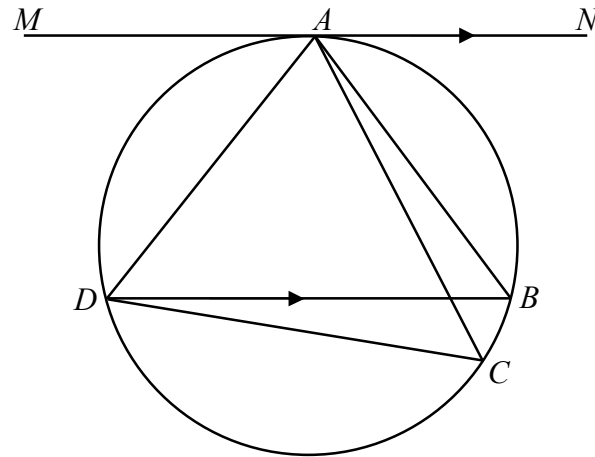
[Turn over

(b) Explain why P cannot be less than -7 .

[2]



[Turn over



The diagram shows a circle passing through the points A , B , C and D . The straight line MAN is a tangent to the circle. DB is parallel to MN and AD bisects angle MAC .

- (a) Show that triangle ACD is an isosceles triangle. [2]

- (b) By identifying another isosceles triangle, show that $(AD)^2 = AB \times CD$. [3]

[Turn over

- 4 (a) Given that the coefficient of x^2 is 100 in the expansion of $(2+mx)^2\left(1-\frac{3x}{2}\right)^6$, find the values of m . [4]

- (b) Determine if the term independent of x exists in the expansion of $\left(x^2 - \frac{1}{2x}\right)^{100}$. [3]

[Turn over

- 5 (a) Solve the simultaneous equations

$$\begin{aligned}x^2 + (y+1)^2 &= 4, \\ 2x + 3y &= 1.\end{aligned}$$

[5]

[Turn over

- (b) Find the set of values of m for which the line $y = m(x-1)$ is a tangent to the curve $y = \frac{5x}{4} + \frac{1}{x}$. [4]



[Turn over

- 6 The volume V of the solid is given by the equation $V = 12 \ln\left(\frac{1}{2}x + 3\right) + 2$,
where x is the length of one of the sides of the solid, in cm.
 x increases from an initial value of 12 cm at a constant rate of 0.008 cm per second.

(a) Find the rate of increase of V after 30 seconds. [4]



(b) Explain why the volume of the solid never reach a maximum value. [2]

[Turn over

- 7 $f(x)$ is such that $f''(x) = -12 \cos 2x$. It is given that $f\left(\frac{\pi}{2}\right) = 4$ and $f'\left(\frac{\pi}{2}\right) = 0$.

(a) Find an expression for $f(x)$. [6]



(b) Explain the significance of the point on the curve where $x = 4\pi$. [2]

[Turn over]

- 8 In an experiment, it was found that the variables x and y are related by the equation $y = \frac{k}{(x+2)^h}$, where h and k are constants. The table below shows some experimental values of x and y . It was believed that an error was made in recording one of the values of y .

x	0.72	5.39	18.09	52.6	146.41
y	32.06	7.91	4.95	0.52	0.14

- (a) On the grid below, plot $\ln y$ against $\ln (x + 2)$ and draw a straight line graph.

[2]

[Turn over]

- (b) Determine which value of y is the incorrect reading. [1]



Use your graph to estimate

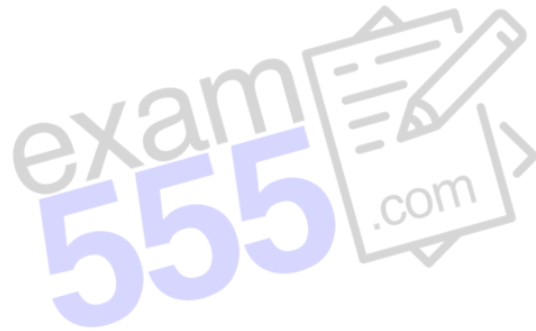
- (c) the value of h and of k . [4]



- (d) the value of y when $x = 31$. [2]

[Turn over

- 9 (a) Show that $\frac{d}{dx}(2x^2\sqrt{x+3}) = \frac{x(5x+12)}{\sqrt{x+3}}$. [3]



[Turn over

(b) Hence find $\int_{-2}^0 \frac{5x^2 + 12x - 1}{\sqrt{x+3}} dx$.

Leave your answer in the form $a + b\sqrt{3}$, where a and b are integers. [6]



[Turn over

- 10 (a) Solve the equation $2e^{2x+3} - e^{x+3} = 3e^3$. [4]

- (b) Sketch the graph of $y = \ln x$. By drawing a suitable straight line on the same axes, find the number of solutions to the equation $x - e^{3-x} = 0$. [3]

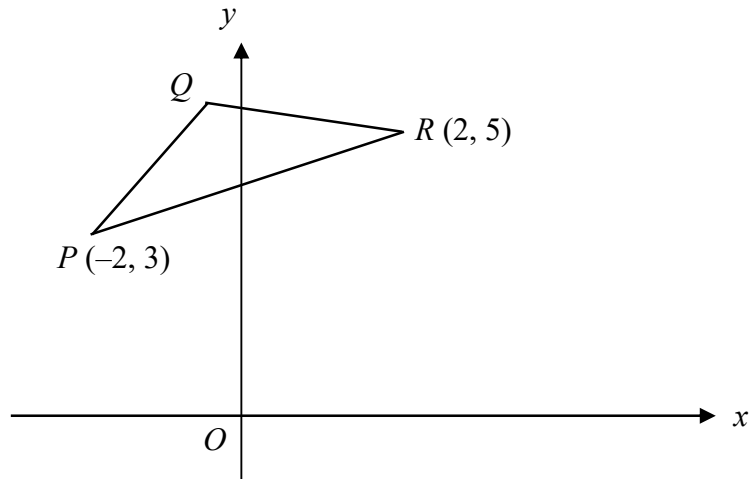
[Turn over]

- (c) An investor invests an amount of money such that its value is given by $M = 40000(1.06)^{kt}$, where M is the value after t years and k is a constant. Its value after 4 years is expected to be \$63700. Calculate the least amount of time, correct to the nearest month, when its expected value will be more than \$100000. [5]



[Turn over

11



The diagram shows a triangle PQR in which the point P is $(-2, 3)$ and the point R is $(2, 5)$. The point Q lies on the perpendicular bisector of PR and the equation of QR is $3y = 17 - x$.

(a) Find the coordinates of Q .

[6]

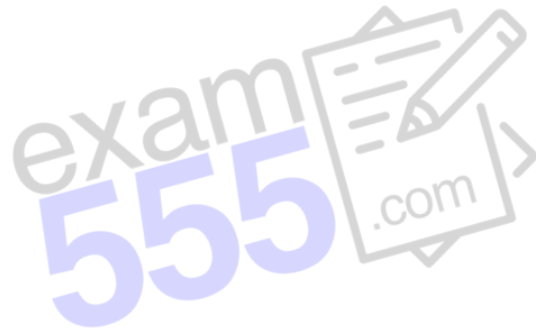
[Turn over

- (b) Given that the point S is $(2, 0)$, explain why $PQRS$ is a kite. [3]

- (c) Calculate the area of $PQRS$. [2]

[Turn over

BLANK PAGE



[Turn over



**ST JOSEPH'S INSTITUTION
PRELIMINARY EXAMINATION 2024
(YEAR 4)**

CANDIDATE
NAME

--

CLASS

--	--	--	--	--

INDEX
NUMBER

--	--

ADDITIONAL MATHEMATICS

4049/02

Paper 2

20 August 2024

Candidates answer on the Question Paper.

**2 hours 15 minutes
(0805 - 1020)**

READ THESE INSTRUCTIONS FIRST

Write your class, index number and name on all the work you hand in.

Write in dark blue or black pen in the space provided.

You may use an HB pencil for any diagrams or graphs.

Do not use staples, paper clips, glue or correction fluid.

Answer all questions.

If working is needed for any question it must be shown with the answer.

Omission of essential working will result in loss of marks.

The use of an approved scientific calculator is expected, where appropriate.

If the degree of accuracy is not specified in the question, and if the answer is not exact, give the answer to three significant figures. Give answers in degrees to one decimal place.

For π , use either your calculator value or 3.142, unless the question requires the answer in terms of π .

The number of marks is given in brackets [] at the end of each question or part question.

The total number of marks for this paper is 90.

This document consists of **19** printed pages and **1** blank page.

1. ALGEBRA

Quadratic Equation

For the equation $ax^2 + bx + c = 0$,

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

Binomial expansion

$$(a + b)^n = a^n + \binom{n}{1}a^{n-1}b + \binom{n}{2}a^{n-2}b^2 + \dots + \binom{n}{r}a^{n-r}b^r + \dots + b^n,$$

where n is a positive integer and $\binom{n}{r} = \frac{n!}{r!(n-r)!} = \frac{n(n-1)\dots(n-r+1)}{r!}$

2. TRIGONOMETRY

Identities

$$\sin^2 A + \cos^2 A = 1$$

$$\sec^2 A = 1 + \tan^2 A$$

$$\operatorname{cosec}^2 A = 1 + \cot^2 A$$

$$\sin(A \pm B) = \sin A \cos B \pm \cos A \sin B$$

$$\cos(A \pm B) = \cos A \cos B \mp \sin A \sin B$$

$$\tan(A \pm B) = \frac{\tan A \pm \tan B}{1 \mp \tan A \tan B}$$

$$\sin 2A = 2 \sin A \cos A$$

$$\cos 2A = \cos^2 A - \sin^2 A = 2 \cos^2 A - 1 = 1 - 2 \sin^2 A$$

$$\tan 2A = \frac{2 \tan A}{1 - \tan^2 A}$$

Formulae for $\triangle ABC$


$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$

$$a^2 = b^2 + c^2 - 2bc \cos A$$

$$\Delta = \frac{1}{2}ab \sin C$$

[Turn over

- 1 Solve the inequality $8 \leq x^2 - 2x < 24$ and represent your solution on a number line. [6]

1	$8 \leq x^2 - 2x < 24$ $8 \leq x^2 - 2x$ $x^2 - 2x - 8 \geq 0$ $(x - 4)(x + 2) \geq 0$ $\therefore x \leq -2 \text{ or } x \geq 4$ $x^2 - 2x < 24$ $x^2 - 2x - 24 < 0$ $(x - 6)(x + 4) < 0$ $\therefore -4 < x < 6$ <p>Combining both inequalities: $-4 < x \leq -2 \text{ or } 4 \leq x < 6$</p> 
---	--

[Turn over

2 The expression $3 \cos 2\theta + 6 \sin 2\theta$ is defined for $0 < \theta < \pi$.

(a) Using $R \cos(2\theta - \alpha)$, where $R > 0$ and $0 < \alpha < \frac{\pi}{2}$, solve the equation

$$3 \cos 2\theta + 6 \sin 2\theta = 5.$$

[6]

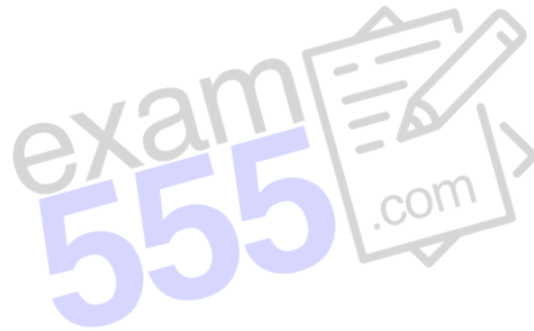
2(a)	$R \cos(2\theta - \alpha) = 3 \cos 2\theta + 6 \sin 2\theta$ $= R \cos 2\theta \cos \alpha + R \sin 2\theta \sin \alpha$ $R \cos \alpha = 3$ $R \sin \alpha = 6$ $R^2 = 3^2 + 6^2$ $R = \sqrt{45}$ $= 3\sqrt{5} \quad \text{or} \quad 6.7082 \quad (6.71)$ $\alpha = \tan^{-1}\left(\frac{6}{3}\right)$ $= 1.1071 \quad (1.11)$ $\underline{3\sqrt{5} \cos(2\theta - 1.11) \quad \text{or} \quad 6.71 \cos(2\theta - 1.11)}$ $3 \cos 2\theta + 6 \sin 2\theta = 5$ $3\sqrt{5} \cos(2\theta - 1.11) = 5$ $\cos(2\theta - 1.11) = \frac{5}{3\sqrt{5}}$ $\alpha = \cos^{-1} \frac{5}{3\sqrt{5}}$ $2\theta - 1.1071 = -0.72972, \quad 0.72972$ $\underline{\theta = 0.189, \quad 0.918}$
------	--

[Turn over

(b) Explain why P cannot be less than -7 .

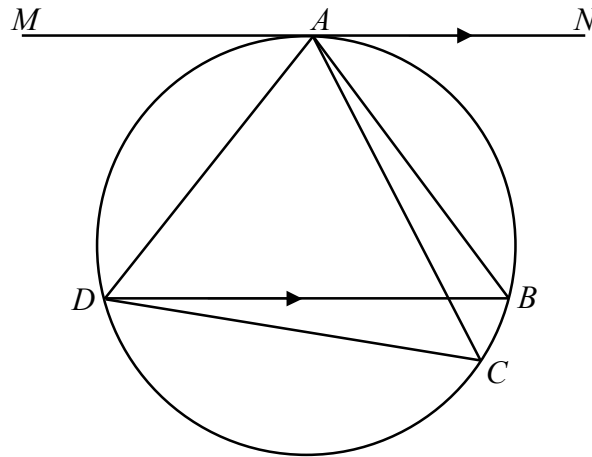
[2]

2(b)	<p>Minimum value of $P = 3\sqrt{5}(-1)$</p> $\approx \underline{-6.71} > -7$ <p>Since the minimum value is greater than -7, $3 \cos 2\theta + 6 \sin 2\theta$ cannot be less than -7.</p>
-------------	---



[Turn over

3



The diagram shows a circle passing through the points A , B , C and D . The straight line MAN is a tangent to the circle. DB is parallel to MN and AD bisects angle MAC .

(a) Show that triangle ACD is an isosceles triangle. [2]

3(a)	<p>Let x be $\angle MAD$ $\angle ACD = x$ (alt. seg theorem) $\angle MAD = \angle CAD = x$ (AD bisects $\angle MAC$) Since $\angle ACD = \angle CAD = x$, <u>triangle ACD is an isosceles triangle</u>, $DA = DC$.</p>
-------------	--

(b) By identifying another isosceles triangle, show that $(AD)^2 = AB \times CD$. [3]

3(b)	<p><u>Triangle DBA</u> with $AD = AB$ $\frac{DA}{DC} = \frac{AD}{AB}$ $DA \times AB = AD \times DC$ since $AD = AB$, <u>$DA \times AD = AB \times DC$</u> $(AD)^2 = AB \times CD$ (shown)</p>
-------------	--

[Turn over

- 4 (a) Given that the coefficient of x^2 is 100 in the expansion of

$$(2+mx)^2 \left(1-\frac{3x}{2}\right)^6, \text{ find the values of } m.$$

[4]

4(a)	$(2+mx)^2 \left(1-\frac{3x}{2}\right)^6 = (4+4mx+m^2x^2) \left[1+6\left(-\frac{3x}{2}\right)+15\left(-\frac{3x}{2}\right)^2+\dots\right]$ $= 4(15)\left(-\frac{3x}{2}\right)^2 + 4m(6)\left(-\frac{3x}{2}\right) + m^2x^2 + \dots$ $4(15)\left(-\frac{3}{2}\right)^2 + 4(6)\left(-\frac{3}{2}\right) + m^2 = 100$ $135 - 36m + m^2 = 100$ $m^2 - 36m + 35 = 0$ $(m-35)(m-1) = 0$ $\underline{m = 1 \text{ or } 35}$
------	---

- (b) Determine if the term independent of x exists in the expansion of $\left(x^2 - \frac{1}{2x}\right)^{100}$.

[3]

4(b)	<p>General Term = ${}^{100}C_r (x^2)^{100-r} \left(-\frac{1}{2x}\right)^r$</p> $= {}^{100}C_r \left(-\frac{1}{2}\right)^r (x^2)^{100-r} (x)^{-r}$ $= {}^{100}C_r \left(-\frac{1}{2}\right)^r x^{200-2r-r}$ $200 - 3r = 0$ $r = \frac{200}{3} = 66\frac{2}{3}$ <p>Since <u>r is not a whole number</u>, the term independent of x does not exist in the expansion.</p>
------	---

[Turn over]

5 (a) Solve the simultaneous equations

$$\begin{aligned}x^2 + (y+1)^2 &= 4, \\ 2x + 3y &= 1.\end{aligned}$$

[5]

5(a)

$$x^2 + (y+1)^2 = 4 \quad \text{--- (1)}$$

$$2x + 3y = 1 \quad \text{--- (2)}$$

From (2), $x = \frac{1-3y}{2} \quad \text{--- (2*)}$

Sub (2*) into (1)

$$\left(\frac{1-3y}{2}\right)^2 + (y+1)^2 = 4$$

$$\frac{1-6y+9y^2}{4} + y^2 + 2y + 1 = 4$$

$$1-6y+9y^2+4y^2+8y+4=16$$

$$13y^2+2y-11=0$$

$$(13y-11)(y+1)=0$$

$$y = \frac{11}{13}$$

or

$$y = -1$$

Sub $y = \frac{11}{13}$ into (2*)

$$\begin{aligned}x &= \frac{1-3\left(\frac{11}{13}\right)}{2} \\ &= \underline{\underline{-\frac{10}{13}}}\end{aligned}$$

Sub $y = -1$ into (2*)

$$\begin{aligned}x &= \frac{1-3(-1)}{2} \\ &= \underline{\underline{2}}\end{aligned}$$

[Turn over]

- (b) Find the set of values of m for which the line $y = m(x-1)$ is a tangent to the curve $y = \frac{5x}{4} + \frac{1}{x}$. [4]

5(b)	$y = m(x-1) \quad \text{--- (1)}$ $y = \frac{5x}{4} + \frac{1}{x} \quad \text{--- (2)}$ <p>Sub (1) into (2)</p> $\frac{5x}{4} + \frac{1}{x} = mx - m$ $5x^2 + 4 = 4mx^2 - 4mx$ $(5-4m)x^2 + 4mx + 4 = 0$ <p>Line touches curve, $\text{discr} = 0$</p> $(4m)^2 - 4(5-4m)(4) = 0$ $16m^2 + 64m - 80 = 0$ $m^2 + 4m - 5 = 0$ $(m+5)(m-1) = 0$ $\underline{m = -5 \quad \text{or} \quad 1}$
-------------	--

[Turn over

- 6 The volume V of the solid is given by the equation $V = 12 \ln\left(\frac{1}{2}x + 3\right) + 2$,
 where x is the length of one of the sides of the solid, in cm.
 x increases from an initial value of 12 cm at a constant rate of 0.008 cm per second.

(a) Find the rate of increase of V after 30 seconds. [4]

6(a)	$V = 12 \ln\left(\frac{1}{2}x + 3\right) + 2$ $\frac{dV}{dx} = \frac{6}{\frac{1}{2}x + 3}$ $= \frac{12}{x + 6}$ <p>After 30s,</p> $x = 12 + 30(0.008)$ $x = 12.24 \text{ cm}$ $\frac{dV}{dt} = \frac{dV}{dx} \times \frac{dx}{dt}$ $\frac{dV}{dt} = \frac{12}{12.24 + 6} \times 0.008$ $\frac{dV}{dt} = \frac{1}{190} \text{ cm}^3/\text{s}$
-------------	--

(b) Explain why the volume of the solid never reach a maximum value. [2]

6(b)	<p>For $x \in \mathbb{R}$, $x > 0$:</p> $\frac{12}{x + 6} \neq 0$ $\frac{dV}{dx} \neq 0$ <p>Since $\frac{dV}{dx} \neq 0$, there are no stationary points.</p> <p>\therefore the volume of the solid will never reach a maximum value.</p>
-------------	---

[Turn over

- 7 $f(x)$ is such that $f''(x) = -12 \cos 2x$. It is given that $f\left(\frac{\pi}{2}\right) = 4$ and $f'\left(\frac{\pi}{2}\right) = 0$.

(a) Find an expression for $f(x)$.

[6]

7(a)	$f''(x) = -12 \cos 2x$ $f'(x) = \frac{-12 \sin 2x}{2} + c_1$ $f'(x) = -6 \sin 2x + c_1$ <p>Sub $f'\left(\frac{\pi}{2}\right) = 0$:</p> $0 = -6 \sin 2\left(\frac{\pi}{2}\right) + c_1$ $c_1 = 0$ $f'(x) = -6 \sin 2x$ $f(x) = 3 \cos 2x + c_2$ <p>Sub. $x = \frac{\pi}{2}, y = 4$:</p> $4 = 3 \cos \pi + c_2$ $c_2 = 7$ $\therefore f(x) = 3 \cos 2x + 7$
------	---

(b) Explain the significance of the point on the curve where $x = 4\pi$.

[2]

7(b)	$y = 3 \cos 8\pi + 7$ $= 10$ <p>As $y = 10$ is the maximum value of y, the point on the curve where $x = 4\pi$ is a maximum point.</p> <p>OR</p> <p>Sub $x = 4\pi$ into $f''(x) = -12 \cos 2x < 0$ and conclude it is a maximum point.</p> <p>[Accept stationary point (TP) with working]</p>
------	--

[Turn over]

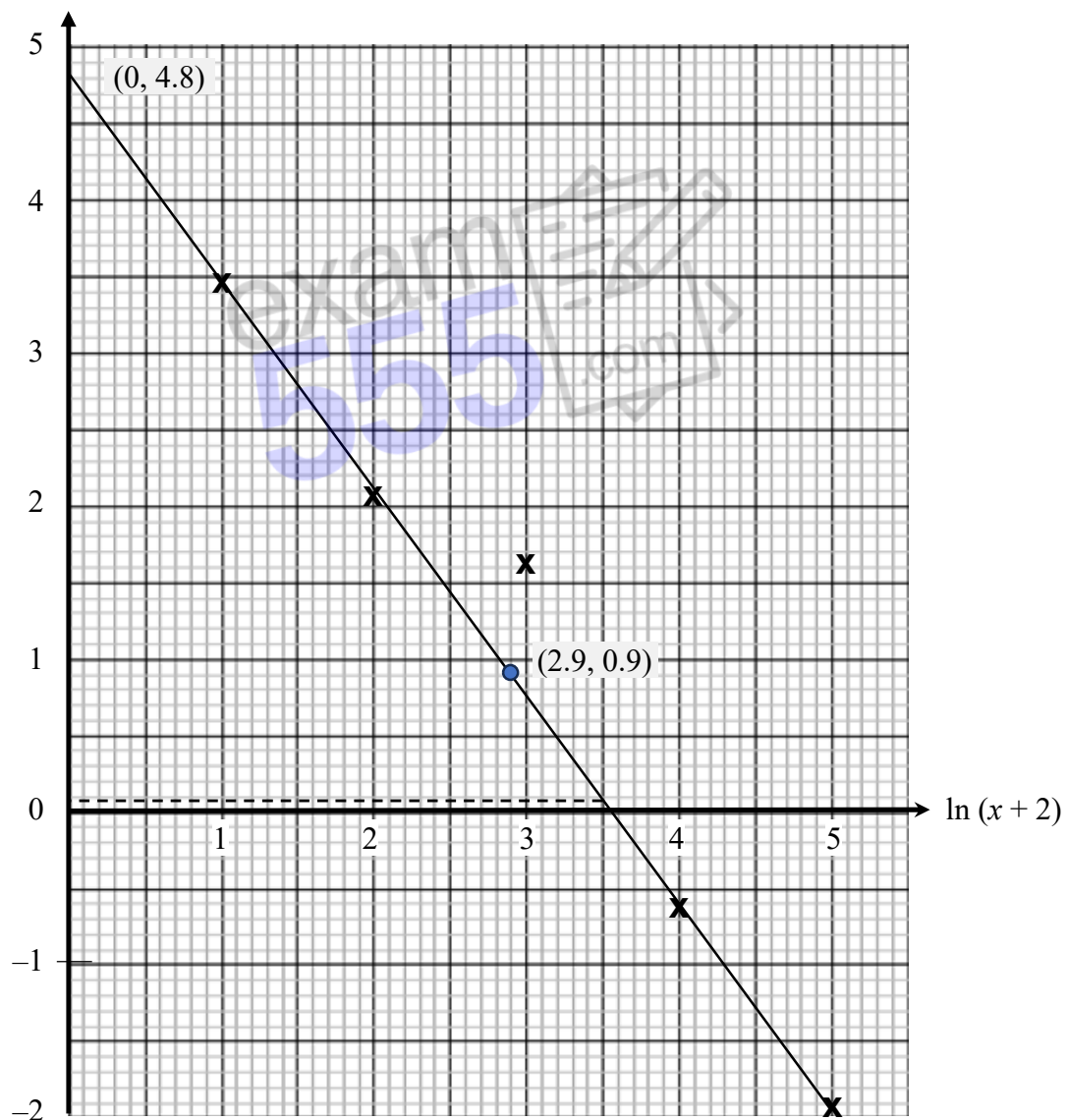
- 8 In an experiment, it was found that the variables x and y are related by the equation $y = \frac{k}{(x+2)^h}$, where h and k are constants. The table below shows some experimental values of x and y . It was believed that an error was made in recording one of the values of y .

x	0.72	5.39	18.09	52.6	146.41
y	32.06	7.91	4.95	0.52	0.14

- (a) On the grid below, plot $\ln y$ against $\ln(x+2)$ and draw a straight line graph.

[2]

$\ln(x+2)$	1.00	2.00	3.00	4.00	5.00
$\ln y$	3.47	2.07	1.60	-0.65	-1.97



[Turn over]

(b) Determine which value of y is the incorrect reading.

[1]

8(b)	$y = 4.95$
-------------	------------

Use your graph to estimate

(c) the value of h and of k .

[4]

8(c)	$y = \frac{k}{(x+2)^h}$ $\ln y = \ln \left[\frac{k}{(x+2)^h} \right]$ $\ln y = \ln k - \ln(x+2)^h$ $\ln y = -h \ln(x+2) + \ln k$ <p>From the graph, $\ln k = 4.8$ $k = e^{4.8}$ $k = 122$ (3 s.f.)</p> $-h = \frac{4.8 - 0.9}{0 - 2.9}$ $h = 1.34$ (3 s.f.)
-------------	--

(d) the value of y when $x = 31$.

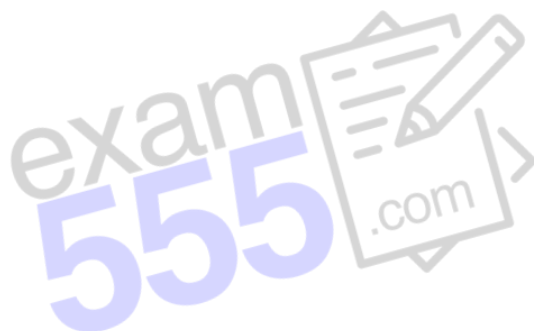
[2]

8(d)	$x = 31, \ln(x+2) = 3.50$ (2 d.p.) From the graph, $\ln y = 0.05$ $y = 1.05$ (3 s.f.)
-------------	--

[Turn over

- 9 (a) Show that $\frac{d}{dx}(2x^2\sqrt{x+3}) = \frac{x(5x+12)}{\sqrt{x+3}}$. [3]

9(a)	$\begin{aligned} & \frac{d}{dx}(2x^2\sqrt{x+3}) \\ &= 2x^2 \left[\frac{1}{2}(x+3)^{-\frac{1}{2}} \right] + (\sqrt{x+3})(4x) \\ &= \frac{x^2}{\sqrt{x+3}} + 4x\sqrt{x+3} \\ &= \frac{x^2 + 4x(x+3)}{\sqrt{x+3}} \\ &= \frac{5x^2 + 12x}{\sqrt{x+3}} \\ &= \frac{x(5x+12)}{\sqrt{x+3}} \text{ (shown)} \end{aligned}$
-------------	--



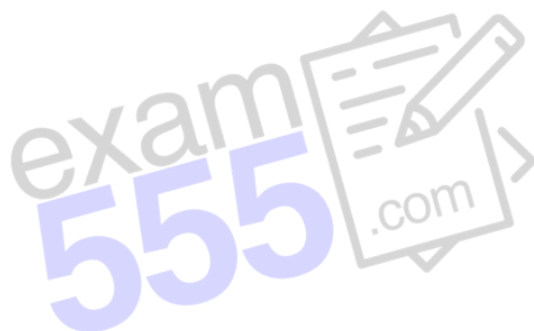
[Turn over

(b) Hence find $\int_{-2}^0 \frac{5x^2 + 12x - 1}{\sqrt{x+3}} dx$.

Leave your answer in the form $a + b\sqrt{3}$, where a and b are integers. [6]

9(b)

$$\begin{aligned}
 & \int_{-2}^0 \frac{5x^2 + 12x - 1}{\sqrt{x+3}} dx \\
 &= \int_{-2}^0 \frac{x(5x+12)}{\sqrt{x+3}} dx - \int_{-2}^0 \frac{1}{\sqrt{x+3}} dx \\
 &= \left[2x^2\sqrt{x+3} \right]_{-2}^0 - \left[(\sqrt{x+3}) \div \frac{1}{2} \right]_{-2}^0 \quad (\text{from (a)}) \\
 &= \left[2x^2\sqrt{x+3} \right]_{-2}^0 - \left[2\sqrt{x+3} \right]_{-2}^0 \\
 &= \left[0 - 2(-2)^2\sqrt{-2+3} \right] - \left[2\sqrt{3} - 2\sqrt{-2+3} \right] \\
 &= -8 - (2\sqrt{3} - 2) \\
 &= -6 - 2\sqrt{3}
 \end{aligned}$$



[Turn over

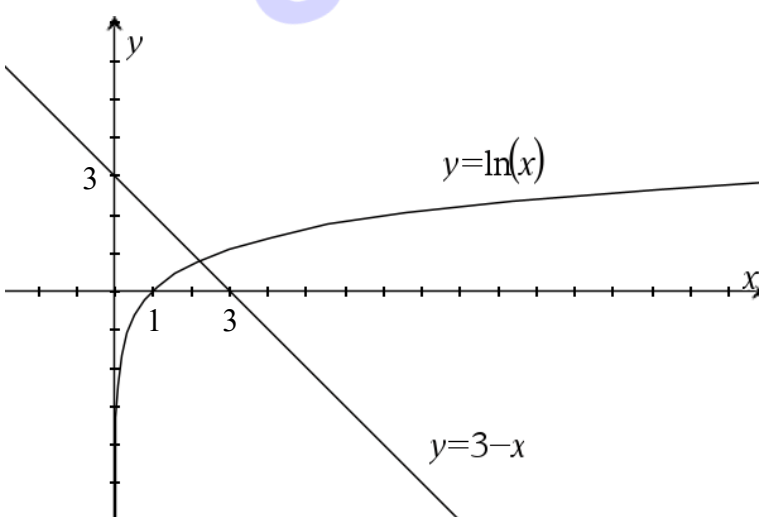
10 (a) Solve the equation $2e^{2x+3} - e^{x+3} = 3e^3$.

[4]

10(a)	$2e^{2x+3} - e^{x+3} = 3e^3$ $2e^{2x}(e^3) - e^x(e^3) = 3e^3$ $2e^{2x} - e^x = 3$ $2(e^x)^2 - e^x - 3 = 0$ $(2e^x - 3)(e^x + 1) = 0$ $e^x = \frac{3}{2} \text{ or } e^x = -1 \text{ (rejected as } e^x > 0)$ $\therefore e^x = \frac{3}{2}$ $x = \ln 1.5 \approx 0.405$
-------	--

(b) Sketch the graph of $y = \ln x$. By drawing a suitable straight line on the same axes, find the number of solutions to the equation $x - e^{3-x} = 0$.

[3]

10(b)	$x - e^{3-x} = 0$ $x = e^{3-x}$ $\ln x = 3 - x$ $y = 3 - x$  $\therefore x - e^{3-x} = 0 \text{ has 1 solution}$
-------	--

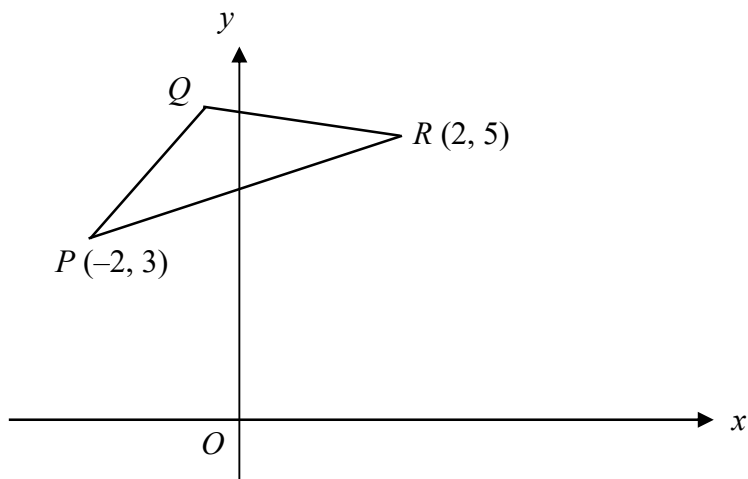
[Turn over]

- (c) An investor invests an amount of money such that its value is given by $M = 40000(1.06)^{kt}$, where M is the value after t years and k is a constant. Its value after 4 years is expected to be \$63700. Calculate the least amount of time, correct to the nearest month, when its expected value will be more than \$100000. [5]

10(c)	$M = 40000(1.06)^{kt}$ <p>Sub. $t = 4$ and $M = 63700$:</p> $63700 = 40000(1.06)^{4k}$ $1.5925 = (1.06)^{4k}$ $4k \lg 1.06 = \lg 1.5925$ $k = \frac{\lg 1.5925}{4 \lg 1.06}$ $k = 1.9964 \text{ (5 s.f.)}$ $k = 2.00 \text{ (3 s.f.)}$ $100000 = 40000(1.06)^{1.9964t}$ $(1.06)^{1.9964t} = 2.5$ $1.9964t \lg 1.06 = \lg 2.5$ $t = 7.88 \text{ years (3 s.f.)}$ $\therefore \text{least amount of time is 7 years and 11 months}$
--------------	--

[Turn over]

11



The diagram shows a triangle PQR in which the point P is $(-2, 3)$ and the point R is $(2, 5)$. The point Q lies on the perpendicular bisector of PR and the equation of QR is $3y = 17 - x$.

(a) Find the coordinates of Q .

[6]

11(a)

$$\text{Midpoint of } PR = \left(\frac{-2+2}{2}, \frac{3+5}{2} \right)$$

$$= (0, 4)$$

$$\text{Gradient of } PR = \frac{5-3}{2-(-2)}$$

$$= \frac{1}{2}$$

$$\text{Gradient of perpendicular bisector} = -1 \div \frac{1}{2} = -2$$

Equation of perpendicular bisector:

$$y - 4 = -2(x - 0)$$

$$y = -2x + 4 \text{ ---(1)}$$

$$3y = 17 - x \text{ ---(2)}$$

Sub. (1) into (2):

$$3(-2x + 4) = 17 - x$$

$$x = -1$$

$$\therefore y = -2(-1) + 4 = 6$$

$$\therefore Q = (-1, 6)$$

[Turn over]

(b) Given that the point S is $(2, 0)$, explain why $PQRS$ is a kite.

[3]

11(b)	<p>Sub. $x = 2$:</p> $y = -2(2) + 4$ $y = 0$ <p>$\therefore S$ lies on the perpendicular bisector</p> $PS = \sqrt{(-2 - 2)^2 + (3 - 0)^2}$ $PS = 5 \text{ units}$ $PQ = \sqrt{(-2 - (-1))^2 + (3 - 6)^2}$ $PQ = \sqrt{10} \text{ units}$ <p>As Q and S lie on the perpendicular bisector of PR, $PQ = QR$ and $PS = SR$. However as $PS \neq PQ$, we can conclude that it is a kite (not a rhombus).</p>
--------------	---

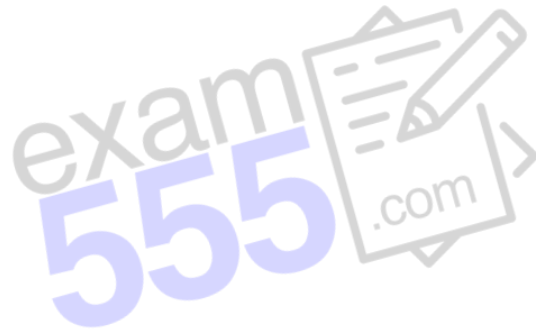
(c) Calculate the area of $PQRS$.

[2]

11(c)	<p>Area of $PQRS$</p> $= \frac{1}{2} \begin{vmatrix} 2 & 2 & -1 & -2 & 2 \\ 0 & 5 & 6 & 3 & 0 \end{vmatrix}$ $= \frac{1}{2} [(10 + 12 - 3 + 0) - (0 - 5 - 12 + 6)]$ $= 15 \text{ sq units}$
--------------	--

[Turn over]

BLANK PAGE



[Turn over