General Certificate of Education Ordinary Level

JUYING SECONDARY SCHOOL, SINGAPORE

Secondary Four Express / Five Normal (Academic)

Preliminary Examination

CANDIDATE NAME											
CENTRE NUMBER	S					INDEX NUMBER					

ADDITIONAL MATHEMATICS 4049/02

Paper 2

31 August 2021

2 hour 15 minutes

Candidates answer on the Question Paper.

No Additional Materials are required.

READ THESE INSTRUCTIONS FIRST

Write your name, index number and class on this cover page.

Write in dark blue or black pen.

You may use an HB pencil for any diagrams or graphs.

Do not use staples, paper clips, glue or correction fluid.

Answer **all** questions.

The number of marks is given in brackets [ ] at the end of each question or part question.

If working is needed for any question it must be shown with the answer.

Omission of essential working will result in loss of marks.

The total number of marks for this paper is **90**.

The use of an approved scientific calculator is expected, where appropriate.

If the degree of accuracy is not specified in the question, and if the answer is not exact, give the answer to three significant figures. Give answers in degrees to one decimal place.

For π, use either your calculator value or 3.142.

Setter: Mdm Mak Wai Han
Vetter: Mdm Norhafiani A Majid

Mathematical Formulae

1. ALGEBRA

Quadratic Equation

For the equation $ax^2 + bx + c = 0$,

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

Binomial expansion

$$(a+b)^n = a^n + \binom{n}{1}a^{n-1}b + \binom{n}{2}a^{n-2}b^2 + \ldots + \binom{n}{r}a^{n-r}b^r + \ldots + b^n,$$

where n is a positive integer and $\binom{n}{r} = \frac{n!}{r!(n-r)!} = \frac{n(n-1)\ldots(n-r+1)}{r!}$

2. TRIGONOMETRY

Identities

$$\sin^2 A + \cos^2 A = 1$$

$$\sec^2 A = 1 + \tan^2 A$$

$$\operatorname{cosec}^2 A = 1 + \cot^2 A$$

$$\sin(A \pm B) = \sin A \cos B \pm \cos A \sin B$$

$$\cos(A \pm B) = \cos A \cos B \mp \sin A \sin B$$

$$\tan(A \pm B) = \frac{\tan A \pm \tan B}{1 \mp \tan A \tan B}$$

$$\sin 2A = 2 \sin A \cos A$$

$$\cos 2A = \cos^2 A - \sin^2 A = 2\cos^2 A - 1 = 1 - 2\sin^2 A$$

$$\tan 2A = \frac{2\tan A}{1 - \tan^2 A}$$

Formulae for ΔABC

$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$

$$a^2 = b^2 + c^2 - 2bc \cos A$$

$$\Delta = \frac{1}{2}ab \sin C$$

1(a) The daily production cost, $\$y$, of making lead crystal bowls in a factory can be represented by the equation $y = \frac{1}{5}x^2 - 10x + 650$, where x is the number of bowls made.

(i) Explain the meaning of the constant term 650 in the equation. [1]

(ii) Mr Tan, the manager, wants to reduce the cost of production to \$500. Show, with clear working, how you would convince him that it is not possible. [3]

(b) Find the range of values of a for which $-ax^2 + 5ax + (a - 4)$ is always negative for all real values of x. [5]

(c) Show that the line $y = -2x - k$ intersects the curve $y = x^2 + kx + 16$ at two distinct points for all real values of k except those in the interval of $-2\sqrt{15} \leq k \leq 2\sqrt{15}$. [5]

2 A rectangular box without the lid is made of thin cardboard of negligible thickness. The sides of the base are $2x$ cm and $3x$ cm and the height of the box is h cm. Given that the total surface area is 200 cm^2, show that $h = \frac{20}{x} - \frac{3x}{5}$. Hence find the dimensions of the box for which the volume is a maximum. [9]

3(a) In the expansion of $(3 - px^2)\left(1 + \frac{x}{2}\right)^7$, the coefficient of the x^4 term is $\frac{525}{16}$. Find the value of p. [4]

(b) In the expansion $\left(\frac{x^3 - 2}{x^2}\right)^n$ in ascending powers of x, the sixth term is independent of x. Find the value of n. [4]

4(a) The points with coordinates $(27, 3)$ and $(1, p)$ lie on the graph of $y = log_c x$.

(i) Find the value of each of the constants $c\ and\ p$. [2]

(ii) Sketch the graph of $y = log_c x$. [2]

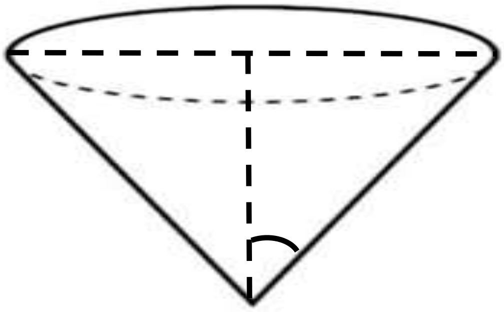
(b) Find the equation of the straight line which must be drawn on the graph of $y = 3 - e^{2x}$ to obtain a solution of the equation $x = ln\ ln\ \sqrt{2 - x}$. [3]

5(i) Using the law for the change of base of logarithm, or otherwise, show that $log_b\, a = \frac{1}{log_a\, b}$. [1]

(ii) Hence, or otherwise, solve the equation $2\, log_9\, x + 1 = 2\, log_x\, 3$. [5]

6 The diagram below shows a hollow cone of semi-vertical angle of 45° held with its axis vertical and vertex downwards. The hollow cone has radius *r* cm and height *h* cm. At the beginning of an

experiment, the cone is filled with 390 cm^3 of liquid. The liquid leaks through a small hole at the vertex at a rate of 2 cm^3 per second.



(i) Calculate, in cm^3, the volume of water that after the start of the experiment. [2]

(ii) Find the rate at which the depth of the liquid is decreasing 3 minutes after the start of the experiment. [4]

7 The number of bacteria, N, in a petri dish, is observed to increase with time, t hours. Measured values of N and t are given in the table given below.

t	2	4	6	8	10
N	61	150	402	915	2250

It is known that N and t are related by the equation $N = N_0 e^{kt}$ where N_0 and k are constants.

(i) By expressing the equation $N = N_0 e^{kt}$ in a form suitable for plotting a straight line graph, plot the graph on the grid given below. Label your axes clearly. [3]

(ii) Us [3]

(iii) Using your graph, estimate the number of hours required for the number of bacteria to increase to 1800. [2]

8 The diagram shows two trigonometric graphs labelled y_1 and y_2 in the interval $0° \leq x \leq 360°$.

(i) State the number of solutions for which $y_1 - y_2 = 0$ for $0° \leq x \leq 360°$. [1]

(ii) Find the equations of the curves y_1 and y_2. [4]

9(a) Express $\int_1^8 \left(3\sqrt{x} + \frac{2}{\sqrt{x}}\right) dx$ in the form of $p + q\sqrt{2}$ where p and q are integers. [6]

(b) A curve has the equation $y = x^3 \ln \ln x$, where $x > 0$. Find $\frac{dy}{dx}$ and hence find $\int x^2 \ln \ln x \, dx$. [5]

10(i) Show that $\frac{d}{dx}\left[(x+4)\sqrt{2x-3}\,\right] = \frac{3x+1}{\sqrt{2x-3}}$. [4]

(ii) Hence find $\int \frac{x}{\sqrt{2x-3}}\,dx$, giving your answer in the form of $\frac{1}{3}(x+k)\sqrt{2x-3}+c$, where k is the constant to be found and c is a constant of integration which cannot be found. [4]

11 The gradient function of a curve is $e^{1-x}+2x$. The curve passes through the point $(1, 1)$.

(i) Find the equation of the curve. [4]

(ii) Show that the equation of the tangent to the curve at $x = 0$ is given as $y = k(x - 1) + 1$, where k is a constant value. [4]

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