Chapter 6

Circular Motion

Image credits to https://en.wikipedia.org/wiki/Singapore_Flyer#/media/File:Singapore_Flyer.JPG

The Singapore Flyer is currently one of the tallest Ferris wheel in the world. Described by its operators as an observation wheel, it reaches 42 stories high. At a total height of 165 m (541 feet) and a diameter of 150 m (492.1 feet), it was the tallest Ferris wheel at its inception, 5 m (16 feet) taller than the previous record holder, the Star of Nanchang. The Flyer was itself surpassed in 2014 by the High Roller in Las Vegas, which stands at 167.6 m (550 feet) with a diameter of 158.5 m (520 feet). The Ain Dubai in Dubai which was launched in 2021 stood at a height of 250 m (820 feet).

TOPIC 6: CIRCULAR MOTION

Circular Motion	**Learning Outcomes** Students should be able to:
Kinematics of uniform circular motion	(a) express angular displacement in radians. (b) show an understanding of and use the concept of angular velocity to solve problems. (c) recall and use $v = r\omega$ to solve problems. (d) derive, from the definitions of velocity and acceleration, equations which represent uniformly accelerated motion in a straight line.
Centripetal acceleration	(e) describe qualitatively motion in a curved path due to a perpendicular force, and understand the centripetal acceleration in the case of uniform motion in a circle. (f) recall and use centripetal acceleration $a = r\omega^2$, $a = v^2/r$ to solve problems.
Centripetal force	(g) recall and use centripetal force $F = mr\omega^2$, $F = mv^2/r$ to solve problems.

Playlist of lecture examples:
https://youtube.com/playlist?list=PL_b5cjrUKDlaAYoOvqyWUXVITtqdkKOSO

Further Readings / References

1) University Physics with Modern Physics, 14th Ed., *Young & Freedman, Chapter 9, Pg 298.*
2) Advanced Level Physics, 6th Ed., *Nelkon & Parker, Chapter 2, Pg 48.*
3) College Physics, 8th Ed., *Young & Geller, Chapter 6, Pg 161.*
4) Physics for Scientists and Engineers with Modern Physics, 6th Ed., *Serway Jewett, Chapter 6, Pg 150.*

Contents

6.1 Introduction

Circular motion is a type of motion we often encounter in our everyday lives, - the motion of a car and its passengers when navigating a bend, the motion of the capsules of a Ferris wheel when in rotation, children on a spinning merry-go-round, and clothes being spun in a spin dryer.

On a larger scale, in astronomy, we know that the moon circles the Earth, which circles the Sun, which circles the centre of the Milky Way.

On the atomic scale, for early models of the hydrogen atom, we picture an electron orbiting around the nucleus at the centre of the atom (a single proton in hydrogen's case).

Have you ever wondered?

- Why must an airplane tilt when executing a turn?
- Why the reading of your weight would be different at the bottom of a Ferris wheel as compared to the top?
- Why do people in a roller coaster not fall out at the top of the loop? (No, it is not because of the belts!)

The exploration of the fascinating world of circular motion begins here.

A Short Revision on basic ideas of Newton's Laws of Motion

Newton's 1st Law:

Describes the motion of any object not subjected to a net external force, specifically that

- an object at rest remains at rest, or
- an object continues with constant speed in a straight line.

In the presence of a net external force, a constant mass experiences an acceleration governed by *Newton's 2nd Law*:

$$\vec{a} = \frac{\vec{F}_{net}}{m}$$

Note:

- Forces which are parallel to the motion of an object result in its motion either speeding up or slowing down, with the direction of motion remaining along the original axis.
- Forces which are perpendicular to the motion of the object cannot affect its speed, but instead will change its direction of motion.
- Forces which are neither wholly parallel nor perpendicular to the direction of motion will affect both the speed and direction of the object upon which the force acts. (Consider projectile motion.)

In this chapter you will learn that objects may move in circular paths when they experience a force or forces acting perpendicularly to their motion.

6.2 Kinematics of Circular Motion

When an extended object such as the wheel of a bicycle is rotating, different parts of the wheel will travel different distances in the same time. The outer parts of the wheel will travel larger distances and hence will have higher speeds than the inner parts of the wheel. However, all parts of the wheel will pass through the same angle in a given time. Hence the speed of rotation of a wheel is better expressed as an angular speed, as this is the same for all parts of the wheel.

6.2.1 How angle is measured

The S.I. unit for angles is not the degree but the radian. An angle measured in radians is actually the ratio of two lengths: arc length and radius. The *arc length s* is the distance travelled along the circular path, and the angle θ is said to *subtend* the arc length at the circle centre (Figure 1).

Hence, note that θ is actually a dimensionless quantity since it is a ratio of 2 lengths.

$$\theta \text{ (in rad)} = \frac{\text{arc length}}{\text{radius}} = \frac{s}{r}$$

$$s = r\,\theta$$

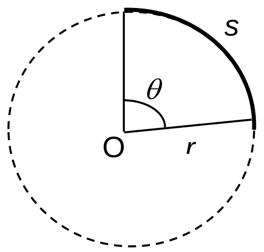


Figure 1

One radian is the angle subtended at the centre of the circle by an arc equal in length to the radius of the circle.

If the circumference of a circle $s = 2\pi r$, $\theta = s/r = 2\pi$ rad .
Since for a complete circle, the angle subtended is 360°,

$$360° \equiv 2\pi \text{ rad}$$

We can deduce that $1 \text{ radian} = \left(\frac{360}{2\pi}\right)° = 57.3°$

Conversion between degrees and radians:

$$180° \equiv \pi \text{ rad}$$

$$Z \text{ (degrees)} \equiv Z \times \frac{\pi}{180} \text{ (radians)}$$

6.2.2 Angular Displacement (*θ*)

The angular displacement θ is the angle an object has turned about a fixed point. In circular motion, the fixed point is taken to be the centre of the circle (Figure 2).

$$\Delta\theta = \theta - \theta_0$$

If $\theta_0 = 0$, then $\Delta\theta = \theta$.

S.I. unit of θ: radian (rad)

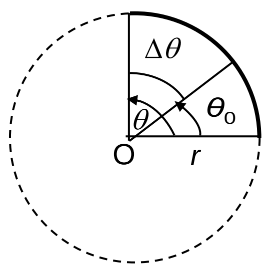


Figure 2

6.2.3 Angular Velocity (ω)

The description of circular motion in angular form is analogous to the description of linear motion. Angular velocity ω (pronounced as *omega*) is defined as the rate of change of angular displacement (Recall: In linear motion, linear velocity is defined as the rate of change of linear displacement).

Angular velocity is the rate of change of angular displacement.

$$\textit{Average angular velocity, } \bar{\omega} = \frac{\text{change in angular displacement}}{\text{Elapsed time}}$$

$$\bar{\omega} = \frac{\theta - \theta_0}{t - t_0} = \frac{\Delta\theta}{\Delta t}$$

The *instantaneous angular velocity* ω is the angular velocity that exists at any given instant. Analogous to instantaneous linear velocity,

$$\textit{Instantaneous angular velocity,} \quad \omega = \frac{d\theta}{dt}$$

If an object has a **constant** angular velocity, the instantaneous value and the average value are **the same** (i.e. $\omega = \bar{\omega}$).

Note:
- Magnitude of the instantaneous angular velocity is called the *instantaneous angular speed*.
- S.I. unit of ω : rad s^{-1}

6.2.4 Relationship between Tangential Speed *v* and Angular Speed ω

A particle moving in a circle has an instantaneous velocity tangential to its circular path (Figure 3). For a **constant** angular speed, the particle's **orbital or *tangential speed*** *v* is also constant. (*v* is also known as ***linear speed***.) At any instant, it is directed tangentially to the circular path at the specific point.

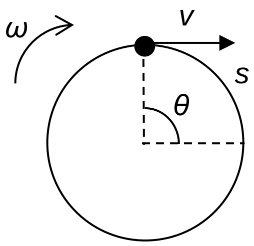


Figure 3

Since $\theta = \frac{s}{r}$ i.e. $s = r\theta$

Taking the time derivative of above and noting that *r* is constant:

$$\frac{ds}{dt} = r\frac{d\theta}{dt} \rightarrow \boxed{v = r\omega}$$

where v is the tangential speed and

ω is the instantaneous angular speed.

Comparing the tangential speed and the angular speed of two circular motion with different radii:
[Angular Velocity – xmPhysics]

6.2.5 Period (T) and Frequency (f)

Period (T) is the time it takes for an object in circular motion to make one complete revolution, or cycle.

Frequency (f) is the number of revolutions, or cycles, made per unit time. The unit of frequency is s^{-1}, which is called the *hertz* (Hz) in the SI.

$$\text{frequency, } f = \frac{1}{T}$$

Note:

- When asked to *define* frequency, students often give it as the number of revolutions *per second*. This is incorrect, as a frequency can also be given as e.g. per minute or per year (frequency of the Earth).

Since an angular displacement of 2π rad is travelled in 1 period,

$$\textit{angular speed, } \omega = \frac{2\pi}{T} = 2\pi f$$

<u>Example 1</u>

A Blu-ray player is spinning a standard Blu-ray Disc (120 mm diameter) at 900 revolutions per minute (RPM). Determine

i) the frequency of the rotation (in revolutions per second)
ii) the period of the rotation
iii) the angular speed

Example 2

Dishes are placed on a Lazy Susan at a reunion dinner during Chinese New Year. When the Lazy Susan is spun around to serve the guests, what can be said about the dishes in the inner circle compared to those in the outer circle in terms of their speed and angular velocity?

6.3 Uniform Circular Motion

Uniform circular motion is the motion of an object travelling at a **constant** (uniform) speed in a circular path.

An object moving in a uniform circular motion (Figure 4) has a *constant angular velocity* (ω). The ***magnitude*** of the velocity vector (or the linear speed, v) is ***constant*** but the ***direction*** of the velocity vector is ***changing***. Thus, the *linear velocity vector is continuously changing* ($\boldsymbol{v_1}$, $\boldsymbol{v_2}$ & $\boldsymbol{v_3}$) as the object moves around the circle. Since there is a change in the velocity with respect to time, the object can be considered to be undergoing acceleration. So the object is travelling at constant speed but is still accelerating!

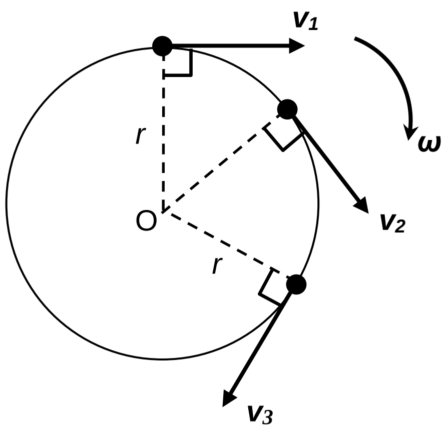


Figure 4

In the next section we derive an expression for the magnitude of this acceleration in terms of the speed and radius of the motion. We also find out the direction of this acceleration which is quite a surprising result!

Centripetal acceleration of a body moving with constant speed in circular motion:
https://xmphysics.com/2022/12/22/6-1-3/

6.3.1 Derivation of the equation of centripetal acceleration

In this derivation we only consider the kinematics of uniform circular motion to arrive at a general expression for the acceleration an object is experiencing when it moves in a circle at constant speed.
Consider a particle moving in a circle of radius r with uniform angular speed (Figure 5a). At one instant the particle is at A, and its instantaneous velocity is v_A in the direction AA'.

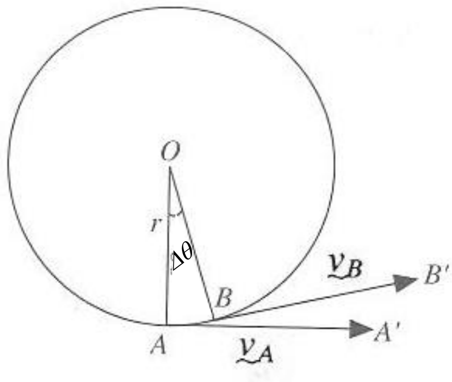


Figure 5a

A short time Δt later, the particle has moved to B, a distance $r\Delta\theta$ along the arc, where $\Delta\theta$ is a small angle, and the particle's velocity has changed to v_B in the direction BB'.

With reference to the vector triangle in Figure 5b, which is rotated 90° with respect to Figure 5a, the arc length can be approximated to a chord. For small angle $\Delta\theta$,

$\Delta v \approx v\Delta\theta$ (note, magnitude of $v_A = v_B = v$)

$\Delta v/\Delta t \approx v\Delta\theta/\Delta t$

$$\frac{dv}{dt} = v\frac{d\theta}{dt}$$

$a = v\,\omega$

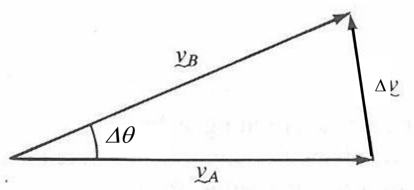


Figure 5b

Similarly,

$$a = (r\omega)\omega = r\omega^2$$

$$a = v\left(\frac{v}{r}\right) = \frac{v^2}{r}$$

As Δt (or $\Delta\theta$) (Figure 5b) becomes smaller, the angle between Δv and either v_A or v_B will tend to 90°. Since v_A and v_B are *tangent* to the circular path, this means Δv will point *towards the centre* of the circular path. From the derivation above, we see that the acceleration a points in the same direction as Δv, so the acceleration also points towards the centre of the circular path. This type of acceleration in uniform circular motion is called ***centripetal acceleration*** (centripetal means centre-seeking).

> The centripetal acceleration is directed **radially inward** towards the centre of the circular path. The direction of the centripetal acceleration is thus **continuously changing.**
>
> For an object in ***uniform*** *circular motion*, there is **no** acceleration component in the tangential direction (i.e. angular acceleration), or else the magnitude of the velocity vector (tangential or linear speed) would change.

The centripetal acceleration is indicated by the subscript c. Hence,

Centripetal acceleration, $a_c = \dfrac{v^2}{r} = r\omega^2 = v\omega$

Deriving the formula for centripetal acceleration $a = \frac{v^2}{r}$:
[Derivation of Formula for Centripetal Acceleration v^2/r - YouTube]

Conceptual Questions:

(a) Is an object travelling at non-uniform speed accelerating?

Yes, there is a change in velocity.

(b) Can an object travelling at constant speed be accelerating?

Yes, an example is an object moving in circular motion.

6.3.2 Centripetal Force (F_c)

In the last section we discovered both the magnitude and direction of the acceleration associated with circular motion. We now move into the dynamics of circular motion, quantifying and identifying forces that can constrain objects to follow such circular paths.
To provide acceleration, there must be a net force. Thus, **to produce a centripetal (inward) acceleration, there must be a resultant force towards the centre of the circular motion**. As the resultant force is centre-seeking, we call it the centripetal force, also indicated by the subscript *c*.

From Newton's second law ($\sum F = ma$),

$F_c = ma_c = \dfrac{mv^2}{r} = mr\omega^2 = mv\omega$, and it is **always directed toward the centre** of the circle.

Note: In general, when a force is continuously applied **at an angle of 90° to the direction of motion** (which in this case centripetal force is), ***only the direction*** of the velocity changes.

Conceptual Question: Is there work done by a centripetal force?

No, the centripetal force is always perpendicular to motion. ($W = Fs\cos\theta$, where $\theta = 90°$)

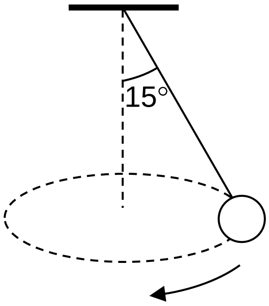
Note: **For uniform circular motion,**

- Centripetal force is not a new type of force, it is provided by the **net force** pointing toward the centre of the circular path, and this net force is the **vector sum of all the force components that point along the radial direction**.
- Forces labelled as "centripetal" or labelled as $\dfrac{mv^2}{r}$ should **never** appear in a free-body diagram!

Some forces that provide the centripetal force necessary for a body to undergo a circular motion:
https://xmphysics.com/2023/01/02/6-2-1-centripetal-force/

Example 3 - Conical pendulum

In a conical pendulum system, a small pendulum bob of mass 0.50 kg is rotating in a fixed horizontal plane. The string is 30 cm long and makes an angle of 15° to the vertical.
Calculate the (a) tension in the string;
(b) linear speed of the bob;
(c) period of rotation of the bob.



A conical problem explained:
https://www.youtube.com/watch?v=7CDwVWMvoGc

Example 4

Explain, with the aid of a diagram, why the mass at the end of a light inelastic string cannot be whirled in uniform circular motion in such a way that the string is horizontal.

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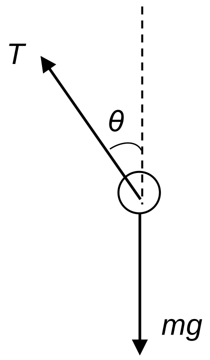

6.4 Vertical Circular Motion

When an object is set to travel in a vertical circular path, (without the use of a motor*) its speed will not be constant as some kinetic energy will be lost to account for potential energy the object gains as it climbs the circle. This results in **non-uniform circular motion**. For non-uniform motion, *the net force is no longer equal to the centripetal force*. We can still resolve the net force into its tangential and radial components. The tangential component changes only the speed of the motion, while the radial component changes only the direction of the motion. Calculations-wise, the equation for centripetal acceleration is still $\frac{v^2}{r}$, just that centripetal acceleration must now vary as the speed changes.

Typical examples of vertical non-uniform motion, are a roller coaster going through a loop-de-loop and an object whirled in a vertical circle at the end of a string. Both will be further explored in Examples 5 and 6 below. **We will assume in both cases, that the object's total energy is conserved**. At the top of the circular motion, the object's gravitational potential energy is larger than at the bottom of the circular motion. By the principle of conservation of energy, the object will be moving faster at the bottom than at the top as it will possess more kinetic energy to compensate for having less potential energy.

Depending on the context, there are two approaches to attack problems involving vertical, non-uniform motion.

In the **FORCES APPROACH,** we follow our **general strategy for solving dynamics problems**: drawing a free-body diagram, carefully identifying the individual forces experienced by the object undergoing circular motion, and determining the net force acting on the object. Subsequently, we resolve the net force into its *tangential* and *centripetal* components.

The **ENERGY APPROACH,** involving vertical, non-uniform circular motion invokes the **principle of conservation of energy**. We know how the gravitational potential energy changes as an object goes from a higher point to a lower point. Hence, if we know the difference in height between two points, we also know the difference in kinetic energy and, thus, the difference in speed.
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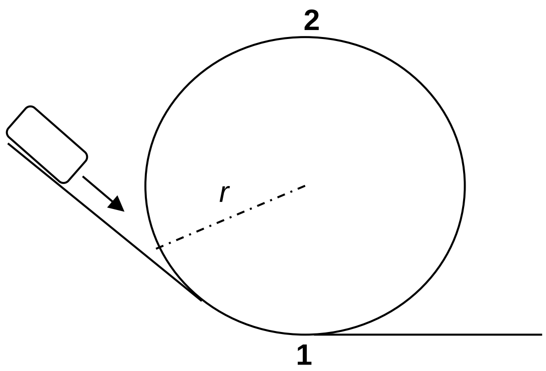
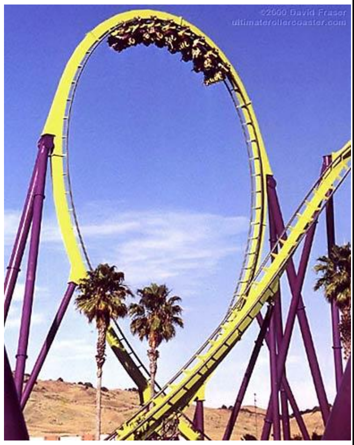
*Take the Singapore Flyer as an example, the carriages are moving in a uniform circular motion although it is a vertical circular path.

Example 5 - Roller coaster

Some roller coasters have several loops along the track. The picture on the right illustrates such a coaster executing a loop-the-loop.





In the question below, assume that the total mass of one car plus its passengers is 170 kg and that friction can be ignored.

(a) A passenger car for a roller coaster enters a loop of radius 19 m at position 1, as indicated in the diagram, with a speed of 33 m s^{-1}. Determine the normal contact force the track exerts on the car at
   i) the bottom (position 1) and
   ii) the top (position 2) of the loop.

(b) Find the minimum speed at which the passenger car must travel while it is at the top of the loop, in order to clear the loop safely.

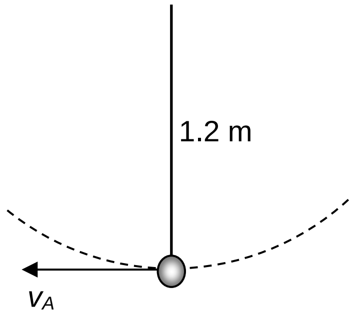
Analysing the motion of the roller coaster:
https://www.youtube.com/watch?v=bEoLhtPif2E

Example 6 – String and bob

A stone of mass 800 g is tied to one end of a string and is whirled in a vertical circle. The string is inextensible and of length 1.2 m. The stone has a certain speed v_A at the lowest point, as shown below.

(i) Determine the minimum speed the stone must have at the top of the circular motion if the string is to be taut at that instant.

(ii) Hence, show that the stone can complete a vertical circular motion if $v_A = 8.0$ m s^{-1}.



Analysing vertical circular motion:
https://www.youtube.com/watch?v=N0ByVmuoIfU

In some other instances of vertical circular motion, the speeds of the objects are kept constant (e.g. Ferris wheel). In these cases, the objects are undergoing uniform circular motion. You will encounter some of these applications in the tutorial.

6.5 Problem solving for Circular Motion

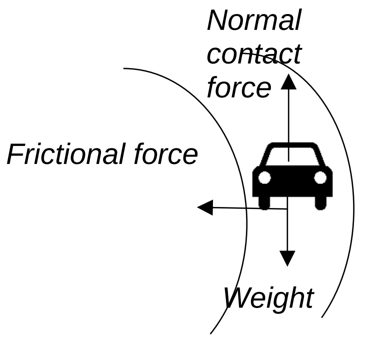
1. Identify the object undergoing circular motion (and the relevant known quantities such as m, v, r, ω…).
2. Draw a **free-body diagram** of the object, carefully identifying the individual forces *experienced by* the object, to determine the net force acting on the object.
3. **Resolve the net force** into relevant perpendicular "directions", i.e., the tangential and centripetal components.
4. The **tangential component** is usually zero. If it is not, applying Newton's 2nd Law *to the tangential component only* will tell you how fast the object is slowing down or speeding up.
5. The **centripetal component** will be equal to $\frac{mv^2}{r}$. You can use this to determine any further unknowns in the question.

Note that you have not learnt a new way of solving physics problems. The main strategy is still to draw a free-body diagram and to resolve the net force into perpendicular components. In the previous topics, it was convenient to take the horizontal and the vertical component, or the component along some slope and the component perpendicular to it. Now it is convenient to choose components parallel and perpendicular to the motion.

6.5.1 Further examples

Example 7 - Car going around a bend

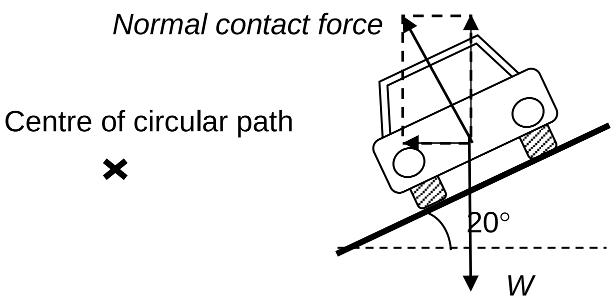
(a) A bend in the road has a 50 m radius of curvature. A car of mass 600 kg takes the bend at 45 km h^{-1}.
   i) What is the centripetal acceleration of the car?
   ii) What is the centripetal force experienced by the car and what provides it?
   iii) What will happen to the car if driver decides to take the bend at 60 km h^{-1} instead? (Given that the maximum friction between the tyres and road surface is 3000 N.)



(b) When the car negotiates a corner on horizontal ground, the frictional force between the tyres and the ground is the only force providing the centripetal force. As there is a limit to this frictional force for a particular road surface, there is a maximum speed which the car can make the turn safely, beyond which skidding will occur.

Hence, some corners (especially at race-tracks) have raised embankments to increase the maximum speed at which a vehicle can turn the corner than if on a level road. It does so by making the **normal contact force** contribute a component to the centripetal force.

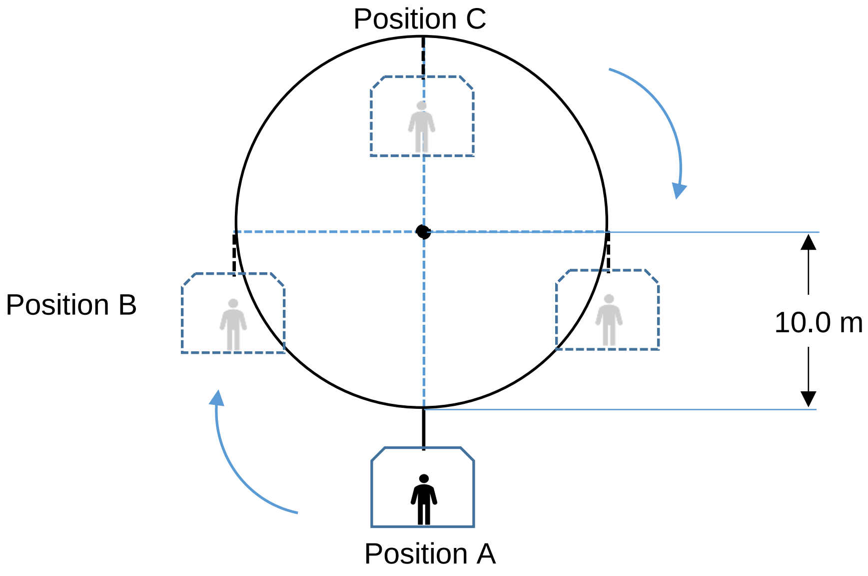
For an embankment inclined at 20° to the horizontal, find the speed at which the normal contact force is able to **completely provide the centripetal force** (i.e. no frictional force is required).



Banking of a slope and the effect on the turning of a car:
https://www.youtube.com/watch?v=KEEqE93nfe8

Example 8 - Person on a Ferris wheel

The figure below is a simplified diagram of a Ferris wheel. A single carriage with a passenger is shown.



The Ferris wheel takes 0.500 mins to make one complete revolution. Each carriage goes through a circular path of radius 10.0 m. The passenger's mass is 65.0 kg.

Determine the normal contact force exerted by the carriage on the passenger at
(a) Position A,
(b) Position B, and
(c) Position C
(d) If the passenger were to be standing on a weighing scale, what can you infer about the readings at Positions A, B and C respectively?

Circular motion of a Ferris Wheel:
https://www.youtube.com/watch?v=XwlMqWEV-pc

Tutorial 6 Circular Motion

Self-Review Questions

S1 (a) An object cannot move in a circle unless there is a resultant force acting ________ the centre of the circle. This is called a ___________ force. If this force is removed, the object will continue moving in a ___________ line, because of Newton's __________ law.

(b) The centripetal force causes a centripetal __________. The larger the linear __________ (in m s^{-1}) and the smaller the __________ of the circle, the larger the acceleration.

(c) The number of revolutions in one second is known as the ___________. This is measured in ___________. The time taken for one complete revolution is called the _________.

S2 *(N96/I/9)* A disc is rotating about an axis through its centre and perpendicular to its plane. A point P on the disc is twice as far from the axis as a point Q.

At a given instant what is the value of $\frac{the\,linear\,velocity\,of\;P}{the\,linear\,velocity\,of\;Q}$?

A 4 **B** 2 **C** ½ **D** ¼

S3 *(edited N91/I/9)* A particle travels in uniform circular motion. Which of the following correctly describes the linear velocity, angular velocity and centripetal acceleration of the particle?

	Linear velocity	*Angular velocity*	*Centripetal acceleration*
A	constant	constant	varying
B	constant	constant	zero
C	constant	varying	constant
D	varying	constant	varying
E	varying	varying	constant

S4 (*N93/I/6)* Which of the following statements is correct for a particle moving in a horizontal circle with constant angular velocity?

A The linear momentum is constant but the kinetic energy varies.

B The kinetic energy is constant but the linear momentum varies.

C Both the kinetic energy and linear momentum are constant.

D Both speed and linear velocity are constant.

E Neither the linear momentum nor the kinetic energy is constant.

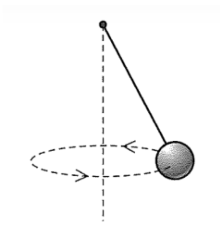
S5 *(N2015/I/11)* The minute hand of a large clock is 3.00 m long. What is the magnitude of its angular velocity?

A 1.39×10^{-4} rads^{-1} **B** 1.75×10^{-3} rads^{-1} **C** 5.24×10^{-3} rads^{-1} **D** 1.05×10^{-1} rads^{-1}

S6 *(J79/II/1)* A mass of 2 kg rotates at constant speed in a horizontal circle of radius 5 m and the time for one complete revolution is 3 s. The force, in N, acting on the mass is

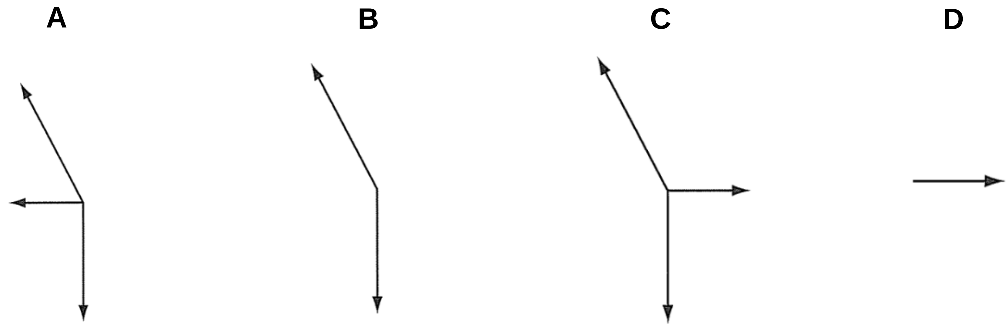
A $\frac{2\pi^2}{9}$ **B** $\frac{4\pi^2}{9}$ **C** $\frac{40\pi^2}{9}$ **D** $\frac{100\pi^2}{9}$ **E** $\frac{400\pi^2}{9}$

S7 *(N2013/I/11)* A small ball suspended from a light thread moves in a horizontal circle at a constant speed.

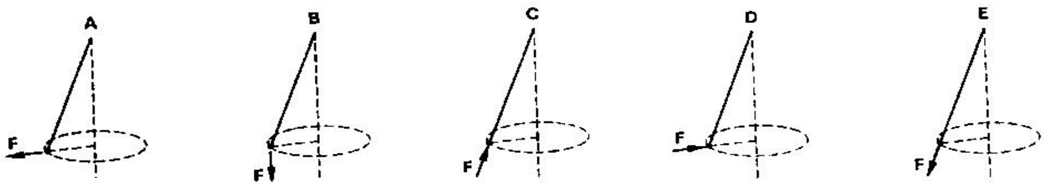


A student draws the forces acting on the ball but fails to label them.

Which diagram shows the correct forces?



S8 (*J91/I/8*) A mass on the end of a string is set in motion so that it describes a circle in a horizontal plane. Which diagram shows the direction of the resultant force acting on the mass at an instant in its motion?



S9 (*J82/II/6; N85/I/4*) A car of mass m moving at a constant speed v passes over a humpback bridge of radius of curvature r. Given that the car remains in contact with the road, what is the net force R exerted by the car on the road when it is at the top of the bridge?

A $R = mg + \dfrac{mv^2}{r}$ **B** $R = \dfrac{mv^2}{r}$

C $R = mg - \dfrac{mv^2}{r}$ **D** $R = mg$

S10 (*J88/I/7*) An artificial satellite travels in a circular orbit about the Earth. Its rocket engine is then fired and produces a force on the satellite exactly equal and opposite to that exerted by the Earth's gravitational field. The satellite would then start to move

A along a spiral path towards the Earth's surface.
B along a tangent to the orbit.
C in a circular orbit with a longer period.
D in a circular orbit with a shorter period.

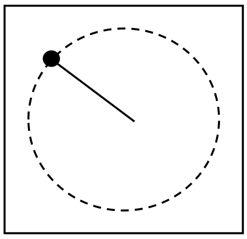
Discussion Questions

D1 A small coin of mass 2.0 g is placed on a flat horizontal turn-table. The turn-table is observed to make three revolutions in 3.14 s

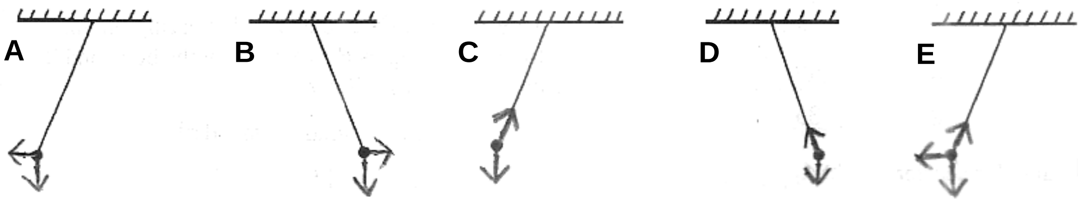
a) What is the angular displacement of the coin after it has rotated for 2.0 seconds?
b) What is the speed of the coin when it rides without slipping at a distance 5.0 cm from the centre of the turn-table?
c) What is the acceleration of the coin in part (b)?
d) What is the frictional force acting on the coin in part (b)?

D2 *(J87/I/9)* A mass of 0.050 kg is attached to one end of a piece of elastic of unstretched length 0.50 m. The force constant of the elastic is 40 N m^{-1}. The mass is rotated steadily on a smooth table in a horizontal circle of radius 0.70 m as shown. What is the approximate speed (in m s^{-1}) of the mass?

A 11
B 15
C 20
D 24
E 28



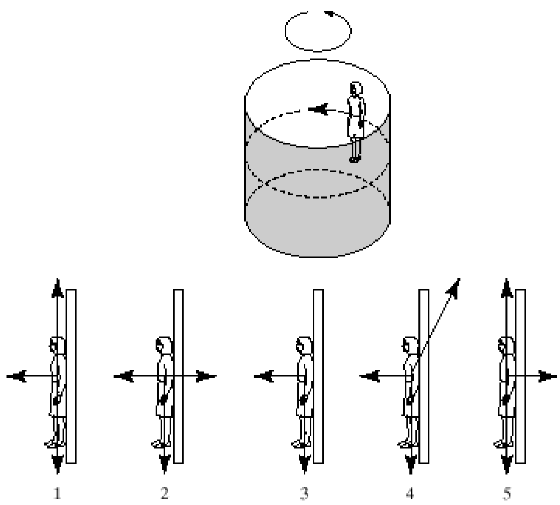
D3 (*J81/II/7)* A passenger is sitting in a railway carriage facing in the direction in which the train is travelling. A pendulum hangs down in front of him from the carriage roof. The train travels along a circular arc bending to the right. Which one of the following diagrams shows the position of the pendulum as seen by the passenger and the directions of the forces acting on it?



D4 *(J97/I/8)* The maximum safe speed of a car rounding a unbanked corner is 20 m s^{-1} when the road is dry. The maximum frictional force between the road surface and the wheels of the car is halved when the road is wet, what is the maximum safe speed (in m s^{-1}) for the car to round the corner?

A $\frac{20}{4}$ **B** $\frac{20}{2\sqrt{2}}$ **C** $\frac{20}{2}$ **D** $\frac{20}{\sqrt{2}}$

D5 In a popular amusement park ride known as the Rotor, people stand against the wall of a cylinder that is rotated. When rotating fast enough, the floor drops away, leaving the riders "pressed" against the wall in a vertical position as shown in the picture below.



For a cylinder of radius 3.00 m rotating at 5.00 rad s^{-1},

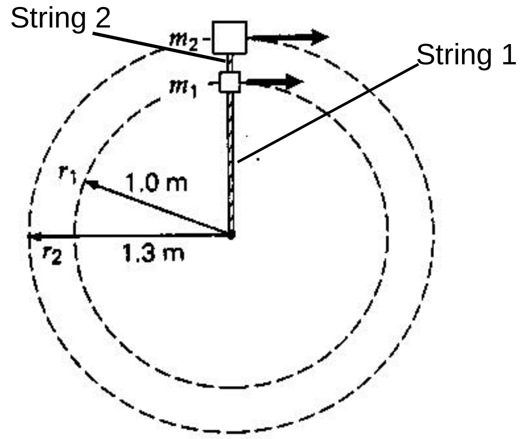
(a) identify which of the diagrams in the figure on the right correctly identifies the forces acting on the person, and suggest what keeps the person from sliding down. What physical force provides for the centripetal acceleration?

Calculate the

(b) centripetal acceleration and

(c) centripetal force, experienced by a 60 kg person.

D6 Suppose that two masses, m_1 = 2.5 kg and m_2 = 3.5 kg, respectively, are connected by light strings and are in uniform circular motion on a horizontal frictionless surface as shown on the right, where r_1 = 1.0 m and r_2 = 1.3 m. The tensions in string 2 and 1 are T_2 = 2.9 N and T_1 = 4.5 N respectively.

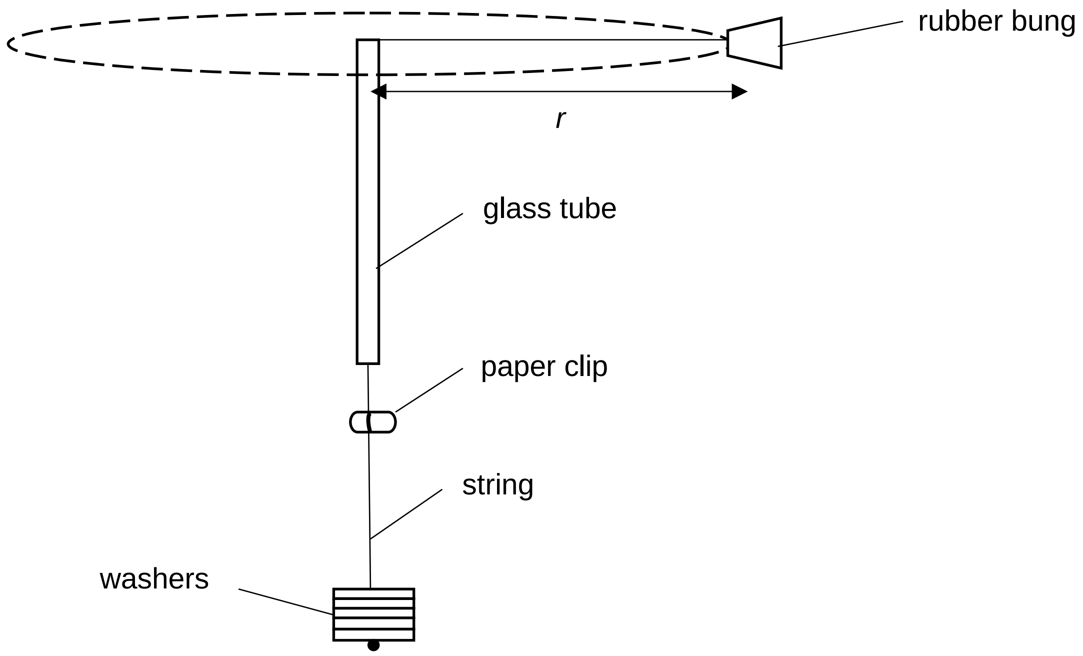


Find the

(a) centripetal accelerations,

(b) magnitude of the tangential velocities of the masses.

D7 *(2017 C2 BT2 P2 Q2)* The setup shown in the figure below is used to investigate how the force required to keep a rubber bung of mass *m* moving in uniform horizontal circular motion at a set radius, *r* varies with the speed of the motion. The rubber bung is set and maintained in motion by the experimenter gripping and twirling the glass tube.



(a) With a set radius *r* of 57.0 cm, washers of total mass 35.0 g allow the bung to perform 20 revolutions in a time of 18.2 seconds.
Show that the mass of the rubber bung is 12.6 g. [3]

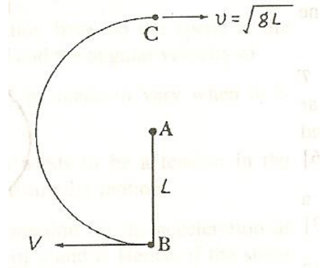
(b) Suggest the purpose of the paper clip in the experimental set up. [1]

(c) (i) Washers of mass 95.0 g are now fixed to the string and the radius of rotation is maintained at 57.0 cm. Calculate the new speed of the bung. [2]

(ii) Explain why in reality, it is not possible for the bung to rotate in a purely horizontal circle. [2]

(iii) Hence, when the bung is performing circular motion with washers of mass 95.0 g attached, calculate the angle θ, the string makes with the horizontal. [2]

D8 (*N81/I/1*) A particle is suspended from a point A by an inextensible string of length L. It is projected from B with a velocity v, perpendicular to AB, which is just sufficient for it to reach the point C.

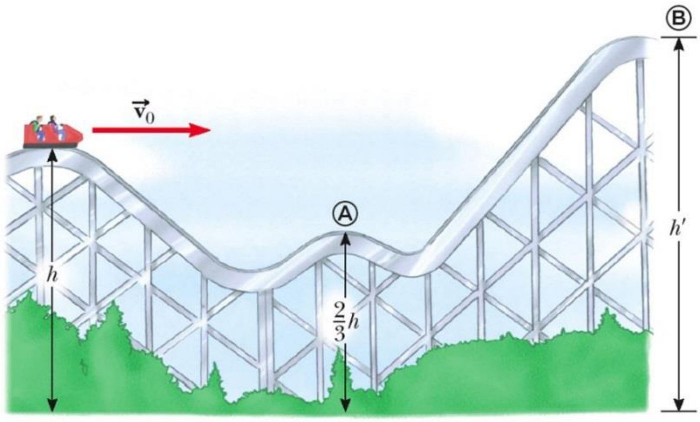


(a) Show that, if the string is just to be taut when the particle reaches C, its speed there is $\sqrt{gL}$.

(b) Find the speed v with which the particle should be projected from B.

D9 **Serway and Faugh. (7E) Page 223. P7.59.**

A frictionless roller coaster is given an initial velocity of v_o at height h, as in the figure below. The radius of curvature of the rack at point A is R.

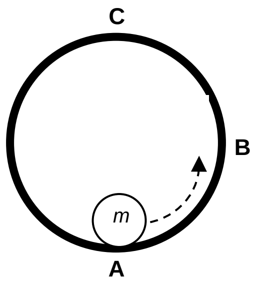


(a) Find the maximum value of v_o so that the roller coaster stays on the track at **A** solely because of gravity.

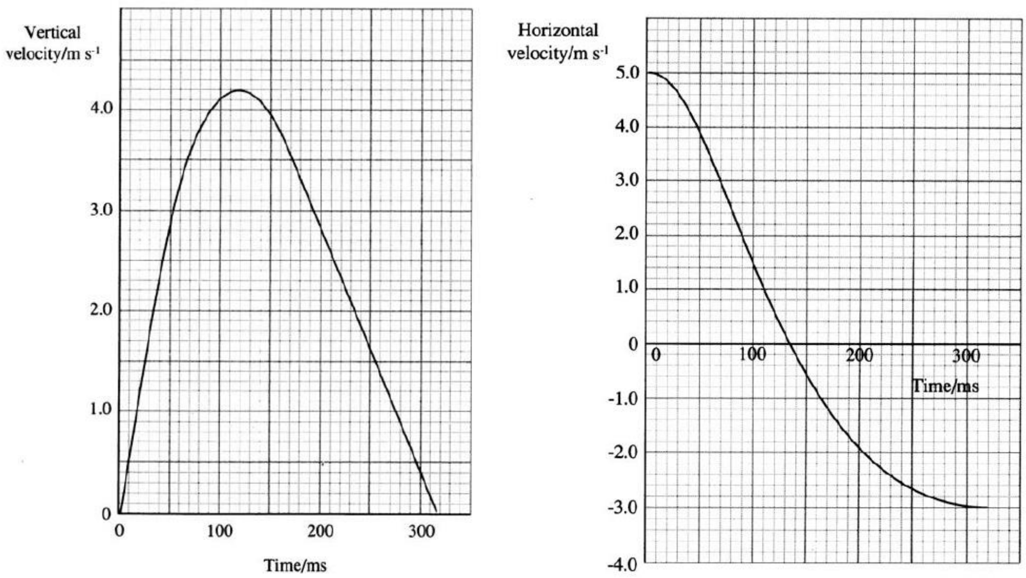
(b) Using the value of v_o calculated in (a), determine the value of h' that is necessary if the roller coaster just makes it to point B.

(c) Consider solution in (b), why do we not use the equations of motion that we learnt in the earlier chapter in kinematics to solve this question?

D10 *(MJC Prelims 10/I/12)* A particle of mass m performs vertical circular motion as shown in the diagram.



The following two graphs show the vertical and horizontal components of the velocity of the particle along path **ABC**.



Calculate the centripetal acceleration at point **C**.

A zero **B** 4.91 m s^{-2} **C** 9.81 m s^{-2} **D** 22.1 m s^{-2}

D11 Sasha's favourite ride at the fair is the Ferris wheel that has a radius of 7.0 m.

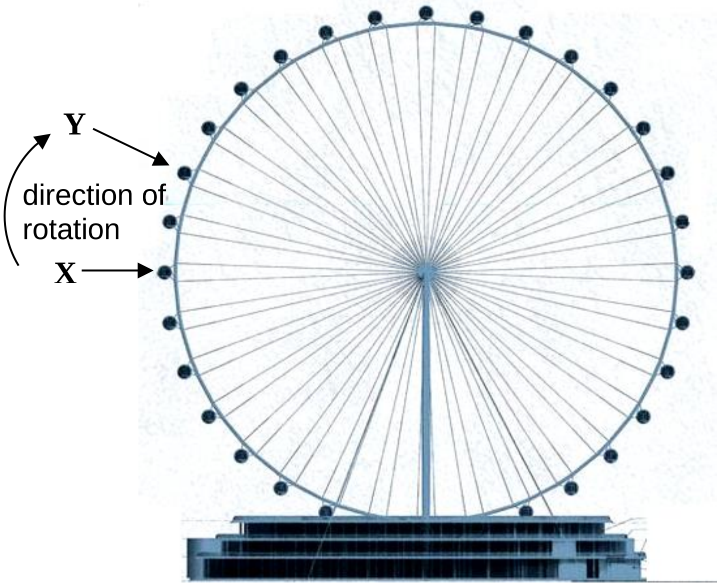
(a) If the ride takes 20.0 s to make one full revolution, what is the linear speed of the wheel?

(b) What is the magnitude of the centripetal force acting on Sasha's 50.0 kg body?

(c) In order for Sasha to feel weightless at the top of the ride, at what linear speed must the Ferris wheel turn?

(d) At this speed, how much will she appear to weigh at the bottom of the Ferris wheel?

D12 The Singapore Flyer (as shown in the figure below) is a giant observation wheel that is Asia's most visible iconic visitor attraction, providing breath-taking, panoramic views of Singapore and beyond. Completed in Mar 2008, it is one of the world's largest man-made moving land objects. It has a height of 178 m, a diameter of 150 m and sits on a 28 m high three-storey terminal building.



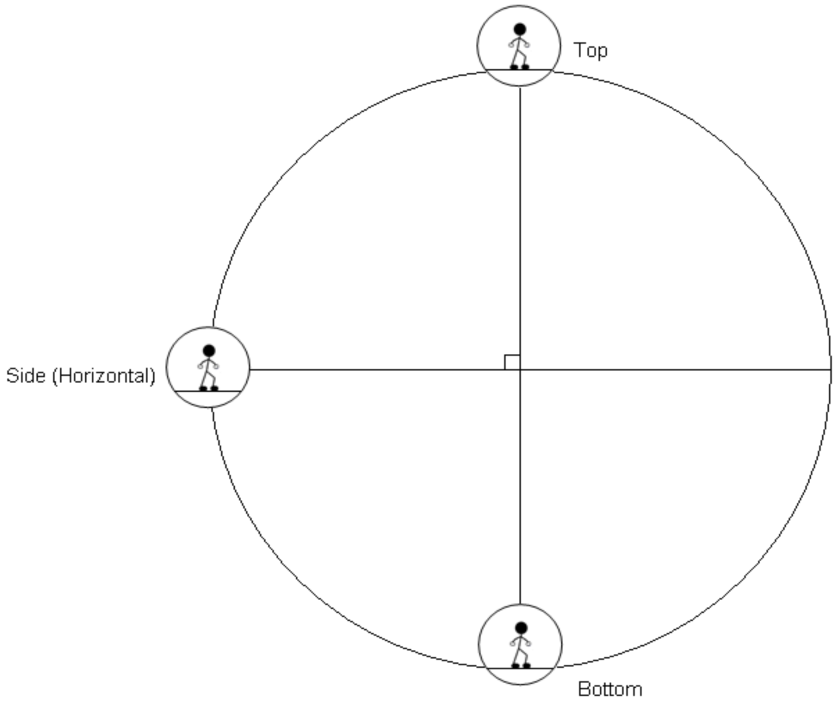
During the testing phase, the Singapore Flyer went through a series of continuous revolutions without stopping. Assume that it revolves at constant angular speed ω and each complete revolution takes 37 minutes.

(a) Calculate the angular speed of the Singapore Flyer. [2]

(b) Hence or otherwise, determine the speed of a passenger capsule which is located at the circumference. [2]

(c) Given that there are 32 equally spaced capsules on the Flyer, find the time taken for a capsule to go from position X to position Y. [2]

(d) The Singapore Flyer now revolves at a much faster rate such that the centripetal force on the passenger becomes about half his weight. Draw and label the forces acting on a passenger standing inside a capsule at the three positions in the figure below. [4]



Answers:

D1	**(a)**	5.7 rad
	(b)	0.30 m s^{-1}
	(c)	1.8 m s^{-2}
	(d)	3.6×10^{-3} N
D5	**(b)**	75.0 m s^{-2}
	(c)	4500 N
D6	**(a)**	a_1 = 0.64 m s^{-2}, a_2 = 0.83 m s^{-2}
	(b)	v_1 = 0.80 m s^{-1}, v_2 = 1.0 m s^{-1}
D7	**(c)(i)**	6.49 ms^{-1}
	(c)(iii)	7.62°
D8	**(b)**	$\sqrt{5gL}$
D9	**(a)**	$v_o = \sqrt{g(R-\frac{2h}{3})}$
	(b)	$h' = \frac{R}{2}+\frac{2h}{3}$
D11	**(a)**	2.20 m s^{-1}
	(b)	34.6 N
	(c)	8.29 m s^{-1}
	(d)	981 N
D12	**(a)**	2.83×10^{-3} rad s^{-1}
	(b)	0.212 m s^{-1}
	(c)	139 s

CIRCULAR MOTION

Definitions:

- Angular displacement, θ: angle swept from a reference position
  Unit: rad
- Period, T: time taken for one complete revolution
- Angular velocity, ω: rate of change of angular displacement wrt time $\left(\omega = \frac{d\theta}{dt}\right)$
  Unit: rad s^{-1}
- Linear/Tangential speed v

Conditions for Uniform Circular Motion:
Constant ω and constant r

Important formulae for uniform circular motion:

- $\omega = \frac{2\pi}{T} = 2\pi f$
- $v = r\omega$
- $a_c = \frac{v^2}{r} = r\omega^2 = v\omega$

Using Newton's Laws to explain why an object moving in a constant speed in a circle experiences a resultant force towards the centre of the circle

- Since object experiences a constant change in direction of motion, by N1L, there must be a resultant force on it.
- Given the linear speed remains constant, by N2L, the resultant force must act perpendicular to the velocity, hence in the radial direction, towards the centre of the circle, to keep changing the directions of the linear velocity.

The resultant force causing the circular motion is called the centripetal force.

$F_c = \sum F = ma_c = m\left(\frac{v^2}{r}\right) = m\left(r\omega^2\right) = m\left(v\omega\right)$

always state "*the _______ (force) provides the centripetal force/acceleration*"

Horizontal Uniform Circular Motion

Conditions:

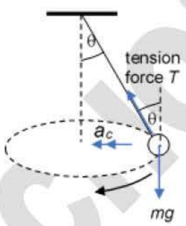
- $\sum F_{radial} = F_{centripetal} = ma_c$
- $\sum F_{tangential} = 0$ (seldom use)

Strategy:

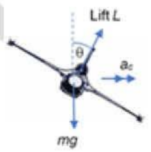
- Draw the FBD for the object undergoing circular motion
- Remember not to draw the centripetal force in the FBD.
- Resolve the forces radially in the horizontal plane
- **Substitute a_c** depending on the variables given.
- Balance the forces in the vertical plane.

See examples below.

For uniform circular motion, there is NO WORK done by the centripetal force since the direction of the force is always perpendicular to the direction of displacement.
Also, kinetic energy of the body remains unchanged. By work-energy theorem, no net work is done on it.



(←) $T \sin\theta = ma_c$
(↑) $T \cos\theta = mg$



(←) $L \sin\theta = ma_c$
(↑) $L \cos\theta = mg$

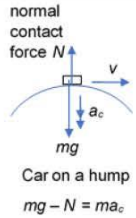
× Centre of circular path

Vertical Circular Motion

- Most examples of vertical circular motion are non-uniform, since the linear speed and ω are not constant. Eg roller coaster. If resistive forces are negligible, we use PCOE to find change in v. (see example below)
- For non-uniform circular motion, except when the object is at the top or bottom of the circular motion, the resultant force no longer points towards the centre of the circle due to presence of tangential acceleration.
- There are situations where ω are fixed eg Ferris wheel, to force vertical circular motion uniform, then extra work has to be done on the system.

Strategy (analyse forces acting at top/ bottom):

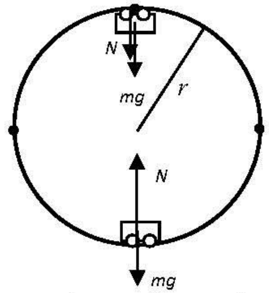
- Draw the FBD for the object in circular motion
- Do not draw the centripetal force in the FBD.
- Write the equation of motion for the object:
  $\sum F_y = F_c = ma_c$
- Substitute a_c using the variables given.
- Solve.



Car on a hump

$mg - N = ma_c$

Example of roller coaster:



At the top of the circular motion:

$F_c = ma_c$

$N + mg = ma_c$

$N = m(a_c - g)$

At the bottom of the circular motion:

$F_c = ma_c$

$N - mg = ma_c$

$N = m(a_c + g)$

Normal contact force, N, decreases as the object goes up and is min. at the top of the circular motion, max at the bottom.
Speed of the roller coaster also decreases as it gains height. There is a minimum velocity at the top that allows the roller coaster to just remain in contact with the track and complete the circular motion.

To find min. speed at the top:

$F_c = ma_c$

$N + mg = ma_c = m\left(\frac{v^2}{r}\right)$

v is minimum when N→ 0, hence

$mg = m\left(\frac{v_{min}^2}{r}\right) \Rightarrow v_{min} = \sqrt{gr}$

Conservation of energy:
$(KE + GPE)_{top} = (KE + GPE)_{bottom}$
If there is no frictional forces