SINGAPORE CHINESE GIRLS' SCHOOL
END-OF-YEAR EXAMINATION 2020
YEAR THREE
INTEGRATED PROGRAMME

CANDIDATE NAME	

CLASS	3			INDEX NUMBER		

ADDITIONAL MATHEMATICS

Wednesday **07 October 2020** **2 hours 15 minutes**

Candidates answer on the Question Paper.
No additional materials are required.

READ THESE INSTRUCTIONS FIRST

Write your name, index number and class in the spaces at the top of this page.
Write in dark blue or black pen.
You may use an HB pencil for any diagrams or graphs.
Do not use staples, paper clips, glue or correction fluid/tape.

Answer **all** the questions.
Give non-exact numerical answers correct to 3 significant figures, or 1 decimal place in the case of angles in degrees, unless a different level of accuracy is specified in the question.
The use of an approved scientific calculator is expected, where appropriate.
You are reminded of the need for clear presentation in your answers.

At the end of the examination, fasten all your work securely together.
The number of marks is given in brackets [ ] at the end of each question or part question.

The total number of marks for this paper is 90.

Mathematical Formulae

ALGEBRA

Quadratic Equation

For the equation $ax^2 + bx + c = 0$,

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

Binomial expansion

$$(a+b)^n = a^n + \binom{n}{1}a^{n-1}b + \binom{n}{2}a^{n-2}b^2 + \ldots + \binom{n}{r}a^{n-r}b^r + \ldots + b^n,$$

where n is a positive integer and $\binom{n}{r} = \frac{n!}{r!(n-r)!} = \frac{n(n-1)\ldots(n-r+1)}{r!}$.

1. **(i)** Factorise $2x^2 + xy - y^2$. [1]

(ii) Hence or otherwise, find the values of x and y which satisfy the equations [3]

$$y^2 - xy - 2x^2 = -8 \ ,$$
$$x + y = 4 \ .$$

2. The curve $y = 2x^2 + x - \frac{1}{2}$ crosses the x-axis at x_1 and x_2, where $x_2 > x_1$. Without using a calculator, find the exact value of $\frac{x_2}{x_1}$, leaving your answer in the form $\frac{\sqrt{p} - q}{2}$, where p and q are integers. [5]

3. **(a)** The equation of a curve is $y = 3x^2 + 6x + c$, where c is a constant. Find the range of values of c for which $y > 2$. [3]

(b) Jason bought some shares in the stock market. The value of the shares can be modelled by the function $y = 4x^2 - 6x + 7$, where \$$y$ is the value of the shares in thousands and x is the time in years after the shares were first bought.

(i) Express $y = 4x^2 - 6x + 7$ in the form $y = a(x-b)^2 + c$, where a, b and c are constants. [1]

(ii) Find the minimum value of the shares and the corresponding time, in months, after the shares were first bought when this occurs. [2]

4. Solve the following equations.

(a) $e^t(e^t + 2) = 48$ [3]

(b) $3^{2x}\left(4^x\right) = 216^{3x-4}$ [3]

(c) $\log_4 x(x+4) = \log_{16} x^6 + \dfrac{1}{\log_3 4}$ [5]

5. **(i)** Factorise $x^3 - 3x + 2$ completely. [3]

(ii) Hence or otherwise, express $\dfrac{2x-3}{x^3-3x+2}$ in partial fractions. [4]

6. In the expansion of $\left(x+\frac{k}{x}\right)^{10}$, the term independent of x is $\frac{70}{3}$ times the coefficient of x^4.

(i) Given that k is a negative constant, find the value of k. [5]

(ii) Using the value of k found in part **(i)**, find the coefficient of x^6 in the expansion of $(4-9x^2)\left(x+\frac{k}{x}\right)^{10}$. [3]

7. The coordinates of P, Q and R are $(2,5)$, $(-3,2)$ and $(0,-3)$ respectively.

(i) Show that triangle PQR is a right-angled triangle. [2]

(ii) Given that the point $S(x,0)$ lies on the perpendicular bisector of PR, find the value of x. [4]

(iii) Find the area of the quadrilateral $PQRS$. [2]

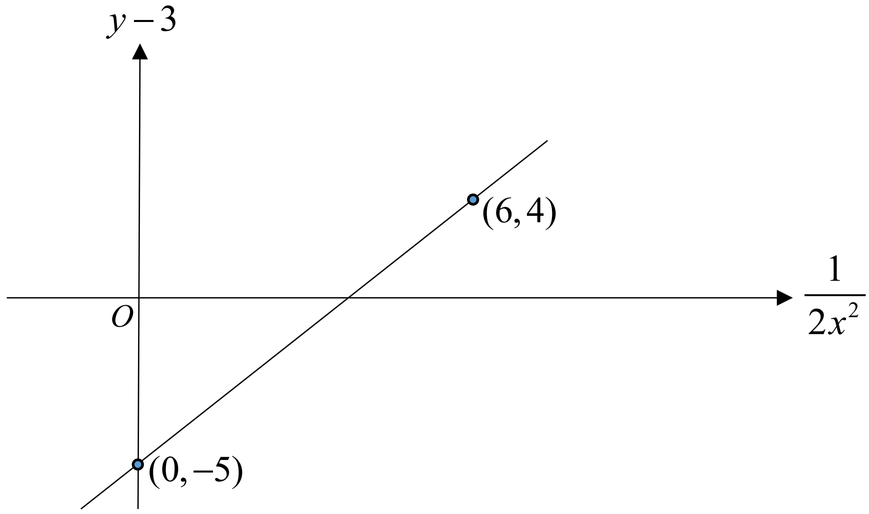
8. The equation of a circle is $x^2 + y^2 + 2x - 4y - 20 = 0$.

(a) Find the coordinates of the centre of the circle and the radius of the circle. [3]

(b) The circle touches the lines $y = a$ and $y = b$, where $a > b$. Find the value of a and of b. [2]

(c) A point A lies above the centre of the circle and on the circumference of the circle. Given that the x-coordinate of A is 3, find the equation of the normal to the circle at the point A. [5]

9.



(a) The diagram shows part of a straight line graph drawn to represent the equation $y = \dfrac{3-ax^2}{bx^2}$, where a and b are constants. Given that the line passes through $(6,4)$ and $(0,-5)$, find the value of a and of b. [3]

(b) The point $(k,7)$ lies on the straight line.

(i) Find the value of k. [1]

(ii) Using the value of k found in part **(i)**, find the corresponding values of x. [2]

10. Given that $\cos 55° = p$, express the following in terms of p.

(a) $\cos(-55°)$ [1]

(b) $\tan 125°$ [2]

11. Given that $\cos A = \frac{4}{5}$ and that A is acute, evaluate $4\tan A + \cos\left(\frac{\pi}{2} - A\right)$. [2]

12. Solve each of the following equations for $0 \le \theta \le 2\pi$.

(a) $\cot 2\theta = -\frac{1}{2}$ [3]

(b) $3\sin\theta\cos\theta = \cos^2\theta$ [4]

13. Using the substitution $y = \sin x$, solve the equation $2\sin x + \operatorname{cosec} x = 3$ for $0° \leq x \leq 180°$. [4]

14. The growth in the number of virus cases, N thousands, t days after observations began, are shown in the table below.

N (thousands)	4.1	8.2	16.4	33.1	68.7
t (days)	1	3	5	7	9

It is known that N and t are related by the equation $N = Ae^{bt}$, where A and b are constants.

(i) On the grid provided, plot ln N against t and draw a straight line graph. [3]

Use your graph to estimate

(ii) the number of cases when the observations began, [2]

(iii) the value of b, [2]

(iv) the time taken for the number of virus cases to rise to 4 times its original number. [2]

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